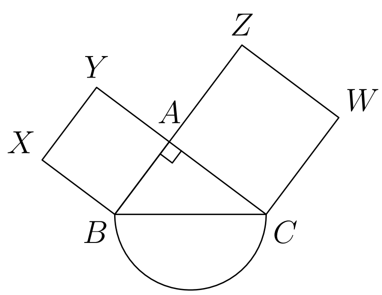
1. What is the slope of the line perpendicular to the the graph $\frac{x}{4}+\frac{y}{9}=1$ at $(0,9)$?

   Solution: Rewriting the equation of the line in slope-intercept form, we get $y=-\frac{9}{4}x+9$. Then, the slope is the coefficient of the x-term, so the answer is $\boxed{-\frac{9}{4}}$.

2. A boy is standing in the middle of a very very long staircase and he has two pogo sticks. One pogo stick allows him to jump 220 steps up the staircase. The second pogo stick allows him to jump 125 steps down the staircase. What is the smallest positive number of steps that he can reach from his original position by a series of jumps?
   Solution: The answer is the result of the Euclidean Algorithm applied to 125 and 220, or the *greatest common divisor* of the two. An empirical solution follows from jumping up the stairs 4 times and down the stairs 7 times, to get an answer of $4\cdot 220-7\cdot 125=\boxed{5}$.

3. If you roll three fair six-sided dice, what is the probability that the product of the results will be a multiple of 3?
   Solution: We find this probability by considering the probability that the product is *not* be a multiple of 3. In this case, none of the rolls can be a multiple of 3, and since two out of the six outcomes for each roll are multiples of 3, the probability that a given roll is not a multiple of 3 is $\frac{4}{6}=\frac{2}{3}$. Therefore, the probability that the product is not a multiple of 3 is $\frac{2}{3}^3=\frac{8}{27}$, so the probability that the product *is* a multiple of 3 is $1-\frac{8}{27}=\boxed{\frac{19}{27}}$

4. Right triangle ABC has squares $ABXY$ and $ACWZ$ drawn externally to its legs and a semicircle drawn externally to its hypotenuse BC. If the area of the semicircle is 18π and the area of triangle ABC is 30, what is the sum of the areas of squares $ABXY$ and $ACWZ$?



   Solution: This question obscures the real problem by adding miscellaneous information. This problem can be solved without using the area of triangle ABC by combining a few small facts. First of all, the area of square $ABXY$ is AB^2, and the area of square $ACWZ$ is CA^2. Thus, the sum of the two areas is AB^2+AC^2, but $\triangle ABC$ is a right triangle, giving $AB^2+AC^2=BC^2$. Finally, the area of the semicircle is equal to $\frac{\pi}{2}\cdot\left(\frac{BC}{2}\right)^2=18\pi$, so $BC^2=18\cdot 8=\boxed{144}$.

5. You have a bag containing 3 types of pens: red, green, and blue. 30% of the pens are red pens, and 20% are green pens. If, after you add 10 blue pens, 60% of the pens are blue pens, how many green pens did you start with?

Solution: 50% of the original set of pens were blue. Let b be the initial number of blue pens and t be the initial number of all pens. Then

$$b = 0.5t$$
$$b + 10 = 0.6(t + 10) \implies b = 0.6t - 4$$
$$0.5t = 0.6t - 4 \implies t = 40$$

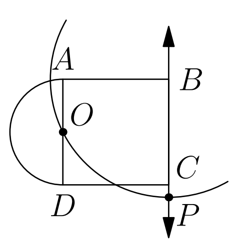
Since there were initially 20% of 40 is 8, the bag originally contained $\boxed{8}$ green pens.

6. Canada gained partial independence from the United Kingdom in 1867, beginning its long role as the headgear of the United States. It gained its full independence in 1982. What is the last digit of 1867^{1982}?
Solution: The last digit of successive powers of 7 cycles modulo 4 over remainders $(7, 9, 3, 1)$. 1982 has a remainder of 2 when divided by 4, so the final digit is the second element, $\boxed{9}$.

