



Algebra B

1. Let the operation \star be defined by $x \star y = y^x - xy$. Calculate $(3 \star 4) - (4 \star 3)$.
2. Let $p(x) = x^2 + x + 1$. Find the fourth smallest prime q such that $p(n)$ is divisible by q for some integer n .
3. Write $\frac{1}{\sqrt[5]{2}-1} = a + b\sqrt[5]{2} + c\sqrt[5]{4} + d\sqrt[5]{8} + e\sqrt[5]{16}$, with $a, b, c, d,$ and e integers. Find $a^2 + b^2 + c^2 + d^2 + e^2$.
4. Let S be the sum of all real x such that $4^x = x^4$. Find the nearest integer to S .
5. Let x be a real root of the polynomial $p(x) = x^3 - 3x + 3$. Find $x^9 + 81x^2$.
6. Define $f(x) = x + \sqrt{x + \sqrt{x + \sqrt{x + \sqrt{x + \dots}}}}$. Find the smallest integer x such that $f(x) \geq 50\sqrt{x}$.
7. Let f be a function such that $f(x) + f(x+1) = 2^x$ and $f(0) = 2010$. Find the last two digits of $f(2010)$.
8. The expression $\sin 2^\circ \sin 4^\circ \sin 6^\circ \cdots \sin 90^\circ$ is equal to $p\sqrt{5}/2^{50}$, where p is an integer. Find p .