PHYSICS

2016 Australian Science Olympiads Exam Solutions

Time Allowed:

Reading Time: 10 minutes

Examination Time: 120 minutes

INSTRUCTIONS

- *Attempt ALL questions in both sections of this paper.*
- Permitted materials: a *non-programmable, non-graphical* calculator, blue and black pens, lead pencils, an eraser, and a ruler.
- Answer SECTION A on the MULTIPLE CHOICE ANSWER SHEET provided.
- Answer SECTION B in the answer booklet provided. Write your answers to each question on the pages indicated. If you need additional space use the spare pages at the back of the booklet. Write in pen and use pencil only for diagrams and graphs.
- You may attempt the questions in Section B in any order. Make sure that you label which parts are for which questions.
- **Do not write on this question paper. It will not be marked.**
- Do **not** staple the multiple choice answer sheet or the writing booklet to anything. They must be returned as they are.
- Ensure that your diagrams are clear and labelled.
- All numerical answers must have correct units.
- Marks will not be deducted for incorrect answers.

MARKS

Section A	10 multiple choice questions	10 marks
Section B	4 written answer questions	50 marks
		60 marks

SECTION A: MULTIPLE CHOICE
USE THE ANSWER SHEET PROVIDED

Throughout, take the acceleration due to gravity to be $9.8\ \mathrm{m\,s^{-2}}$.

Question 1

Why does a raindrop fall with near-constant speed during the later stages of its descent?

a. The gravitational force is constant.

b. Air resistance just balances the force of gravity.

c. The height from which the raindrop started falling is fixed in space.

d. The force of gravity is negligible for objects as small as a raindrop.

e. Gravity cannot increase the speed of a falling object to more than $9.8\ \mathrm{m\,s^{-1}}$.

Solution: b. For an object to travel with constant speed the total force acting on it must be zero. There is a gravitational force acting on the raindrop, so if it is travelling with near constant speed there must be another force acting on it in the opposite direction.

Question 2

The Moon orbits the Earth. What is the Newton's Third Law reaction force to the gravitational force of the Earth on the Moon?

a. The gravitational force of the Earth on the Moon

b. The normal force of the Earth on the Moon

c. The normal force of the Moon on the Earth

d. The gravitational force of the Moon on the Earth

e. The friction force acting on the Moon

Solution: d. Newton's Third Law means for each force there is a corresponding force of the same type, which acts on the body which exerts the original force and is caused by the body experiencing the original force. In this case the reaction force must be gravitational and act on the Earth due to the Moon.

Question 3

"Slice" is the name given to the angular deviation from the straight-through direction that a golfer produces when they hit a golf ball off the tee. David derives the following formula for his slice:

$$\phi = \frac{(1-P)}{(H-L)\sin\theta} \quad .$$

In this formula, David's slice, ϕ, depends on three positive values (P, H and L) that are all less than one, with $H > L$, and an angle θ (measured in degrees) that can range between 0° and 180°.
Which of the following options will minimise David's slice?

a. Make P as small as possible, keeping the other variables constant.

b. Make H as small as possible, keeping the other variables constant.

c. Make L as small as possible, keeping the other variables constant.

d. Make θ as small as possible, keeping the other variables constant.

e. None of the above will help David minimise his slice.

Solution: c. Reducing L increases $(H-L)$ which is in the denominator of the expression for David's slice. By increasing the denominator the value of the slice is decreased. Hence, making L as small as possible while keeping other variables constant will reduce David's slice.

Question 4

A glass-walled elevator is moving upwards with constant acceleration. At some point in the elevator's motion, a bolt breaks loose and drops from the ceiling. What is the motion of the bolt as seen by an **external** observer (i.e. one located outside the elevator)?

a. The bolt moves upwards at a constant speed.

b. The bold initially moves upwards, then slows, reverses direction and moves downwards.

c. The bolt appears to remain stationary.

d. The bolt immediately moves downwards, accelerating as it goes.

e. The bolt immediately moves downwards, at constant speed.

Solution: b. Immediately after breaking loose the bolt will be moving at the same speed as the elevator, which is upwards. Since the bolt is no longer attached to the elevator, the only force it experiences is the gravitational force downwards. This means that the bolt moves upwards initially, slows and then reverses direction.

Question 5

The Earth and the Moon both exert gravitational forces on objects in their vicinity. Imagine a line joining the Earth to the Moon, and extending to either side, as shown below (not to scale). Consider placing an object along this line. Where along this line is the net gravitational force on the object due to the Earth and the Moon equal to zero?

a. On the far side of the Earth from the Moon.

b. Between the Earth and the Moon, but closer to the Earth than to the Moon.

c. Halfway between the Earth and the Moon.

d. Between the Earth and the Moon, but closer to the Moon than to the Earth.

e. Nowhere along the line.

Solution: d. The gravitational force exerted by a spherical object like the Earth or the Moon acts towards the centre of that object. For the two gravitational forces to cancel each other they must be equal in magnitude and opposite in direction. The forces are only opposite in direction along the segment of the line between the two bodies. The gravitational force towards a more massive object is greater, and the gravitational force is larger closer to an object. For the forces to have equal magnitudes an object must be closer to the less massive Moon than to the Earth.