7. Bacon, Meat, and Tomato are dealing with paperwork. Bacon can fill out 5 forms in 3 minutes, Meat can fill out 7 forms in 5 minutes, and Tomato can staple 3 forms in 1 minute. A form must be filled out and stapled together (in either order) to complete it. How long will it take them to complete 105 forms?
Solution: In this rates problem, two separate processes occur simultaneously. Because the workers staple and fill out forms at the same time, whichever process finishes last will determine the time it takes to complete the forms. Bacon and Meat together work at a rate of $\frac{5}{3} + \frac{7}{5} = \frac{46}{15}$ forms per minute, and Tomato works at a rate of 3 forms per minute. Since Tomato works at a slower rate than Bacon and Meat, he will finish his work after them. Therefore, our answer is the time it takes him to finish: $\frac{105}{3} = \boxed{45}$ minutes.

8. *Nice* numbers are defined to be 7-digit palindromes that have no 3 identical digits (e.g., 1234321 or 5610165 but not 7427247). A *pretty* number is a *nice* number with a 7 in its decimal representation (e.g., 3781873). What is the 7^{th} *pretty* number?
Solution: The first seven are 1027201, 1037301, 1047401, 1057501, 1067601, 1072701, and $\boxed{1073701}$.

9. Let O be the center of a semicircle with diameter AD and area 2π. Given square $ABCD$ drawn externally to the semicircle, construct a new circle with center B and radius BO. If we extend BC, this new circle intersects BC at P. What is the length of CP?



Solution: Since the semicircle's diameter is a side of the square, the side of the square is $s = 2r$, where r is the radius of the circle, and by the Pythagorean theorem, $BO = \sqrt{r^2 + (2r)^2} = r\sqrt{5}$. Because B is the center of the new circle and P is on the circle, $BP = BO = r\sqrt{5}$. But BC is a side of the square, so $BC = 2r$ and $CP = BP - BC = (\sqrt{5} - 2)r$. Thus, CP has length $\boxed{2\sqrt{5} - 4}$.

10. Derek has 10 American coins in his pocket, summing to a total of 53 cents. If he randomly grabs 3 coins from his pocket, what is the probability that they're all different?
Solution: By guess and check, determine that the 10 coins must be 3 pennies, 4 nickels, and 3 dimes. In order for the three coins to be different, Derek must select precisely one penny, one nickel, and one dime. He can choose these in $3 \cdot 4 \cdot 3$ ways, and there are $\binom{10}{3}$ ways in which he may choose any collection of coins. Thus, the probability all coins are different is $\frac{36}{120} = \boxed{\frac{3}{10}}$.

11. What is the sum of the whole numbers between $6\sqrt{10}$ and 7π?
Solution: Because $18 = 6 \cdot 3 < 6\sqrt{10} = \sqrt{360} < \sqrt{361} = 19 < 20 < 21 < 7\pi = 7 \cdot 3.14159\ldots < 7 \cdot 3.142857\ldots = 7 \cdot \frac{22}{7} = 22$, the some of the whole numbers is $19 + 20 + 21 = \boxed{60}$.

12. What is the volume of a cylinder whose radius is equal to its height and whose surface area is numerically equal to its volume?
Solution: The volume of a cylinder is $\pi r^2 h = \pi r^3$, and the surface area is $2\pi r \cdot h + 2\pi r^2 = 4\pi r^2$. Since volume and surface area are numerically equivalent, $r = 4$, so the volume (and surface area) is $\boxed{64\pi}$.

13. 15 people, including Luke and Matt, attend a Berkeley Math meeting. If Luke and Matt sit next to each other, a fight will break out. If they sit around a circular table, all positions equally likely, what is the probability that a fight doesn't break out?
Solution: Consider Jake's seat. After Jake sits down, Matt can sit in any open seat aside from the two seats immediately adjacent to Jake's seat. Therefore, the probability that a fight does not break out is equal to the probability that Matt does not sit in a certain 2 seats. There are 12 remaining possible seats out of the available 14, so the probability a fight does not break out is $\frac{12}{14} = \boxed{\frac{6}{7}}$.

