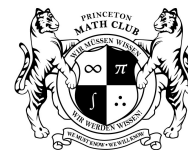# PUMaC 2008-9

## Individual Finals A

1.      Find all positive real numbers $b$ for which there exists a positive real number $k$ such that $n - k \le \lfloor bn \rfloor \le n$ for all positive integers $n$.

2.  A *hypergraph* consists of a set of vertices $V$ and a set of subsets of those vertices, each of which is called an edge. (Intuitively, it's a graph in which each edge can contain multiple vertices). Suppose that in some hypergraph, no two edges have exactly one vertex in common. Prove that one can color this hypergraph's vertices such that every edge contains both colors of vertices.

3.  A sequence $\{a_i\}$ is defined by $a_1 = c$ for some $c > 0$ and $a_{n+1} = a_n + \frac{n}{a_n}$. Prove that $\frac{a_n}{n}$ converges and find its limit.