Question 6

The electrostatic force F_e that an object of charge Q exerts on another object of charge q is given by Coulomb's law:

$$F_e = \frac{1}{4\pi\varepsilon_0}\frac{Qq}{r^2} \quad ,$$

where the charges of the two objects, Q and q, are measured in units of coulombs (C), and r is the distance separating the two objects.
What are the units of the constant ε_0?

a. $\mathrm{kg.m^3.s^{-2}.C^2}$

b. $\mathrm{kg.m^{-3}.s^2.C^{-2}}$

c. $\mathrm{kg.m.s^2.C^{-2}}$

d. $\mathrm{s^2.C^2.kg^{-1}.m^{-3}}$

e. $\mathrm{s^2.C^2.kg^{-1}.m^{-1}}$

Solution: d. Coulomb's Law can be rearranged to give

$$\varepsilon_0 = \frac{1}{4\pi}\frac{Qq}{r^2 F_e} \quad .$$

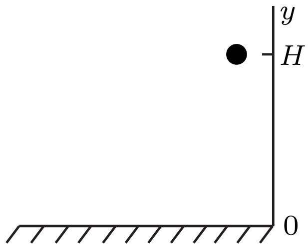
Since the units of both sides of the equation must be equal, the S.I. unit of charge is C, the S.I. unit of distance is m and the S.I. unit of force can be written $\mathrm{kg.m.s^{-2}}$ the unit of ε_0 is

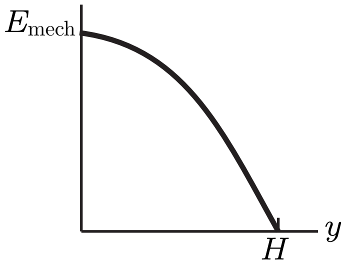
$$\frac{\mathrm{C^2}}{\mathrm{m^2.kg.m.s^{-2}}} \quad ,$$

and index laws can be used to simplify this to give the expression in response d. above.

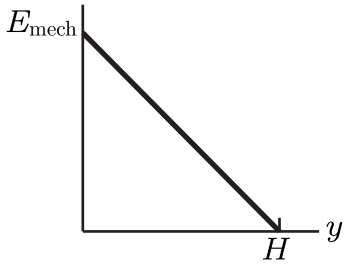
Question 7

A ball is held at a height H above a floor, as sketched in the diagram on the right. It is then released and falls to the floor. If air resistance can be ignored, which of the five graphs below (labelled a. to e. beneath each graph) correctly gives the mechanical energy E_{mech} of the Earth-ball system as a function of the altitude y of the ball?

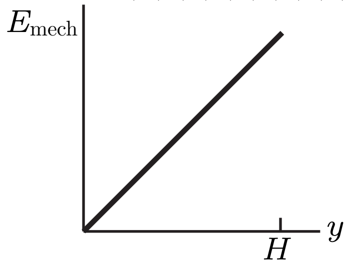




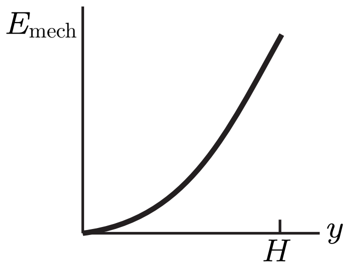
a.



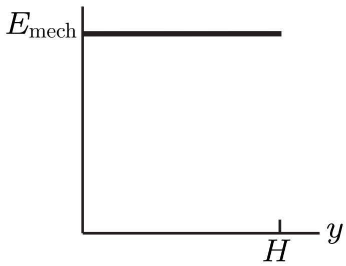
b.



c.



d.



e.

Solution: e. As the questions asks about the energy of the *Earth-ball* system, we must include both kinetic energy and gravitational potential energy as part of the mechanical energy. As the ball falls gravitational potential energy is converted to kinetic energy but the total energy of the Earth-ball system remains constant as energy is conserved.

Question 8

Some students are doing an experiment to calculate the spring constant of a particular spring. As part of this experiment, they suspend an object from the spring, pull it downwards, and let it go. The object then oscillates up and down, with a period of about 2 seconds. The students use a stopwatch to measure the period of oscillation. The best way to get an accurate and precise measurement of the period would be to:

a. measure the time taken for a single oscillation 10 times, and average the results.

b. measure the time taken for 10 oscillations, and divide by 10.

c. measure the time taken for a single oscillation 10 times, and average the results. Repeat this measurement three times, and average the results.

d. measure the time taken for 10 oscillations, and divide by 10. Repeat this measurement three times, and average the results.

e. All of the methods above are equivalent, and will result in the same precision in the measurement of the period of oscillation.

Solution: d. By measuring for 10 oscillations and dividing by 10 the size of the error introduced when starting and stopping timing is much smaller compared to the period of around 2 s than if the measurement is made for only one period. Even if an average of ten measurements of a single period is taken the error due to starting and stopping the timing 10 times will be greater than for one measurement of 10 periods. By then repeating the measurement and averaging the precision in the measurement of the period can be increased further.

Question 9

A roller-coaster car is full of water, and is moving at constant velocity along a frictionless horizontal track. Someone removes a plug in the bottom of the floor, allowing the water to drain out. Ignoring air resistance, as the water drains out the car will:

a. slow down.

b. continue to move at constant velocity.

c. speed up.

d. initially slow down, then speed up.

e. initially speed up, then slow down.

Solution: b. As the water flows out the bottom of the cart it will have some momentum in the horizontal direction. If the cart remains at the same speed the momentum of the cart plus contents will reduce due to the mass of the cart plus contents decreasing. The amount of this decrease is equal to the horizontal momentum carried by the water which has left the cart. This means that momentum is conserved without any changes to the velocity of the cart.

Question 10

The velocity of a particle moving along the x-axis changes from v_i to v_f. For which of these values of v_i and v_f does the particle's kinetic energy increase the most?

a. $v_i = 5\ \mathrm{m\,s^{-1}}$, $v_f = 2\ \mathrm{m\,s^{-1}}$

b. $v_i = 5\ \mathrm{m\,s^{-1}}$, $v_f = -2\ \mathrm{m\,s^{-1}}$

c. $v_i = -5\ \mathrm{m\,s^{-1}}$, $v_f = -2\ \mathrm{m\,s^{-1}}$

d. $v_i = -5\ \mathrm{m\,s^{-1}}$, $v_f = 2\ \mathrm{m\,s^{-1}}$

e. $v_i = 2\ \mathrm{m\,s^{-1}}$, $v_f = -5\ \mathrm{m\,s^{-1}}$

Solution: e. The kinetic energy is given by $K = mv^2/2$. Since the velocity is squared the kinetic energy does not depend on the sign, or direction, of the velocity. This means that the only option where the kinetic energy increases is when the initial velocity is $v_i = 2\ \mathrm{m\,s^{-1}}$ and the final velocity is $v_f = -5\ \mathrm{m\,s^{-1}}$.

