

Individual Finals A

- 1. Suppose that A is a set of integers. Denote the number of elements in A by |A|. Define $A + A = \{a_1 + a_2 : a_1, a_2 \in A\}$ and $A A = \{a_1 a_2 : a_1, a_2 \in A\}$. Prove or disprove: for any set A, we have the inequality $|A A| \ge |A + A|$.
- 2. We have a 2007×2007 square table filled with nonnegative integers. For each entry of 0 in the table, the sum of the elements that are in the same row or column as that entry is at least 2007. Find the minimum sum of all the elements of such a table.
- 3. Is there a set of distinct integers X containing all the primes less than 2007 such that the product of the elements of X equals the sum of the squares of those elements?