Team

1. If

$$\begin{aligned} x+y+\quad z+w \quad &= 20 \\ y+\quad 2z-3w \quad &= 28 \\ x-2y+\quad z \quad &= 36 \\ -7x-y+\quad 5z+3w \quad &= 84 \end{aligned}$$

then what is (x, y, z, w)?

2. Find the biggest non-integer x such that $(x+2)^2+(x+3)^3+(x+4)^4=2$.

3. In triangle ABC, let O and I_A be the centers of the circumcircle and the circle tangent to AB and AC and externally tangent to BC, and let R and R_A be their radii. Find $\frac{I_AA \cdot I_AB \cdot I_AC}{R \cdot R_A^2}$.

4. Find all x such that $6^x+27^{x-1}=8^x-1$.

5. Integers x_1, x_2, $\ldots$, x_{100} satisfy

$$\frac{1}{\sqrt{x_1}}+\frac{1}{\sqrt{x_2}}+\cdots+\frac{1}{\sqrt{x_{100}}}=20.$$

Find $\prod_{i \neq j}(x_i-x_j)$.

6. If integers a, b, c, and d satisfy $bc+ad=ac+2bd=1$, find all possible values of $\frac{a^2+c^2}{b^2+d^2}$.

7. In triangle ABC, Q and R are points on segments AC and AB, respectively, and P is the intersection of CR and BQ. If $AR=RB=CP$ and $CQ=PQ$, find $\angle BRC$.

8. Find the biggest $n<2007$ such that there exists a partition of the integers from 1 to n into two sets the sums of the squares of whose elements are equal.

9. Find $p+r$ if p and q are primes and r is a positive integer such that

$$(r^2+pr+1)(r^2+(p^2-q)r-p)=pq.$$

10. If x, y, and z are real numbers such that $x^2+z^2=1$ and $y^2+2y(x+z)=6$, find the maximum value of $y(z-x)$.