PUMaC 2010



C Standard Contraction

Geometry B

- 1. In a polygon, every external angle is one sixth of its corresponding internal angle. How many sides does the polygon have?
- 2. On rectangular coordinates, point A = (1, 2), B = (3, 4). P = (a, 0) is on x-axis. Given that P is chosen such that AP + PB is minimized, compute 60a.
- 3. As in the following diagram, square ABCD and square CEFG are placed side by side (i.e. C is between B and E and G is between C and D). Now CE = 14, AB > 14, compute the minimal area of $\triangle AEG$.



4. Unit square ABCD is divided into four rectangles by EF and GH, with $BF = \frac{1}{4}$. EF is parallel to AB and GH parallel to BC. EF and GH meet at point P. Suppose BF + DH = FH, calculate the nearest integer to the degree of $\angle FAH$.



5. In a rectangular plot of land, a man walks in a very peculiar fashion. Labeling the corners ABCD, he starts at A and walks to C. Then, he walks to the midpoint of side AD, say A_1 . Then, he walks to the midpoint of side CD say C_1 , and then the midpoint of A_1D which is A_2 . He continues in this fashion, indefinitely. The total length of his path if AB = 5 and BC = 12 is of the form $a + b\sqrt{c}$. Find $\frac{abc}{4}$.





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6. In regular hexagon ABCDEF, AC, CE are two diagonals. Points M, N are on AC, CE respectively and satisfy AC : AM = CE : CN = r. Suppose B, M, N are collinear, find $100r^2$.



7. A cuboctahedron is a solid with 6 square faces and 8 equilateral triangle faces, with each edge adjacent to both a square and a triangle (see picture). Suppose the ratio of the volume of an octahedron to a cuboctahedron with the same side length is r. Find $100r^2$.



8. Point P is in the interior of $\triangle ABC$. The side lengths of ABC are AB = 7, BC = 8, CA = 9. The three foots of perpendicular lines from P to sides BC, CA, AB are D, E, F respectively. Suppose the minimal value of $\frac{BC}{PD} + \frac{CA}{PE} + \frac{AB}{PF}$ can be written as $\frac{a}{b}\sqrt{c}$, where gcd(a, b) = 1 and c is square free, calculate *abc*.