SECTION B: WRITTEN ANSWER QUESTIONS
USE THE ANSWER BOOKLET PROVIDED

Note: Suggested times are given for Section B as a general guide only. You may take more or less time on any question – everyone is different.

Question 11

Suggested Time: 25 min

The following parts of this question are unrelated and so may be completed in any order. Note that the data provided on the next page may be used for any part, as required. You must explain your reasoning for each part.

a) Calculate the amount of energy per gram of fat.

Solution: From the supplied data: 30% of 8700 kJ is supplied by 70 g. Hence, fat has an energy content of $30\% \times 8700/70\ \text{kJ/g} = 37\ \text{kJ/g}$.

Markers' comments: This part of this question was well done in general. Some students were not consistent with their units.

b) The typical office computer is only used one third of the time during business hours. Today a typical office computer consumes around 130 W of power.
Estimate the cost savings and reduction in greenhouse gas emissions over one year if everyone in an office of 60 people turns off their computers when they are not using them instead of leaving them on at all times, even over the weekend.

Solution: The number of hours in a year that computers are used is approximately

$$\begin{aligned} t_{\text{in use}} &= \frac{1}{3} \times 40\,\frac{\text{hours}}{\text{week}} \times (52-4)\,\frac{\text{weeks}}{\text{year}} \\ &= 640\,\text{hours} \quad . \end{aligned}$$

The energy saved by turning 60 computers off when not in use is

$$\begin{aligned} E_s &= 130\ \text{W} \times 60 \times \left(365\,\frac{\text{days}}{\text{year}} \times 24\,\frac{\text{hours}}{\text{day}} - 640\,\frac{\text{hours}}{\text{year}}\right) \\ &= 63\,\frac{\text{MWh}}{\text{year}} \quad . \end{aligned}$$

Over one year, the cost saved is $63000\,\text{kWh}\,\text{year}^{-1} \times 1\,\text{year} \times \$0.15\,\text{kWh}^{-1} = \$9500$
and the reduction in greenhouse gas emissions is
$63000\,\text{kWh}\,\text{year}^{-1} \times 1\,\text{year} \times 0.87\,\text{kg}\,(\text{CO}_2\ \text{equiv.})\,\text{kWh}^{-1} = 55000\,\text{kg}\,(\text{CO}_2\ \text{equiv.})$.

Markers' comments: The solution provided above is an estimate and a range of answers was accepted. Students were required to account for the number of hours worked in a day, weekends and holidays, as well as only being in use one third of the time. The number of school hours per day and days per year were considered too low to be a reasonable estimate of business hours. As long as the assumptions were reasonable and ensuing calculations were correct full marks were given. Once again, units caused many students some difficulty.

c) Estimate the energy efficiency of the conversion of sunlight into sugarcane.

Solution: The mass of sugar required to give 100% of the recommended daily energy intake is around $0.23\,\mathrm{kg}/0.45 = 0.51\,\mathrm{kg}$ or $0.31\,\mathrm{kg}/0.65 = 0.48\,\mathrm{kg}$. Hence, the energy obtained from growing $1\,\mathrm{m}^2$ of sugar cane is

$$\begin{aligned} E_{\mathrm{sc}} &= 9.2\frac{\mathrm{kg}}{\mathrm{m}^2} \times \left(\frac{8.7\,\mathrm{MJ}}{0.5\,\mathrm{kg}} \times 0.15 + 19.2\frac{\mathrm{MJ}}{\mathrm{kg}} \times 0.30\right) \\ &= 77\,\mathrm{MJ}/\mathrm{m}^2 \quad . \end{aligned}$$

Assuming that it takes one year to grow a sugarcane crop, the energy input from sunlight over $1\,\mathrm{m}^2$ of ground is

$$\begin{aligned} E_{\mathrm{Sun}} &= 1366\,\mathrm{W}/\mathrm{m}^2 \times 3600\,\mathrm{s}/\mathrm{hour} \times 8\,\mathrm{hours}/\mathrm{day} \times 365\,\mathrm{days} \\ &= 1.4 \times 10^{10}\,\mathrm{J}/\mathrm{m}^2 \quad . \end{aligned}$$

Note that the assumption has been made that each day the ground receives the equivalent of 8 hours of direct sunlight as the sun is up for an average of 12 hours per day, but is not always directly overhead, so the intensity is lower than the stated $1366\ \mathrm{W}/\mathrm{m}^2$.

The efficiency of conversion of sunlight into sugar cane is

$$\eta = 77 \times 10^6\,\mathrm{J}/\mathrm{m}^2/1.4 \times 10^{10}\,\mathrm{J}/\mathrm{m}^2 = 0.5\% \quad .$$

Markers' comments: Once again, students were required to estimate some values to answer this question and a range of answers was accepted. In particular, students had to estimate the number of hours of direct sunlight incident on the ground and also the energy content of sugar by using the supplied dietary data.

Some common errors were to assume that sunlight is incident on the ground 24 hours per day and to include the energy of only one of sugar or bagasse in the calculation of the energy content of bagasse. Other common errors related to units and also the use of percentages.

d) "Kilogram per kilogram humans produce more heat than the Sun." Is this claim true?

Solution: The power of the heat flow out of the Sun is $1366\ \mathrm{W}/\mathrm{m}^2 \times 4\pi \times (1.5 \times 10^{11}\,\mathrm{m})^2 = 3.86 \times 10^{26}\,\mathrm{W}$, since all the energy emitted from the Sun is radiated outwards in a sphere and we are given the intensity of sunlight at the distance of the Earth from the Sun. This means that the power per kg of the Sun is $3.86 \times 10^{26}\,\mathrm{W}/2.0 \times 10^{30}\,\mathrm{kg} = 1.93 \times 10^{-4}\,\mathrm{W}/\mathrm{kg}$.

