STEP MATHEMATICS 2019

Notation and Required Formulae

Introduction

The **notation** for STEP follows the notation for the A level examinations[1] with some minor additions and omissions. STEP papers are set in LaTeX fonts, which are not the same as the usual fonts used for A level.

The **required formulae** for each STEP paper are the formulae that candidates must be able to use without them being provided. If other formulae are required for a particular question, they will be given in the question (or candidates will be asked to derive them); there are no Formulae Booklets for STEP examinations.

The required formulae are those required for the corresponding AS or A level as set out in the Department for Education's guidance documents,[1] with some additions and omissions. Nearly all the additions can be found in the AS or A level Formulae Booklets provided by the individual examination boards, but candidates are not expected to know all the formulae in these booklets. Throughout the tables that follow, notation and formulae that do not appear in the Department for Education's corresponding guidance documents are indicated by a '•' in the 'Papers' column.

The formulae are usually given in their simplest forms. For example, the derivative of $\sin x$ rather than $\sin kx$ is given, the latter being easily derivable from the former.

Some formulae are omitted because it is better not to learn them. For example, the derivative of $\sin^{-1} x$ is included, but the derivative of $\cos^{-1} x$ is not; it is better to understand that (for acute angles) $\cos^{-1} x = \frac{1}{2}\pi - \sin^{-1} x$ so that the only difference in the derivatives is a minus sign.

[1] See https://www.gov.uk/government/publications/gce-as-and-a-level-mathematics and https://www.gov.uk/government/publications/gce-as-and-a-level-further-mathematics

NOTATION

Set notation			
Notation	**Meaning**	**Comment**	**Papers**
$\in$	is an element of		1, 2, 3
$\notin$	is not an element of		1, 2, 3
$\subseteq$	is a subset of		1, 2, 3
$\subset$	is a proper subset of		1, 2, 3
$\{x_1, x_2, \ldots, x_n\}$	the set with elements x_1, x_2, $\ldots$, x_n		1, 2, 3
$\varnothing$	the empty set	only used with explanation	1, 2, 3
A'	the complement set of the set A	only used with explanation	1, 2, 3
$\mathbb{N}$	the set of natural numbers $\{1, 2, 3, \ldots\}$		1, 2, 3
$\mathbb{Z}$	the set of integers $\{\ldots, -2, -1, 0, 1, 2, \ldots\}$		1, 2, 3
$\mathbb{Q}$	the set of rational numbers $\{\frac{p}{q} : p \in \mathbb{Z}, q \in \mathbb{N}\}$		1, 2, 3
$\mathbb{R}$	the set of real numbers		1, 2, 3
$\mathbb{C}$	the set of complex numbers		2, 3
$\cup$	union of sets		1, 2, 3
$\cap$	intersection of sets		1, 2, 3
(x, y)	the ordered pair x, y	for example, coordinates	1, 2, 3

Miscellaneous symbols			
Notation	**Meaning**	**Comment**	**Papers**
$=$	is equal to		1, 2, 3
$\neq$	is not equal to		1, 2, 3
$\equiv$	is identical to, or is equivalent to, or is congruent to		1, 2, 3
$\approx$	is approximately equal to		1, 2, 3
∞	infinity		1, 2, 3
$\propto$	is proportional to		1, 2, 3
$<$	is less than		1, 2, 3
$\leqslant$	is less than or equal to		1, 2, 3
$>$	is greater than		1, 2, 3
$\geqslant$	is greater than or equal to		1, 2, 3
$\therefore$	therefore		1, 2, 3
$p \Rightarrow q$	p implies q (if p then q)		1, 2, 3
$p \Leftarrow q$	p is implied by q (if q then p)		1, 2, 3
$p \Leftrightarrow q$	p is equivalent to q (p if and only if q)		1, 2, 3
S_n	the sum to n terms of a progression		1, 2, 3
S_∞	the sum to infinity of a progression		1, 2, 3
$x \longrightarrow \infty$	x tends to ∞		1, 2, 3
$x_n \longrightarrow a$	x_n tends to a	for sequences when $n \longrightarrow \infty$	1, 2, 3

