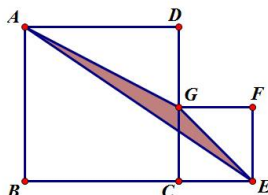# PUMaC 2010

## Geometry A

1. As in the following diagram, square $ABCD$ and square $CEFG$ are placed side by side (i.e. $C$ is between $B$ and $E$ and $G$ is between $C$ and $D$). If $CE = 14$, $AB > 14$, compute the minimal area of $\triangle AEG$.



2. In a rectangular plot of land, a man walks in a very peculiar fashion. Labeling the corners $ABCD$, he starts at $A$ and walks to $C$. Then, he walks to the midpoint of side $AD$, say $A_1$. Then, he walks to the midpoint of side $CD$ say $C_1$, and then the midpoint of $A_1D$ which is $A_2$. He continues in this fashion, indefinitely. The total length of his path if $AB = 5$ and $BC = 12$ is of the form $a + b\sqrt{c}$. Find $\dfrac{abc}{4}$.

3. Triangle $ABC$ has $AB = 4$, $AC = 5$, and $BC = 6$. An angle bisector is drawn from angle $A$, and meets $BC$ at $M$. What is the nearest integer to $100\frac{AM}{CM}$?

4. In regular hexagon $ABCDEF$, $AC$, $CE$ are two diagonals. Points $M$, $N$ are on $AC$, $CE$ respectively and satisfy $AC : AM = CE : CN = r$. Suppose $B, M, N$ are collinear, find $100r^2$.
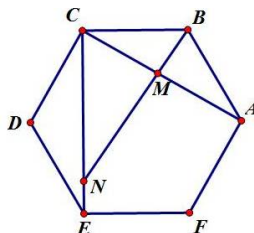


5. A cuboctahedron is a solid with 6 square faces and 8 equilateral triangle faces, with each edge adjacent to both a square and a triangle (see picture). Suppose the ratio of the volume of an octahedron to a cuboctahedron with the same side length is $r$. Find $100r^2$.



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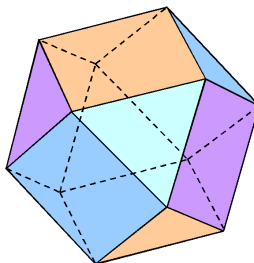
6. In the following diagram, a semicircle is folded along a chord $AN$ and intersects its diameter $MN$ at $B$. Given that $MB : BN = 2 : 3$ and $MN = 10$. If $AN = x$, find $x^2$.
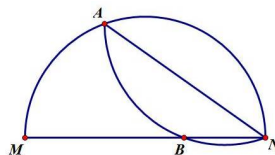


7. Square $ABCD$ is divided into four rectangles by $EF$ and $GH$. $EF$ is parallel to $AB$ and $GH$ parallel to $BC$. $\angle BAF = 18°$. $EF$ and $GH$ meet at point $P$. The area of rectangle $PFCH$ is twice that of rectangle $AGPE$. Given that the value of $\angle FAH$ in degrees is $x$, find the nearest integer to $x$.
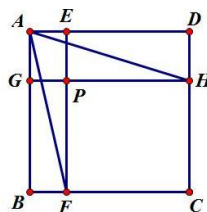


8. There is a point source of light in an empty universe. What is the minimum number of solid balls (of any size) one must place in space so that any light ray emanating from the light source intersects at least one ball?