The majority of energy that adults consume is dissipated as heat, so the power of the heat flow from a typical person is $8700 \times 10^3\,\mathrm{J}/\mathrm{day}/(24\,\mathrm{hours}/\mathrm{day} \times 3600\,\mathrm{s}/\mathrm{hour}) = 100\,\mathrm{W}$. Thus, the power per kg of a typical human is around $100\,\mathrm{W}/65\,\mathrm{kg} = 1.5\,\mathrm{W}/\mathrm{kg}$.

These results mean that the claim is correct and humans produce around 10000 times as much heat per kilogram than the Sun.

Markers' comments: Fewer students attempted this part of the question. Students who attempted this part mostly made reasonable estimates of the heat produced per kg by a person. However, many students had difficulty calculating the total heat flow out of the Sun.

Typical Australian Adult:

Typical adult mass	65 kg
Typical adult height	1.7 m

Dietary Guidelines:

Recommended energy intake	8700 kJ/day
Recommended daily intake of fat	30% of energy (70 g/day)
Recommended daily intake of carbohydrates (sugars, flour, etc.)	45–65% of energy (230–310 g/day)

Electricity Data:

Typical price	15 c/kWh
Typical greenhouse emissions	0.87 kg(CO_2 equiv.)/kWh

Note: 1 kWh = 1 kilowatt hour

Australian Sugar Cane Data:

Average yield of sugar cane	$9.2\ \mathrm{kg/m^2}$
Typical sugar content	15%
Typical bagasse content	30%
Typical water content	55%
Typical energy content of bagasse	19.2 MJ/kg

Note: bagasse is the by-product of crushing sugar cane to extract sugar and is often used as a fuel.

Solar System Data

Earth-Sun distance	1.5×10^{11} m
Radius of Sun	7.0×10^{5} km
Mass of Sun	2.0×10^{30} kg
Radius of Earth	6.4×10^{3} km
Mass of Earth	6.0×10^{24} kg
Typical intensity of sunlight at Earth	$1366\ \mathrm{W/m^2}$

Question 12

Suggested Time: 25 min

Angus the accountant decides to change careers and starts a bicycle building business. To make his bicycles stand out he builds them with square wheels, making sure that both wheels can sit with a side flat on the ground at the same time.

Angus rides his first bicycle proudly out of his shed turning his pedals at a constant rate.

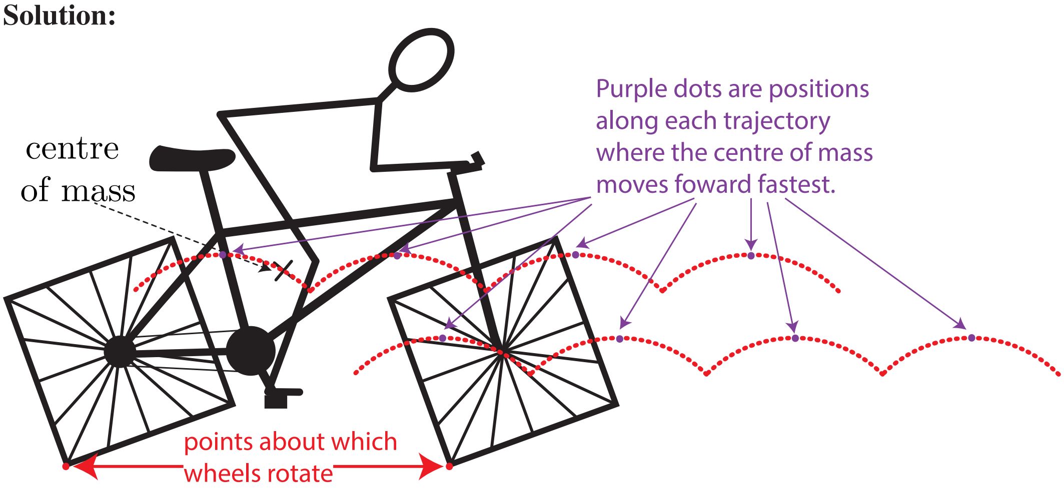
a) (i) Sketch the trajectory of the centre of the front wheel in the space on p. 4 of the answer booklet.

(ii) On the same diagram sketch the trajectory of the centre of mass of the bicycle. In the space below the diagram explain how the trajectory of the centre of mass is related to the trajectory of the centre of the front wheel and why this is so.

(iii) For each wheel label the point about which it is rotating.

(iv) Label the points on the trajectory where the centre of mass of the bicycle is moving forward fastest. Explain your reasoning in the space provided below the diagram.

Solution:



(i) See above.

(ii) See above. The centre of mass of the bike is a fixed distance from the centre of the front wheel. This means that the trajectory is the same shape but shifted by a fixed distance up and to the left.

(iii) See above.

(iv) The bike is moving forward fastest when it is at its highest point. Since the pedals are turned at a constant rate, the wheels also turn at a constant rate. This means that the speed of the bike is the same at all times, but the direction of the velocity follows the trajectory and this is most forwards at the highest points.

Markers' Comments:

Most students correctly identified that the bike moves up and down and could relate the trajectories of the centre of the front wheel and the centre of mass. Some common mistakes were having a trajectory which changed direction smoothly when the went from rotating about one corner to rotating about the next corner. Identifying that the wheels rotated around the corner in contact with the ground and the points where the bike was moving forward fastest were more difficult.

b) Calculate the ratio of the length of the path followed by the centre of mass to the distance travelled along the road.