Operations			
Notation	**Meaning**	**Comment**	**Papers**
$a \pm b$	a plus or minus b		1, 2, 3 •
$a \mp b$	a minus or plus b		1, 2, 3 •
$a \times b$, ab, $a.b$	a multiplied by b	$a.b$ not usually used	1, 2, 3
$a \div b$, $\frac{a}{b}$, a/b	a divided by b	a/b is not given in the A level notation list; $a \div b$ not usually used	1, 2, 3 •
$\sum_{i=m}^{n} a_i$	$a_m + a_{m+1} + \cdots + a_n$	only the case $m = 1$ is given in the A level notation list	1, 2, 3 •
$\prod_{i=m}^{n} a_i$	$a_m a_{m+1} \cdots a_n$	only the case $m = 1$ is given in the A level notation list	1, 2, 3 •
$\sqrt{a}$	the positive square root of a	$a \in \mathbb{R}$, $a \geqslant 0$	1, 2, 3
$\lvert a \rvert$	the modulus of a	$a \in \mathbb{R}$	1, 2, 3
$n!$	n factorial, $n \in \mathbb{N}$	$0! = 1$, by definition	1, 2, 3
$\binom{n}{r}$ or ${}^nC_r$	the binomial coefficient $\frac{n!}{r!\,(n-r)!}$		1, 2, 3

Functions			
Notation	**Meaning**	**Comment**	**Papers**
$\lim_{x \to a} \mathrm{f}(x)$	the limit of $\mathrm{f}(x)$ as x tends to a		1, 2, 3
$\frac{\mathrm{d}y}{\mathrm{d}x}$	the first derivative of y with respect to x		1, 2, 3
$\frac{\mathrm{d}^n y}{\mathrm{d}x^n}$	the nth derivative of y with respect to x		1, 2, 3
$\dot{x}$ and $\ddot{x}$	the first and second derivatives of x with respect to t	where t is time, unless otherwise specified	1, 2, 3
$\mathrm{f}'(x)$	the first derivative of f evaluated at x		1, 2, 3
$\mathrm{f}''(x)$	the second derivative of f evaluated at x		1, 2, 3
$\mathrm{f}^{(n)}(x)$	the nth derivative of f evaluated at x		1, 2, 3
$\int \mathrm{f}(x)\,\mathrm{d}x$	the indefinite integral of $\mathrm{f}(x)$ with respect to x		1, 2, 3
$\int_a^b \mathrm{f}(x)\,\mathrm{d}x$	the definite integral of $\mathrm{f}(x)$ with respect to x between the limits of $x = a$ and $x = b$		1, 2, 3
e	base of natural logarithms		1, 2, 3
e^x, $\exp x$	exponential function of x		1, 2, 3
$\log_a x$	logarithm to base a of x		1, 2, 3
$\ln x$, $\log_{\mathrm{e}} x$	natural logarithm of x		1, 2, 3
sin, cos, tan, cosec, sec, cot	the trigonometric functions		1, 2, 3
$\sin^{-1}$, etc	the inverse trigonometric functions	arcsin, etc, will not be used	1, 2, 3
sinh, cosh, tanh, cosech, sech, coth	the hyperbolic functions		3
$\sinh^{-1}$, etc	the inverse hyperbolic functions	arsinh, etc, will not be used	3

Complex numbers			
Notation	**Meaning**	**Comment**	**Papers**
i	square root of -1	j will not be used	2, 3
$x + \mathrm{i}y$	complex number with real part x and imaginary part y		2, 3
$r(\cos\theta + \mathrm{i}\sin\theta)$	complex number with modulus r and argument θ	$r \geqslant 0$; the range of θ will be given if required	2, 3
$\mathrm{Re}(z)$	the real part of z	$\mathrm{Re}(z) = x$ if $z = x + \mathrm{i}y$	2, 3
$\mathrm{Im}(z)$	the imaginary part of z	$\mathrm{Im}(z) = y$ if $z = x + \mathrm{i}y$	2, 3
$\arg(z)$	the argument of z	$\arg(z) = \theta$ if $z = r(\cos\theta + \mathrm{i}\sin\theta)$; the range of $\arg(z)$ will be given if required	2, 3
$\lvert z\rvert$	the modulus of z	$\lvert z\rvert = r$ if $z = r(\cos\theta + \mathrm{i}\sin\theta)$	2, 3
z^*	the complex conjugate of z	$z^* = x - \mathrm{i}y$ if $z = x + \mathrm{i}y$	2, 3

