



Algebra A

1. If a , b , and c are real numbers such that $a + b + c = 6$ and $ab + bc + ca = 9$, find the sum of all possible values of the expression $\lfloor a \rfloor + \lfloor b \rfloor + \lfloor c \rfloor$.
2. Positive reals p and q are such that the graph of $y = x^2 - 2px + q$ does not intersect the x -axis. Find q if there is a unique pair of points A , B on the graph with AB parallel to the x -axis and $\angle AOB = \frac{\pi}{2}$, where O is the origin.
3. For how many rational numbers p is the area of the triangle formed by the intercepts and vertex of $f(x) = -x^2 + 4px - p + 1$ an integer?
4. Find the all values of a such that $x^6 - 6x^5 + 12x^4 + ax^3 + 12x^2 - 6x + 1$ is nonnegative for all real x .
5. Find the values of a such that $\log(ax + 1) = \log(x - a) + \log(2 - x)$ has a unique real solution.
6. If a , b , c , and d are reals with $a \geq b \geq c \geq d \geq 0$ and $b(b - a) + c(c - b) + d(d - c) \leq 2 - \frac{a^2}{2}$, find the minimum value of the expression

$$\frac{1}{b + 2006c - 2006d} + \frac{1}{a + 2006b - 2006c - d} + \frac{1}{2007a - 2006b - c + d} + \frac{1}{a - b + c + 2006d}.$$

7. Given two sequences x_n and y_n defined by $x_0 = y_0 = 7$,

$$x_n = 4x_{n-1} + 3y_{n-1}, \text{ and}$$

$$y_n = 3y_{n-1} + 2x_{n-1},$$

Find $\lim_{n \rightarrow \infty} \frac{x_n}{y_n}$.

8. $f(x) = x^3 + 3x^2 - 1$. Find the number of real solutions of $f(f(x)) = 0$.
9. Find the value of $x + y$ for which the expression

$$\frac{6x^2}{y^6} + \frac{6y^2}{x^6} + 9x^2y^2 + \frac{4}{x^6y^6}$$
 is minimized.
10. Find the real root of $x^5 + 5x^3 + 5x - 1$. Hint: Let $x = u + k/u$.