PUMaC 2012





Algebra B

- 1. [3] Find the largest n such that the last nonzero digit of n! is 1.
- 2. [3] Define a sequence a_n such that $a_n = a_{n-1} a_{n-2}$. Let $a_1 = 6$ and $a_2 = 5$. Find $\sum_{n=1}^{1000} a_n$.
- 3. [4] Evaluate $\sqrt[3]{26+15\sqrt{3}}+\sqrt[3]{26-15\sqrt{3}}$.
- 4. [4] If x, y, and z are real numbers with $\frac{x-y}{z} + \frac{y-z}{x} + \frac{z-x}{y} = 36$, find

$$2012 + \frac{x-y}{z} \cdot \frac{y-z}{x} \cdot \frac{z-x}{y}$$

- 5. [5] Considering all numbers of the form $n = \lfloor \frac{k^3}{2012} \rfloor$, where $\lfloor x \rfloor$ denotes the greatest integer less than or equal to x, and k ranges from 1 to 2012, how many of these n's are distinct?
- 6. **[6**] Compute

$$\sum_{n=1}^{\infty} \frac{n+1}{n^2(n+2)^2}$$

Your answer in simplest form can be written as a/b, where a,b are relatively-prime positive integers. Find a+b.

- 7. [7] Let f be a polynomial of degree 3 with integer coefficients such that f(0) = 3 and f(1) = 11. If f has exactly 2 integer roots, how many such polynomials f exist?
- 8. [8] Let a_n be a sequence such that $a_1 = 1$ and $a_{n+1} = \lfloor a_n + \sqrt{a_n} + \frac{1}{2} \rfloor$. What are the last four digits of a_{2012} ?