Matrices			
Notation	**Meaning**	**Comment**	**Papers**
$\mathbf{M}$	the matrix $\mathbf{M}$		2, 3
M_{ij}	the entry in the ith row and jth column of the matrix $\mathbf{M}$	only used with explanation	2, 3 •
$\mathbf{0}$	matrix with all entries 0		2, 3
$\mathbf{I}$	identity matrix		2, 3
$\mathbf{M}^{-1}$	the inverse of the (square) matrix $\mathbf{M}$		2, 3
$\mathbf{M}^{\mathrm{T}}$	the transpose of the matrix $\mathbf{M}$		2, 3
$\det \mathbf{M}$	determinant of the (square) matrix $\mathbf{M}$		2, 3
$\mathbf{Mr}$	image of the column vector $\mathbf{r}$ under the transformation associated with the matrix $\mathbf{M}$		2, 3

Vectors			
Notation	**Meaning**	**Comment**	**Papers**
$\mathbf{a}$	the vector $\mathbf{a}$		1, 2, 3
$\overrightarrow{AB}$	the vector represented by the directed line segment AB		1, 2, 3
$\widehat{\mathbf{a}}$	the unit vector in the direction of $\mathbf{a}$	only used with explanation	1, 2, 3
$\mathbf{i}$, $\mathbf{j}$, $\mathbf{k}$	unit vectors in the directions of the Cartesian axes		1, 2, 3
$\lvert\mathbf{a}\rvert$	the magnitude of $\mathbf{a}$		1, 2, 3
$\lvert\overrightarrow{AB}\rvert$	the magnitude of $\overrightarrow{AB}$		1, 2, 3
$\mathbf{r}$	position vector		1, 2, 3
$\mathbf{s}$	displacement vector		1, 2, 3
$\mathbf{a}\,.\,\mathbf{b}$	the scalar product of vectors $\mathbf{a}$ and $\mathbf{b}$		2, 3
$\mathbf{a}\times\mathbf{b}$	the vector product of vectors $\mathbf{a}$ and $\mathbf{b}$		3 •

Probability/Statistics			
Notation	**Meaning**	**Comment**	**Papers**
A, B, C, etc	events		1, 2, 3
$A \cup B$	union of events A and B		1, 2, 3
$A \cap B$	intersection of events A and B		1, 2, 3
$\mathrm{P}(A)$	probability of the event A		1, 2, 3
A'	complement of event A	only used with explanation	1, 2, 3
$\mathrm{P}(A \mid B)$	probability of the event A conditional on (i.e. given) the event B		1, 2, 3
x, y, r, etc	values of the random variables X, Y, R, etc		1, 2, 3
$\mathrm{P}(X = x)$	probability function of a discrete random variable X		1, 2, 3
$\mathrm{f}(x)$	probability density function (p.d.f.) of a continuous random variable		2, 3
$\mathrm{F}(x)$	cumulative distribution function (c.d.f.) of a continuous random variable		1, 2, 3
$\mathrm{E}(X)$	expectation of the random variable X		1, 2, 3
$\mathrm{Var}\,(X)$	variance of the random variable X		1, 2, 3
$\sim$	has the distribution		1, 2, 3
$\mathrm{B}(n, p)$	Binomial distribution with parameters n and p, where n is the number of trials and p is the probability of success in any trial	$q = 1 - p$	1, 2, 3
$\mathrm{N}(\mu, \sigma^2)$	Normal distribution with mean μ and variance σ^2		1, 2, 3
$\mathrm{N}(0, 1)$	the standard Normal distribution		1, 2, 3
ϕ, Φ	probability density function and cumulative distribution function of a random variable with standard Normal distribution	knowledge of formulae is not required; only used with explanation	1, 2, 3

