PUMaC 2010

Number Theory B

1. Find the positive integer less than 18 with the most positive divisors.

2. Let $f(n)$ be the sum of the digits of n. Find $\displaystyle\sum_{n=1}^{99} f(n)$.

3. Find the smallest positive integer n such that $n^4 + (n+1)^4$ is composite.

4. Find the sum of the first 5 positive integers n such that $n^2 - 1$ is the product of 3 distinct primes.

5. Given that x, y, and z are positive integers such that $\dfrac{x}{y} + \dfrac{y}{z} + \dfrac{z}{x} = 2$. Find the sum of all possible x values.

6. Given that x, y are positive integers with $x(x+1)|y(y+1)$, but neither x nor $x+1$ divides either of y or $y+1$, and $x^2 + y^2$ as small as possible, find $x^2 + y^2$.

7. Find the numerator of

$$\frac{1010\overbrace{11\ldots11}^{2011\ \text{ones}}0101}{1100\underbrace{11\ldots11}_{2011\ \text{ones}}0011}$$

when reduced.

8. Let N be the number of (positive) divisors of 2010^{2010} ending in the digit 2. What is the remainder when N is divided by 2010?