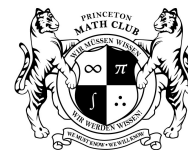


# PUMaC 2008-9



## Algebra

---

- (2 points) Solve for  $x$ :  $x = 2 + \frac{4(2^6)}{11-3}$
- (2 points) What is  $3(2 \log_4 (2(2 \log_3 9)))$ ?
- (3 points) Given the sequence 1, 2, 1, 2, 2, 1, 2, 2, 2, 1, 2, 2, 2, 2, 1, ..., find  $n$  such that the sum of the first  $n$  terms is 2008 or 2009.
- (3 points) Find the product of the minimum and maximum values of  $\frac{3x+1}{9x^2+6x+2}$ .
- (4 points) How many real roots do  $x^5 + 3x^4 - 4x^3 - 8x^2 + 6x - 1$  and  $x^5 - 3x^4 - 2x^3 + 10x^2 - 6x + 1$  share?
- (4 points) Let  $f(n) = 9n^5 - 5n^3 - 4n$ . Find the greatest common divisor of  $f(17)$ ,  $f(18)$ , ...,  $f(2009)$ .
- (5 points) What's the greatest integer  $n$  for which the system  $k < x^k < k + 1$  for  $k = 1, 2, \dots, n$  has a solution?
- (5 points) Let  $H_k = \sum_{i=1}^k \frac{1}{i}$  for all positive integers  $k$ . Find an closed-form expression for  $\sum_{k=1}^n H_k$  in terms of  $n$  and  $H_n$ .
- (7 points) Find the polynomial  $f$  with the following properties:
  - its leading coefficient is 1,
  - its coefficients are nonnegative integers,
  - $72|f(x)$  if  $x$  is an integer,
  - if  $g$  is another polynomial with the same properties, then  $g - f$  has a nonnegative leading coefficient.
- (7 points) If  $p(x)$  is a polynomial with integer coefficients, let  $q(x) = \frac{p(x)}{x(1-x)}$ . If  $q(x) = q\left(\frac{1}{1-x}\right)$  for every  $x \neq 0$ , and  $p(2) = -7$ ,  $p(3) = -11$ , find  $p(10)$ .