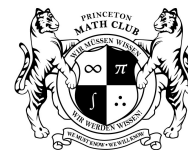


PUMaC 2008-9



Number Theory A

1. (2 points) How many zeros are there at the end of $792!$ when written in base 10?
2. (3 points) Find all integral solutions to $x^y - y^x = 1$.
3. (3 points) Find the largest integer n , where 2009^n divides $2008^{2009^{2010}} + 2010^{2009^{2008}}$.
4. (3 points) $f(n)$ is the sum of all integers less than n and relatively prime to n . Find all integers n such that there exist integers k and l such that $f(n^k) = n^l$.
5. (4 points) If $f(x) = x^{x^{x^x}}$, find the last two digits of $f(17) + f(18) + f(19) + f(20)$.
6. (4 points) What is the largest integer which cannot be expressed as $2008x + 2009y + 2010z$ for some positive integers x, y , and z ?
7. (5 points) Find the smallest positive integer n such that $32^n = 167x + 2$ for some integer x .
8. (5 points) Find all sets of three primes p, q , and r such that $p + q = r$ and $(r - p)(q - p) - 27p$ is a perfect square.
9. (7 points) Find the number of positive integer solutions of $(x^2 + 2)(y^2 + 3)(z^2 + 4) = 60xyz$.
10. (7 points) What is the smallest number n such that you can choose n distinct odd integers a_1, a_2, \dots, a_n , none of them 1, with $\frac{1}{a_1} + \frac{1}{a_2} + \dots + \frac{1}{a_n} = 1$?