Individual Finals B

1. Let C and D be two points, not diametrically opposite, on a circle C_1 with center M. Let H be a point on minor arc CD. The tangent to C_1 at H intersects the circumcircle of CMD at points A and B. Prove that CD bisects MH iff $\angle AMB = \frac{\pi}{2}$.

2. Suppose that A is a set of positive integers less than N and that no two distinct elements of A sum to a perfect square. That is, if $a_1, a_2 \in A$ and $a_1 \neq a_2$ then $|a_1 + a_2|$ is not a square of an integer. Prove that the maximum number of elements in A is at least $\lfloor \frac{11}{32} N \rfloor$.

3. Find the minimum number of colors necessary to color the integers from 1 to 2007 such that if distinct integers a, b, and c are the same color, then $a \nmid b$ or $b \nmid c$.