Mechanics			
Notation	**Meaning**	**Comment**	**Papers**
kg	kilogram		1, 2, 3
m	metre		1, 2, 3
km	kilometre		1, 2, 3
$\mathrm{m\,s^{-1}}$	metres per second		1, 2, 3
$\mathrm{m\,s^{-2}}$	metres per second per second	acceleration	1, 2, 3
N	newton		1, 2, 3
N m	newton metre	moment of a force, for example	1, 2, 3
J	joule		1, 2, 3
t	time		1, 2, 3
s	displacement		1, 2, 3
u	initial speed		1, 2, 3
v	speed or final speed		1, 2, 3
a	acceleration		1, 2, 3
g	acceleration due to gravity		1, 2, 3
μ	coefficient of friction		1, 2, 3
e	coefficient of restitution		2, 3
k	stiffness		2, 3
λ	modulus of elasticity		2, 3
ω	angular speed		3

REQUIRED FORMULAE

Roots of polynomials		
Formula	**Comment**	**Papers**
$ax^2 + bx + c = 0$ has roots $x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$		1, 2, 3
For $ax^2 + bx + c = 0$ with roots α and β: $\alpha + \beta = -b/a, \quad \alpha\beta = c/a$		1, 2, 3
For $ax^3 + bx^2 + cx + d = 0$ with roots α, β and γ: $\alpha + \beta + \gamma = -b/a, \quad \alpha\beta + \beta\gamma + \gamma\alpha = c/a, \quad \alpha\beta\gamma = -d/a$	The pattern is the same for polynomial equations of higher degree	2, 3

Laws of indices		
Formula	**Comment**	**Papers**
$a^x a^y = a^{x+y}$		1, 2, 3
$a^0 = 1$	$a \neq 0$	1, 2, 3
$(a^x)^y = a^{xy}$		1, 2, 3
$a^x = \mathrm{e}^{x \ln a}$	defines a^x when x is not an integer	1, 2, 3 •

Laws of logarithms		
Formula	**Comment**	**Papers**
$x = a^n \Leftrightarrow n = \log_a x$	$x > 0$, $a > 0$ $(a \neq 1)$	1, 2, 3
$\log_a x + \log_a y = \log_a(xy)$		1, 2, 3
$\log_a x - \log_a y = \log_a(x/y)$		1, 2, 3
$k \log_a x = \log_a x^k$	for $x > 0$	1, 2, 3

Sequences and series		
Formula	**Comment**	**Papers**
General (nth) term of an arithmetic progression: $u_n = a + (n-1)d$	d is the common difference	1, 2, 3
General (nth) term of a geometric progression: $u_n = ar^{n-1}$	r is the common ratio	1, 2, 3
Sum of an arithmetic progression: $S_n = \frac{1}{2}n\{2a + (n-1)d\}$	or: $S_n = an + \frac{1}{2}n(n-1)d$	1, 2, 3 •
Sum of a geometric progression: $S_n = \dfrac{a(1-r^n)}{1-r}$		1, 2, 3 •
Sum to infinity of a geometric progression: $S_\infty = \dfrac{a}{1-r}$	$\lvert r\rvert < 1$	1, 2, 3 •
${}^nC_r = \dfrac{n!}{(n-r)!r!}$		1, 2, 3 •
$(a+b)^n = \displaystyle\sum_{r=0}^{n} {}^nC_r a^{n-r}b^r$	Binomial expansion, $n \in \mathbb{N}$	1, 2, 3 •
$(1+x)^k = 1 + kx + \frac{k(k-1)}{2!}x^2 + \cdots + \frac{k(k-1)\cdots(k-r+1)}{r!}x^r + \cdots$	$\lvert x\rvert < 1$, $k \in \mathbb{Q}$	1, 2, 3 •
$\displaystyle\sum_{r=1}^{n} r = \frac{1}{2}n(n+1)$		1, 2, 3 •
$\mathrm{f}(x) = \displaystyle\sum_{r=0}^{\infty} \frac{1}{r!}\,\mathrm{f}^{(r)}(0)\,x^r$	Maclaurin series	3 •
$\mathrm{e}^x = \displaystyle\sum_{r=0}^{\infty} \frac{x^r}{r!}$	converges for all x	2, 3 •
$\ln(1+x) = \displaystyle\sum_{r=1}^{\infty} (-1)^{r+1}\frac{x^r}{r}$	converges for $-1 < x \leqslant 1$	3 •
$\sin x = \displaystyle\sum_{r=0}^{\infty} (-1)^r \frac{x^{2r+1}}{(2r+1)!}$	converges for all x	3 •
$\cos x = \displaystyle\sum_{r=0}^{\infty} (-1)^r \frac{x^{2r}}{(2r)!}$	converges for all x	3 •