14. A non-degenerate square has sides of length s, and a circle has radius r. Let the area of the square be equal to that of the circle. If we have a rectangle with sides of lengths r, s, and its area has an integer value, what is the smallest possible value for s?
Solution: The area of the square is s^2, and the area of the circle is $\pi \cdot r^2$. If a rectangle with minimum width has integer value, then the value of its area will be 1. Thus, s is the solution to the simultaneous equations $\pi \cdot r^2 = s^2$ and $r \cdot s = 1$, or $s = \boxed{\sqrt[4]{\pi}}$.

15. How many ways can you arrange the letters of the word "BERKELEY" such that no two E's are next to each other?
Solution: First, choose one of the 5! permutations of (B, R, K, L, Y). Then, place each of the remaining 3 E's in one of the 6 gaps (represented by a parentheses or comma above), for a total of $5! \times^6 \mathrm{C}_3 = \boxed{2400}$ arrangements.

16. Kim, who has a tragic allergy to cake, is having a birthday party. She invites 12 people but isn't sure if 11 or 12 will show up. However, she needs to cut the cake before the party starts. What is the least number of slices (not necessarily of equal size) that she will need to cut to ensure that the cake can be equally split among *either* 11 or 12 guests with no excess?
Solution: Kim can first cut the cake into 12 pieces. Then, she can cut one of those 12 pieces into 11 pieces (10 more cuts). If 11 people show up, each receives one big piece and one small piece. If 12 people show up, 11 people receive a large piece, and the 12^{th} receives the remaining small pieces. Therefore, the answer is $12 + 10 = \boxed{22}$ cuts.

17. Tom has 2012 blue cards, 2012 red cards, and 2012 boxes. He distributes the cards in such a way such that each box has at least 1 card. Sam chooses a box randomly, then chooses a card randomly. Suppose that Tom arranges the cards such that the probability of Sam choosing a blue card is maximized. What is this maximum probability?
Solution: Generalize 2012 to N. We put all the red cards in one box, and put one blue card in each box. This will maximize the probability, since choosing the box with all red cards is lowered. We can calcuate the maximum probability by calculating the probability of picking a red card and taking the complement. Thus, our answer is $1 - \frac{1}{N} \cdot \frac{N}{N+1} = \frac{N}{N+1} = \boxed{\frac{2012}{2013}}$.

18. Allison wants to bake a pie, so she goes to the discount market with a handful of dollar bills. The discount market sells different fruit each for some integer number of cents and does not add tax to purchases. She buys 22 apples and 7 boxes of blueberries, using all of her dollar bills. When she arrives back home, she realizes she needs more fruit, though, so she grabs her checkbook and heads back to the market. This time, she buys 31 apples and 4 boxes of blueberries, for a total of 60 cents more than her last visit. Given she spent less than 100 dollars over the two trips, how much (in dollars) did she spend on her first trip to the market?
Solution: Let a represent the cost of one apple and b the cost of one blueberry. Then, $22a + 7b = 100d = 31a + 4b - 60$, where d is a positive integer. Thus, $b = 3a - 20$, so $43a \equiv 40 \mod 100$. The smallest possible value of a is 80 cents, for a total of $\boxed{33}$ dollars her first visit.

19. Consider a parallelogram $ABCD$. Let k be the line passing through A and parallel to the bisector of $\angle ABC$, and let l be the bisector of $\angle BAD$. Let k intersect line CD at E and l intersect line CD at F. If $AB = 13$ and $BC = 37$, find the length EF.
Solution: Drawing and angle chasing makes it clear $\triangle DAF$ is an isosceles triangle with $DA = DF$, and similarly for $\triangle DAE$. Thus, $EF = 2BC = \boxed{74}$.

20. Given for some real a, b, c, d,

$$P(x) = ax^4 + bx^3 + cx^2 + dx$$

$$P(-5) = P(-2) = P(2) = P(5) = 1$$

Find $P(10)$.
Solution: We know $P(x) - 1 = \alpha(x-5)(x-2)(x+2)(x+5)$ for some α. We notice the constant term is -1, so we must have $\alpha \cdot 5 \cdot 2 \cdot 5 \cdot 2 = -1$, so $\alpha = -\frac{1}{100}$. Thus, $P(x) = 1 - \frac{(x-5)(x-2)(x+5)(x+2)}{100}$, and plugging in $x = 10$ gives us $P(10) = \boxed{-71}$.