Solution: The bike wheels are rotating about the corners and the centre of the wheel is fixed distance from the corner so it follows the arc of a circle. If the length of the side of the wheel is s, then the centre is a distance

$$\begin{aligned} d &= \sqrt{\left(\left(\frac{s}{2}\right)^2 + \left(\frac{s}{2}\right)^2\right)} \\ &= \frac{s}{\sqrt{2}} \ , \end{aligned}$$

from the corner. In one quarter turn the length of the path followed by the centre of the wheel is $l = \pi s/(2\sqrt{2})$ and the wheel has moved horizontally by s. Since the trajectories of the centre of the wheel and the centre of mass are the same but offset, the ratio of the length of the path to the distance along the road for the centre of mass is

$$\frac{\pi s/(2\sqrt{2})}{s} = \frac{\pi}{2\sqrt{2}} \quad .$$

Markers' Comments:
Students had difficulty identifying the trajectory as being made of arcs of circles, and also with calculating the length of a circular arc.

Angus generously gives Sam one of his square wheeled bicycles for her birthday. Sam is very lazy and teaches herself to ride the bicycle so that she is only just pedalling hard enough to replace energy lost from the bike. The bike then has constant energy.

c) Describe the differences and similarities between the motion of the bicycle when Sam rides compared to when Angus rides.

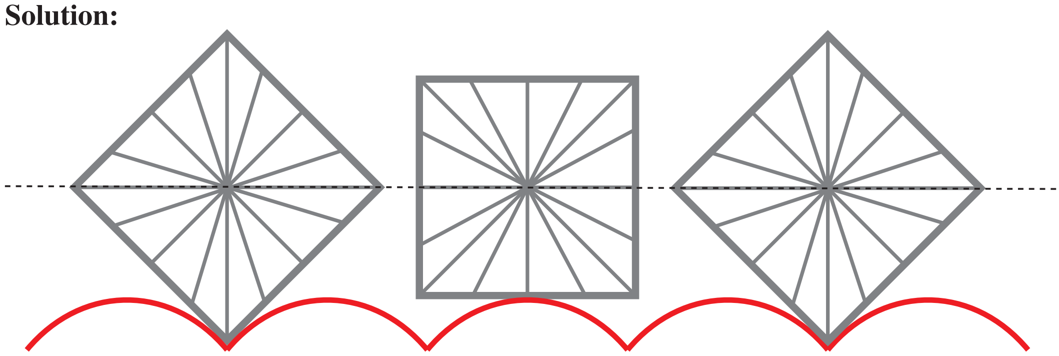
Solution:
Regardless of how the pedals are turned the centre of the wheel and the centre of mass will both still follow the same path. However, when Sam pedals she keeps the energy of the bike the same and this will include gravitational potential energy as well as kinetic energy. At the tops of the trajectory Sam's speed will be lowest as she has greatest gravitational potential energy and the kinetic energy must be lower to compensate. Sam's speed will be greatest just before and after the lowest points of the trajectory when the sides of the wheels are flat on the ground.

Markers' Comments:
Many students did not attempt this part of this question. Those who did often did not communicate their reasoning clearly.

After watching Sam ride the bicycle Angus realises the error of his ways, and decides to construct a racetrack specifically for his square bicycles.

d) Draw a side view of the racetrack's surface shape to ensure the smoothest ride on one of the square wheeled bicycles.

Solution:



Markers' Comments:

The important features of the sketch were the cusps which make a right angle at the lowest points of the track, and that the distance between the cusps is less than s, so that the length along the surface of the tracks can be equal to s. The height difference between the cusps and maxima must be $(\sqrt{2}-1)s/2$ in order that the centre of mass of the bike remain at a constant height.

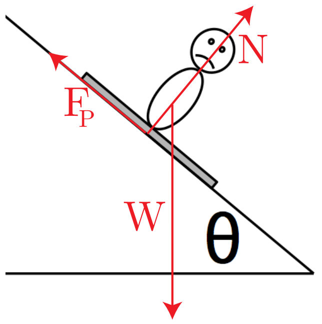
Note: only the red line in the diagram is a required part of the solution, but the grey wheels and dashed black line are drawn in to show how the shape of the path allows for a smooth ride with centre of the wheel at a constant height.

Question 13

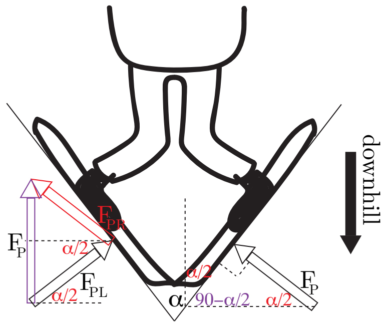
Suggested Time: 25 min

Having missed the boat for Rio 2016, Emily and Joshua are determined to qualify instead for the Winter Olympics in 2018, to be held in PyeongChang, South Korea.

Emily, being a novice skier, is sliding down a slope and desperately trying to slow down using a 'snowplough' stop. The ski slope makes an angle θ with the horizontal. By digging the inside edges of her skis into the snow while keeping them in a V-shape, Emily can stop completely. The forces due to this ploughing motion act on each ski perpendicular to the ski and along the slope. Each of these forces has magnitude F_P. Emily has a mass m.



Emily skiing down a slope.



Snowplough stop

a) (i) Which direction will the total force due to the ploughing motion be in?

Solution:

The total force will be uphill as it is the sum of the ploughing force on each ski. This is shown in the snowplough stop diagram above, where the force on the right ski is added to that on the left ski to give a resultant force uphill.

Markers' Comments:

Many students answered the question vaguely with phrases such as "behind Emily" which were not clear enough to be given credit.

(ii) Explain why the magnitude of the total force due to the ploughing motion is $2F_P \sin\frac{\alpha}{2}$ when the angle between Emily's skis is α.

Solution:

The angle between uphill and each ski is $\alpha/2$, so the angle between the each ploughing force and the horizontal is also $\alpha/2$. This is also shown in red and purple on the right hand diagram above. The ploughing force on the left ski has a component to the right and the force on the right ski has a component to the left, and, since the forces are symmetric these are equal so the total force in the horizontal direction is zero.

