PUMaC 2011

Individual Finals A

1. Find, with proof, all triples of positive integers (x, y, z) satisfying the equation $3^x - 5^y = 4z^2$.
2. Define the sequence of real numbers $\{x_n\}_{n\geq 1}$, where x_1 is any real number and

$$x_n = 1 - x_1 x_2 \dots x_{n-1} \text{ for all } n > 1.$$

   Show that $x_{2011} > \frac{2011}{2012}$.

3. Let ABC be an equilateral triangle having sides of length 1, and let P be a point in the interior of ΔABC such that $\angle ABP = 15°$. Find, with proof, the minimum possible value of $AP + BP + CP$.

Please write complete, concise and clear proofs. Have fun! – PUMaC Problem Writers