Coordinate geometry		
Formula	**Comment**	**Papers**
The straight line graph with gradient m passing through the point (x_1, y_1) has equation $y - y_1 = m(x - x_1)$		1, 2, 3
Straight lines with non-zero gradients m_1 and m_2 are perpendicular if and only if $m_1 m_2 = -1$		1, 2, 3

Trigonometry		
Formula	**Comment**	**Papers**
Sine rule for the triangle ABC: $\dfrac{a}{\sin A} = \dfrac{b}{\sin B} = \dfrac{c}{\sin C}$		1, 2, 3
Cosine rule in the triangle ABC: $a^2 = b^2 + c^2 - 2bc\cos A$		1, 2, 3
Area of triangle ABC: $\frac{1}{2}ab\sin C$		1, 2, 3
$\cos^2 A + \sin^2 A = 1$		1, 2, 3
$\sec^2 A = 1 + \tan^2 A$		1, 2, 3
$\operatorname{cosec}^2 A = 1 + \cot^2 A$		1, 2, 3
$\sin 2A = 2\sin A\cos A$		1, 2, 3
$\cos 2A = \cos^2 A - \sin^2 A$		1, 2, 3
$\tan 2A = \dfrac{2\tan A}{1 - \tan^2 A}$	$A \neq (k + \frac{1}{2})\frac{\pi}{2}$, $k \in \mathbb{Z}$	1, 2, 3
$\sin(A \pm B) = \sin A\cos B \pm \cos A\sin B$		1, 2, 3
$\cos(A \pm B) = \cos A\cos B \mp \sin A\sin B$		1, 2, 3
$\tan(A \pm B) = \dfrac{\tan A \pm \tan B}{1 \mp \tan A\tan B}$	$A \pm B \neq (k + \frac{1}{2})\pi$, $k \in \mathbb{Z}$	1, 2, 3 •
$\sin\theta \approx \theta$, $\cos\theta \approx 1 - \frac{1}{2}\theta^2$, $\tan\theta \approx \theta$	θ small (compared with 1); θ in radians	1, 2, 3 •

Hyperbolic functions		
Formula	**Comment**	**Papers**
$\sinh x = \dfrac{\mathrm{e}^{x} - \mathrm{e}^{-x}}{2}$	by definition	3
$\cosh x = \dfrac{\mathrm{e}^{x} + \mathrm{e}^{-x}}{2}$	by definition	3
$\tanh x = \dfrac{\sinh x}{\cosh x}$	by definition	3
$\cosh^2 A - \sinh^2 A = 1$		3 •
$\operatorname{sech}^2 A = 1 - \tanh^2 A$		3 •
$\operatorname{cosech}^2 A = \coth^2 A - 1$		3 •
$\sinh 2A = 2 \sinh A \cosh A$		3 •
$\cosh 2A = \cosh^2 A + \sinh^2 A$		3 •
$\tanh 2A = \dfrac{2 \tanh A}{1 + \tanh^2 A}$		3 •
$\sinh(A \pm B) = \sinh A \cosh B \pm \cosh A \sinh B$		3 •
$\cosh(A \pm B) = \cosh A \cosh B \pm \sinh A \sinh B$		3 •
$\tanh(A \pm B) = \dfrac{\tanh A \pm \tanh B}{1 \pm \tanh A \tanh B}$		3 •