The ratio of the uphill component of the ploughing force due to each ski to the magnitude of the ploughing force on each ski is

$$\frac{F_{P\ \mathrm{uphill}}}{F_P} = \sin\frac{\alpha}{2} \quad .$$

Rearranging the equation gives $F_{P\ \mathrm{uphill}} = F_P \sin\frac{\alpha}{2}$ as the contribution from each ski, so the total force due to the ploughing motion is $2F_P \sin\frac{\alpha}{2}$ uphill.

Markers' Comments:

Some students explained why the angle was $\alpha/2$ and why there was a factor of 2 at the start of the expression but did not do any calculation, which was required to explain why there is a sine function in the expression. Some students resolved the skis into components rather than forces.

b) Draw all the forces as vectors acting on Emily as she ploughs downhill on the diagram on p. 6 of the answer booklet. Include the combined effect of the ploughing forces as a single vector.

Solution:

See the left hand diagram above. The forces are marked with red arrows.

Markers' Comments:

Many students included extra information which was not required in this diagram. When marking particular attention was paid to the direction of the forces: the ploughing force should be uphill, the normal force should be perpendicular to the hill and the gravitational force should be downwards, not downhill.

c) At what angle α between the skis will Emily continue down the hill at a constant speed?

Solution:

For Emily to travel at a constant speed the total force acting on her needs to be zero. Considering the total force perpendicular to the slope will always be zero so we only need consider the forces along the slope. The component of the weight force along the slope is $mg\sin\theta$ downhill. Combining this with the expression for the total force due to the ploughing motion gives

$$0 = 2F_P \sin\frac{\alpha}{2} - mg\sin\theta \quad .$$

Rearranging this equation gives

$$\sin\frac{\alpha}{2} = \frac{mg\sin\theta}{2F_P} \quad .$$

Inverting the sine function then gives the angle between the skis required for Emily to move at a constant speed

$$\alpha = 2\arcsin\left(\frac{mg\sin\theta}{2F_P}\right) \quad .$$

Markers' Comments:

Not many students made much progress with this part of the question.

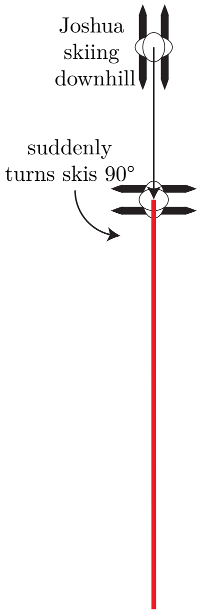
Joshua thinks he might have a go at the slalom event so has been practising sharp turns.

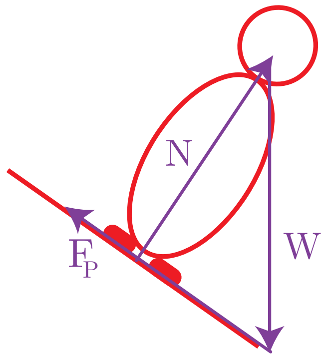
d) Joshua is skiing down the slope with his skis pointing downhill and suddenly turns his skis so that they are pointing across the slope.

(i) Sketch a diagram showing Joshua's path once he has turned his skis on the diagram on p. 7 of the answer booklet.

(ii) Briefly explain your reasoning. Note: another diagram might help your explanation.

Solution:





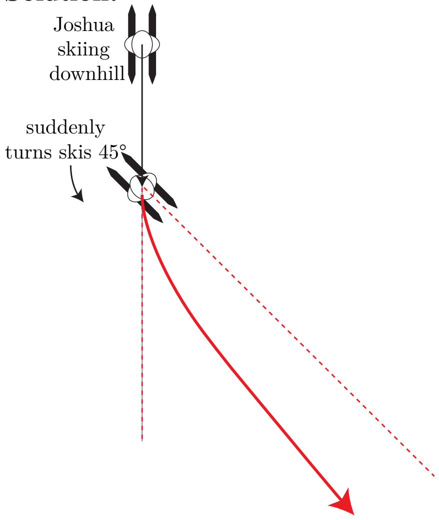
As Josh's skis are both across the slope the forces on the skis are perpendicular to this and uphill. This means that all the forces on Josh are acting directly uphill or downhill and Josh will not turn. He will continue in a straight line down the hill but slow down and reach a stop. A diagram of the forces acting on Josh is included to the right above.

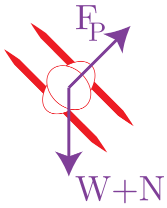
Markers' Comments:

Many students felt that Josh would turn and credit was not given for paths which had changes of direction. To get full credit students had to include explanations based on the forces acting on Josh and their directions.

e) Joshua decides to practice a different turn. He skis down the same hill and then suddenly turns his skis by 45° and then keeps them pointing in the same direction. Draw a diagram of his path on p. 7 of the answer booklet. Please point out any important features of the path.

Solution:





The important features of the path are that it is initially still directly downhill but curves so that Josh is moving to the right as viewed from above. Josh's path will become linear again, but at an angle of less than 45° to downhill. A diagram of the directions of forces along the slope which act on Josh is shown to the right above. The total force will have components down the hill and across the hill.

In the instant after Josh turns he will still be moving downhill but starts accelerating somewhat to the right as viewed from above. This means that the path curves. As the path curves the forces on the skis will decrease as Josh is moving in a direction closer to the direction that the skis are pointing. However, there is always a component of the gravitational force which is downhill, and his skis remain in the same direction. This means that there is an angle between directly downhill and the direction in which his skis point where the total force is in the direction of motion. Josh's direction will asymptotically approach this direction and his path will become linear again.

