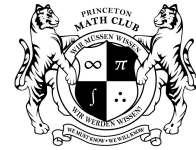


PUMaC 2008-9



Number Theory B

- (2 points) What is the remainder, in base 10, when $24_7 + 364_7 + 43_7 + 12_7 + 3_7 + 1_7$ is divided by 6?
- (2 points) How many zeros are there at the end of $792!$ when written in base 10?
- (3 points) Find all integral solutions to $x^y - y^x = 1$.
- (3 points) Find the largest integer n , where 2009^n divides $2008^{2009^{2010}} + 2010^{2009^{2008}}$.
- (4 points) How many integers n are there such that $0 \leq n \leq 720$ and $n^2 \equiv 1 \pmod{720}$?
- (4 points) $f(n)$ is the sum of all integers less than n and relatively prime to n . Find all integers n such that there exist integers k and l such that $f(n^k) = n^l$.
- (5 points) In this problem, we consider only polynomials with integer coefficients. Call two polynomials p and q *really close* if $p(2k+1) \equiv q(2k+1) \pmod{2^{10}}$ for all $k \in \mathbb{Z}^+$. Call a polynomial p *partial credit* if no polynomial of lesser degree is *really close* to it. What is the maximum possible degree of partial credit?
- (5 points) What is the largest integer which cannot be expressed as $2008x + 2009y + 2010z$ for some positive integers x , y , and z ?
- (7 points) Find all sets of three primes p , q , and r such that $p + q = r$ and $(r - p)(q - p) - 27p$ is a perfect square.
- (7 points) What is the smallest number n such that you can choose n distinct odd integers a_1, a_2, \dots, a_n , none of them 1, with $\frac{1}{a_1} + \frac{1}{a_2} + \dots + \frac{1}{a_n} = 1$?