Derivatives			
Function	**Derivative**	**Comment**	**Papers**
$\sin x$	$\cos x$		1, 2, 3
$\cos x$	$-\sin x$		1, 2, 3
$\tan x$	$\sec^2 x$		1, 2, 3 •
$\cot x$	$-\mathrm{cosec}^2 x$		1, 2, 3 •
$\sec x$	$\sec x \tan x$		1, 2, 3 •
$\mathrm{cosec}\, x$	$-\mathrm{cosec}\, x \cot x$		1, 2, 3 •
$\sin^{-1} x$	$\dfrac{1}{\sqrt{1-x^2}}$		2, 3 •
$\tan^{-1} x$	$\dfrac{1}{1+x^2}$		2, 3 •
$\sinh x$	$\cosh x$		3
$\cosh x$	$\sinh x$		3
$\tanh x$	$\mathrm{sech}^2 x$		3 •
$\coth x$	$-\mathrm{cosech}^2 x$		3 •
$\mathrm{sech}\, x$	$-\mathrm{sech}\, x \tanh x$		3 •
$\sinh^{-1} x$	$\dfrac{1}{\sqrt{1+x^2}}$		3 •
$\tanh^{-1} x$	$\dfrac{1}{1-x^2}$		3 •
e^x	e^x		1, 2, 3
$\ln x$	$\dfrac{1}{x}$		1, 2, 3
$\mathrm{f}(x) + \mathrm{g}(x)$	$\mathrm{f}'(x) + \mathrm{g}'(x)$		1, 2, 3
$\mathrm{f}(x)\mathrm{g}(x)$	$\mathrm{f}'(x)\mathrm{g}(x) + \mathrm{f}(x)\mathrm{g}'(x)$	product rule	1, 2, 3
$\dfrac{\mathrm{f}(x)}{\mathrm{g}(x)}$	$\dfrac{\mathrm{f}'(x)\mathrm{g}(x) - \mathrm{f}(x)\mathrm{g}'(x)}{\big(\mathrm{g}(x)\big)^2}$	quotient rule	1, 2, 3 •
$\mathrm{f}\big(\mathrm{g}(x)\big)$	$\mathrm{f}'\big(\mathrm{g}(x)\big)\mathrm{g}'(x)$	chain rule	1, 2, 3

Integrals			
Function	**Integral**	**Comment**	**Papers**
x^n	$\frac{1}{n+1}\,x^{n+1}+c$	$n \neq -1$	1, 2, 3
x^{-1}	$\ln\lvert x\rvert + c$		1, 2, 3
$\cos x$	$\sin x + c$		1, 2, 3
$\sin x$	$-\cos x + c$		1, 2, 3
$\sinh x$	$\cosh x + c$		3
$\cosh x$	$\sinh x + c$		3
$\dfrac{1}{\sqrt{1-x^2}}$	$\sin^{-1} x + c$	$-1 < x < 1$	2, 3 •
$\dfrac{1}{1+x^2}$	$\tan^{-1} x + c$		2, 3 •
e^x	$\mathrm{e}^x + c$		1, 2, 3
$\mathrm{f}'(x) + \mathrm{g}'(x)$	$\mathrm{f}(x) + \mathrm{g}(x) + c$		1, 2, 3
$\mathrm{f}'\big(\mathrm{g}(x)\big)\,\mathrm{g}'(x)$	$\mathrm{f}\big(\mathrm{g}(x)\big) + c$		1, 2, 3
$\dfrac{\mathrm{f}'(x)}{\mathrm{f}(x)}$	$\ln\lvert\mathrm{f}(x)\rvert + c$		1, 2, 3 •
$\big(\mathrm{f}(x)\big)^n\,\mathrm{f}'(x)$	$\frac{1}{n+1}\big(\mathrm{f}(x)\big)^{n+1} + c$	$n \neq -1$	1, 2, 3 •
$u\dfrac{\mathrm{d}v}{\mathrm{d}x}$	$uv - \displaystyle\int v\frac{\mathrm{d}u}{\mathrm{d}x}\,\mathrm{d}x$	integration by parts	1, 2, 3 •