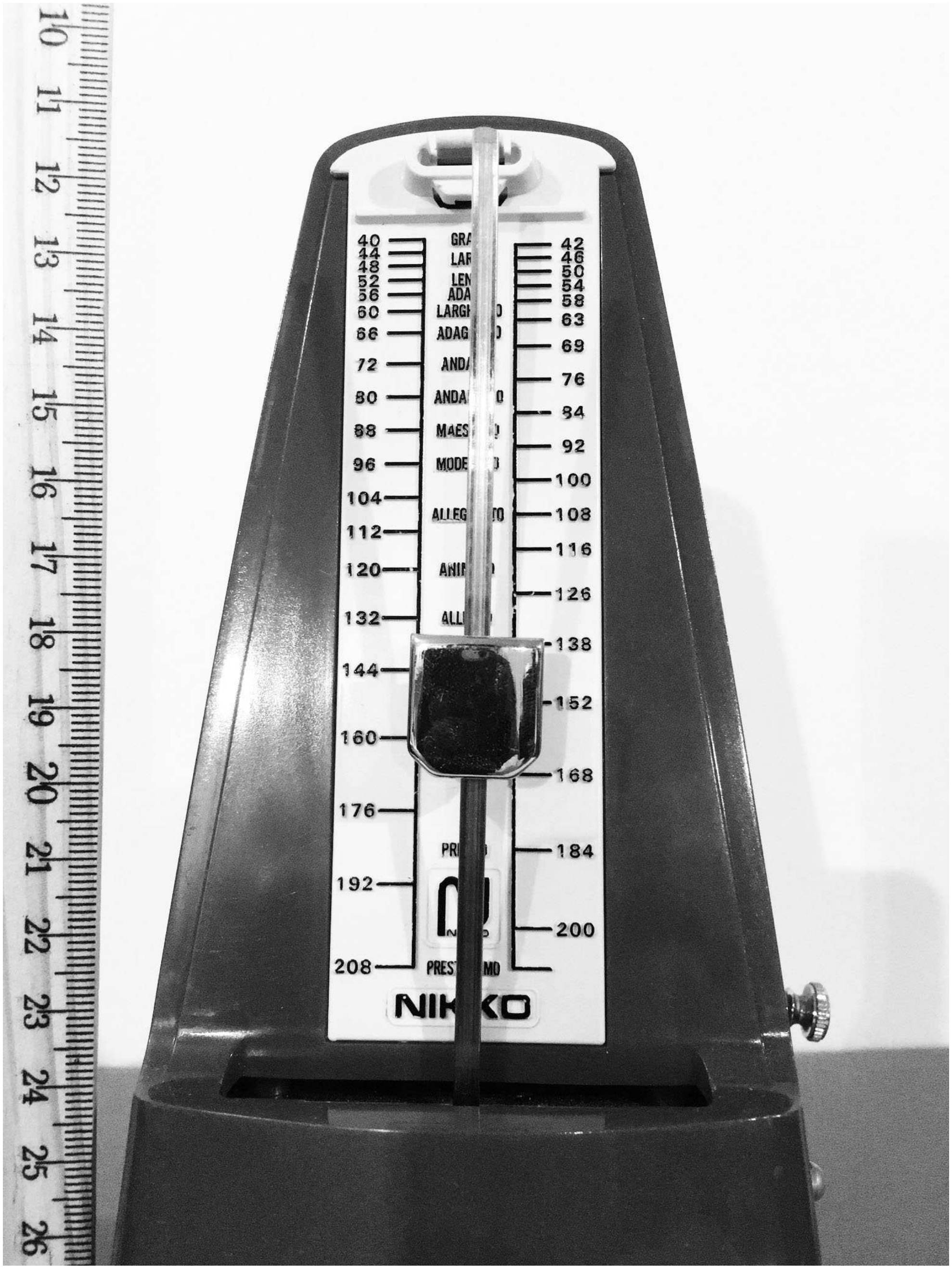
Markers' Comments:

Students needed to consider the directions of forces and how these resolve into components across and along the hill.

Question 14

Suggested Time: 25 min

Metronomes are tools sometimes used by musicians to help make their tempo more consistent when playing a piece of music. The tempo is the number of beats per minute. The metronome pictured below is set to 138 beats per minute. When the vertical arm swings back and forth it produces clicks at either end of the swing. The tempo is changed by sliding the weight up or down the metal arm until the top of the weight aligns with the marking for the desired tempo.



a) Explain how you could use the image above to determine y, which is the distance from the top of the vertical arm to the top of the sliding weight, for any given tempo setting of the metronome.

Solution:

With the metronome set to 138 beats per minute the top of the weight is aligned with the 138 marking on the scale. This means that y is also the distance from the tempo marking to the top of the vertical arm.

Measuring the ruler in the printed photo from 10.0cm to 26.0cm is a distance of 16.6cm. This means that we can measure distances with a ruler an multiply by a scaling factor of 16.0/16.6 to find y.

Markers' Comments:

There are acceptable alternative solutions including drawing horizontal lines for each marking. Regardless of the method used students needed to explain clearly how measurements were made. Any method which included constructing horizontal lines was required to include an explanation of how it was ensured that the lines were actually horizontal. Methods which involved some calculation of y based on some model without first measuring for a range of values were not accepted.

b) Use this method to complete the table of tempos and distances y on p. 8 of the answer booklet.

Solution:

Tempo (beats per minute)	Measured length (mm)	y (mm)
40	15	14
60	25	24
80	37	36
126	64	62
200	110	106

