PUMaC 2008-9

Number Theory A

1. (2 points)How many zeros are there at the end of 792! when written in base 10?

2. (3 points)Find all integral solutions to $x^y - y^x = 1$.

3. (3 points) Find the largest integer n, where 2009^n divides $2008^{2009^{2010}} + 2010^{2009^{2008}}$.

4. (3 points)$f(n)$ is the sum of all integers less than n and relatively prime to n. Find all integers n such that there exist integers k and l such that $f(n^k) = n^l$.

5. (4 points)If $f(x) = x^{x^{x^x}}$, find the last two digits of $f(17) + f(18) + f(19) + f(20)$.

6. (4 points)What is the largest integer which cannot be expressed as $2008x + 2009y + 2010z$ for some positive integers x, y, and z?

7. (5 points)Find the smallest positive integer n such that $32^n = 167x + 2$ for some integer x.

8. (5 points)Find all sets of three primes p, q, and r such that $p + q = r$ and $(r - p)(q - p) - 27p$ is a perfect square.

9. (7 points)Find the number of positive integer solutions of $(x^2 + 2)(y^2 + 3)(z^2 + 4) = 60xyz$.

10. (7 points)What is the smallest number n such that you can choose n distinct odd integers $a_1, a_2, \ldots a_n$, none of them 1, with $\frac{1}{a_1} + \frac{1}{a_2} + \ldots + \frac{1}{a_n} = 1$?