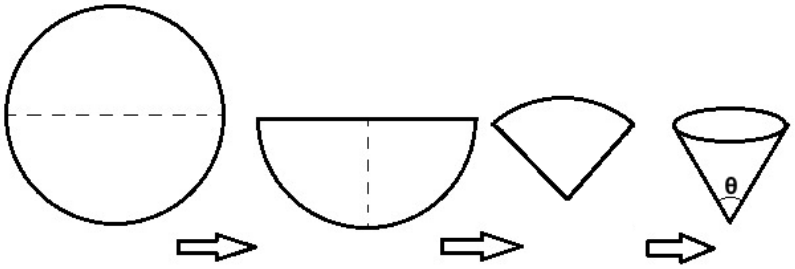
PUMaC 2012

Geometry B

1. [**3**] During chemistry labs, we oftentimes fold a disk-shaped filter paper twice, and then open up a flap of the quartercircle to form a cone shape, as in the diagram. What is the angle θ, in degrees, of the bottom of the cone when we look at it from the side?



2. [**3**] A 6-inch-wide rectangle is rotated 90 degrees about one of its corners, sweeping out an area of 45π square inches, excluding the area enclosed by the rectangle in its starting position. Find the rectangle's length in inches.

3. [**4**] Let A be a regular 12-sided polygon. A new 12-gon B is constructed by connecting the midpoints of the sides of A. The ratio of the area of B to the area of A can be written in simplest form as $(a+\sqrt{b})/c$, where a, b, c are integers. Find $a+b+c$.

4. [**4**] Three circles, ω_1, ω_2, and ω_3, are externally tangent to each other, with radii of $1, 1$, and 2 respectively. Quadrilateral $ABCD$ contains and is tangent to all three circles. Find the minimum possible area of $ABCD$. Your answer will be of the form $a+b\sqrt{c}$ where c is not divisible by any perfect square. Find $a+b+c$.

5. [**5**] Two circles centered at O and P have radii of length 5 and 6 respectively. Circle O passes through point P. Let the intersection points of circles O and P be M and N. The area of triangle $\triangle MNP$ can be written in the form a/b, where a and b are relatively prime positive integers. Find $a+b$.

6. [**6**] A square is inscribed in an ellipse such that two sides of the square respectively pass through the two foci of the ellipse. The square has a side length of 4. The square of the length of the minor axis of the ellipse can be written in the form $a+b\sqrt{c}$ where a, b, and c are integers, and c is not divisible by the square of any prime. Find the sum $a+b+c$.

7. [**7**] Assume the earth is a perfect sphere with a circumference of 60 units. A great circle is a circle on a sphere whose center is also the center of the sphere. There are three train tracks on three great circles of the earth. One is along the equator and the other two pass through the poles, intersecting at a 90 degree angle. If each track has a train of length L traveling at the same speed, what is the maximum value of L such that the trains can travel without crashing?

8. [**8**] A cyclic quadrilateral $ABCD$ has side lengths $AB=3$, $BC=AD=5$, and $CD=8$. The radius of its circumcircle can be written in the form $a\sqrt{b}/c$, where a, b, c are positive integers, a, c are relatively prime, and b is not divisible by the square of any prime. Find $a+b+c$.