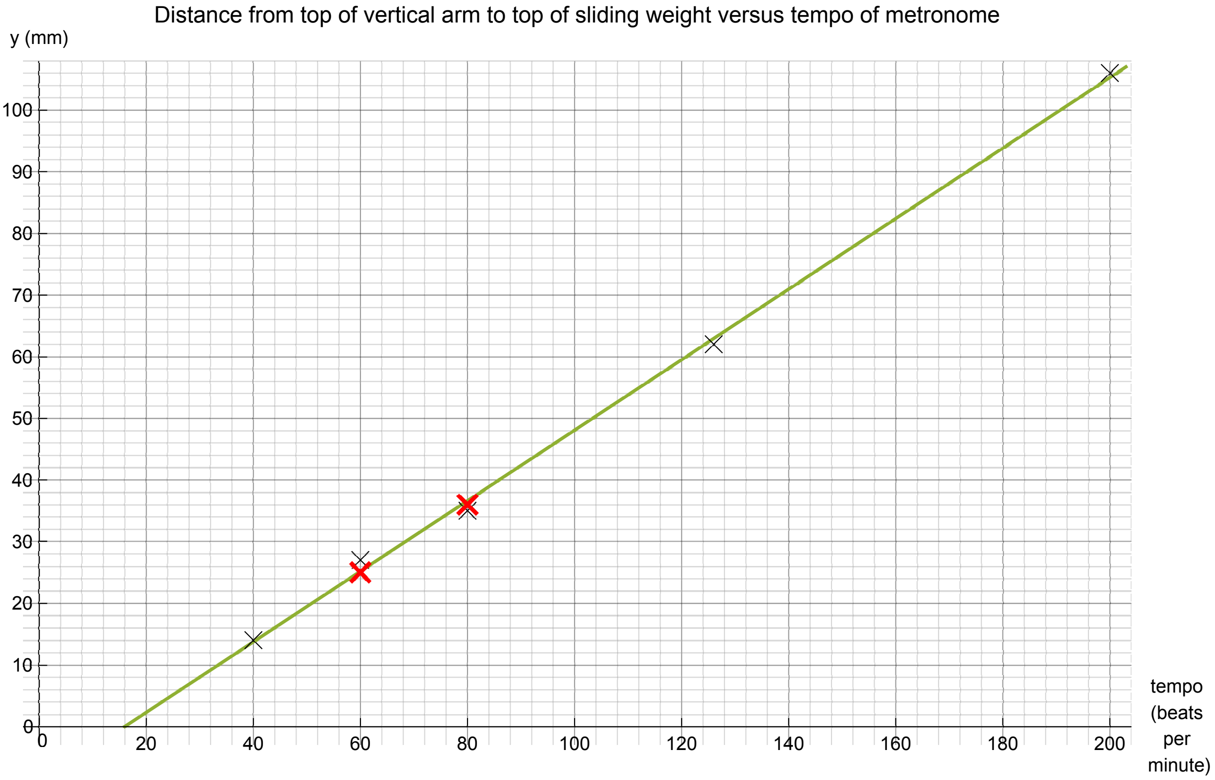
Markers' Comments:

The blank column was provided to record direct measurements before any calculation was completed. The vast majority of methods involved making a measurement such as the position of a tempo marking on the ruler, before calculating y, such as by subtracting the position of the top of the vertical arm. Many students made measurements and performed calculations without recording the intermediate step. This was not given full credit as it is not good practice.

c) A graph of y versus tempo has been plotted on p. 9 of the answer booklet. However, one or two of the points are not plotted correctly.

(i) Complete the graph by adding any necessary labels and markings.
(ii) Correct any points which have not been correctly plotted.
(iii) Draw a line of best fit to the data.

Solution:



(i) Scales, axis labels including units and a title have been added in the graph above.
(ii) Two points needed correction. The correct points have been marked as thick red crosses.
(iii) A line of best fit is drawn in green on the graph above.

Markers' Comments:
Many students had difficulty reading the scales, especially where points did not fall exactly on grid lines. Some students forced their line of best fit to pass through the origin. There is no reason to expect it to pass through that point, so it should not be forced to do so.

d) Calculate the slope of the graph and explain what information this gives about the metronome.

Solution:
The slope of the graph is

$$\begin{aligned} m &= \frac{\text{rise}}{\text{run}} \\ &= \frac{94\text{ mm} - 16\text{ mm}}{180\text{ beats/minute} - 44\text{ beats/minute}} \\ &= 0.57\text{ mm minutes/beat} \quad . \end{aligned}$$

This tells us that the slider must be moved downwards by 0.57mm to increase the tempo by 1 beat per minute.

Markers' Comments:

When calculating the slope it is important to pick two points from the line of best fit, not two data points, otherwise there is not value in drawing the line of best fit. When calculating the slope it is important to also calculate the units for the slope, which are the units of the rise divided by the units of the run. Values which differed a little from that given above, but with calculations which were correct and made it possible to follow the reasoning were given full credit. Many students did not provide an interpretation of the meaning of the slope.

e) What is the lowest tempo to which the metronome could be set? Explain your reasoning.

Solution:

The x-intercept of the graph is the point where the weight has reached the top of the vertical arm. From the graph above the tempo of the x-intercept is 16 beats per minute.

Markers' Comments:

Some students made this part overly complicated by trying to use a different point and a value for the slope to calculate the x-intercept. The simplest method was to extrapolate the line of best fit to find the intercept.

f) The graphed data points do not lie exactly on the line of best fit. Suggest the most likely reason(s) why this may be so and give a detailed but concise explanation of the reasons for your suggestions in the allocated space on p. 9 of the answer booklet.

Solution:

There are two main reasons why points may not lie on a line of best fit. The first is that there is a non-linear relationship, which is the most likely reason in this case. The end points are both above the line of best fit with the three middle points below. Of those, the central point is the furthest below the line of best fit. This suggests that random errors are low and that the relationship is actually slightly non-linear.

The other acceptable reason is due to errors which may have related to how the camera, ruler and metronome were aligned in the photo, or perhaps random errors in the measurements of the distances.

Markers' Comments:

Many students had difficulty with the technical language to describe these possible causes. When discussing the non-linearity of the data many students used the word "exponential" instead of the term "non-linear" when describing a relationship which is not a straight line. Whilst an exponential relationship is non-linear, it is not the only type of non-linear relationship.

When discussing possible sources of experimental error many students attributed it to the metronome being old and broken, however, the markings are unlikely to move and the length of the arm is not likely to change as the metronome ages. Good possible sources of error needed to relate to how the photo was taken, e.g. the ruler and/or vertical arm being tilted, or not in the same plane or the method of taking measurements from the photo used by the student.

Integrity of Competition

If there is evidence of collusion or other academic dishonesty, students will be disqualified. Markers' decisions are final.