PUMaC 2008-9

Number Theory B

1. (2 points)What is the remainder, in base 10, when $24_7 + 364_7 + 43_7 + 12_7 + 3_7 + 1_7$ is divided by 6?

2. (2 points)How many zeros are there at the end of 792! when written in base 10?

3. (3 points)Find all integral solutions to $x^y - y^x = 1$.

4. (3 points) Find the largest integer n, where 2009^n divides $2008^{2009^{2010}} + 2010^{2009^{2008}}$.

5. (4 points)How many integers n are there such that $0 \leq n \leq 720$ and $n^2 \equiv 1 \pmod{720}$?

6. (4 points)$f(n)$ is the sum of all integers less than n and relatively prime to n. Find all integers n such that there exist integers k and l such that $f(n^k) = n^l$.

7. (5 points)In this problem, we consider only polynomials with integer coeffients. Call two polynomials p and q *really close* if $p(2k+1) \equiv q(2k+1) \pmod{2^{10}}$ for all $k \in \mathbb{Z}^+$. Call a polynomial p *partial credit* if no polynomial of lesser degree is *really close* to it. What is the maximum possible degree of partial credit?

8. (5 points)What is the largest integer which cannot be expressed as $2008x + 2009y + 2010z$ for some positive integers x, y, and z?

9. (7 points)Find all sets of three primes p, q, and r such that $p + q = r$ and $(r-p)(q-p) - 27p$ is a perfect square.

10. (7 points)What is the smallest number n such that you can choose n distinct odd integers $a_1, a_2, \ldots a_n$, none of them 1, with $\frac{1}{a_1} + \frac{1}{a_2} + \ldots + \frac{1}{a_n} = 1$?