



Individual Finals A

1. Let p be a prime number greater than 5. Prove that there exists a positive integer n such that p divides $20^n + 15^n - 12^n$.
2. Let a, b, c be real numbers such that $a + b + c = abc$. Prove that $\frac{1}{a^2+1} + \frac{1}{b^2+1} + \frac{1}{c^2+1} \geq \frac{3}{4}$.
3. Let ABC be a triangle with incenter I , and let D be the foot of the angle bisector from A to BC . Let Γ be the circumcircle of triangle BIC , and let PQ be a chord of Γ passing through D . Prove that AD bisects $\angle PAQ$.

Please write complete, concise and clear proofs. Have fun! – PUMaC Problem Writers