General calculus		
Formula	**Comment**	**Papers**
$\mathrm{f}'(x) = \lim\limits_{h\to 0} \dfrac{\mathrm{f}(x+h) - \mathrm{f}(x)}{h}$	first principles definition	1, 2, 3 •
$\dfrac{\mathrm{d}y}{\mathrm{d}x} = \dfrac{\mathrm{d}y}{\mathrm{d}t} \Big/ \dfrac{\mathrm{d}x}{\mathrm{d}t}$	for the parameterised curve $y = y(t)$, $x = x(t)$	1, 2, 3 •
Area under the curve $y = \mathrm{f}(x)$ and above the x-axis: $\displaystyle\int_a^b \mathrm{f}(x)\,\mathrm{d}x$		1, 2, 3
Volume of revolution about the x-axis: $\pi \displaystyle\int_a^b \big(\mathrm{f}(x)\big)^2\,\mathrm{d}x$		3
$\displaystyle\int_a^b y\,\mathrm{d}x \approx \tfrac{1}{2}h(y_0 + y_n) + h(y_1 + y_2 + \cdots + y_{n-1})$	$h = \dfrac{b-a}{n}$, $y_r = y(a + rh)$, trapezium rule	1, 2, 3 •
$\ddot{x} = -\omega^2 x \Rightarrow x = R\sin(\omega t + \alpha)$ or $x = R\cos(\omega t + \beta)$ or $x = A\cos\omega t + B\sin\omega t$	simple harmonic motion	3 •

Circles		
Formula	**Comment**	**Papers**
Length of an arc of a circle of radius r: $r\theta$	θ is angle subtended in radians	1, 2, 3
Area of a sector of a circle of radius r: $\frac{1}{2}r^2\theta$	θ is angle subtended in radians	1, 2, 3

Complex numbers		
Formula	**Comment**	**Papers**
$e^{i\theta} = \cos\theta + i\sin\theta$		3
$z = r(\cos\theta + i\sin\theta) \Rightarrow z^n = r^n(\cos n\theta + i\sin n\theta)$	de Moivre's theorem	3 •
$z^n = 1$ has roots $z = e^{2\pi ki/n}$ where $k = 0, 1, \ldots, (n-1)$	Roots of unity	3 •
Half line with end-point a: $\arg(z-a) = \theta$	θ is the angle between the line and a line parallel to the positive real axis	2, 3
Circle, centre a and radius r: $\lvert z-a\rvert = r$		2, 3

Vectors		
Formula	**Comment**	**Papers**
$\lvert x\mathbf{i} + y\mathbf{j} + z\mathbf{k}\rvert = \sqrt{x^2+y^2+z^2}$		1, 2, 3
$\mathbf{a}.\mathbf{b} = a_1b_1 + a_2b_2 + a_3b_3 = \lvert\mathbf{a}\rvert\,\lvert\mathbf{b}\rvert\cos\theta$	scalar product, $\mathbf{a} = a_1\mathbf{i} + a_2\mathbf{j} + a_3\mathbf{k}$	2, 3
$\mathbf{a}\times\mathbf{b} = (a_2b_3 - a_3b_2)\mathbf{i} + (a_3b_1 - a_1b_3)\mathbf{j} + (a_1b_2 - a_2b_1)\mathbf{k}$	vector product	3 •
$\lvert\mathbf{a}\times\mathbf{b}\rvert = \lvert\mathbf{a}\rvert\,\lvert\mathbf{b}\rvert\lvert\sin\theta\rvert$	θ is the acute angle between the vectors	3 •
Equation of the line through the point with position vector $\mathbf{a}$ parallel to $\mathbf{b}$: $\mathbf{r} = \mathbf{a} + t\mathbf{b}$		2, 3
Equation of the plane containing the point with position vector $\mathbf{a}$ and with normal $\mathbf{n}$: $(\mathbf{r}-\mathbf{a}).\mathbf{n} = 0$		3

Matrices		
Formula	**Comment**	**Papers**
For $\mathbf{A} = \begin{pmatrix} a & b \\ c & d \end{pmatrix}$, $\det \mathbf{A} = ad - bc$		2, 3
For $\mathbf{A} = \begin{pmatrix} a & b \\ c & d \end{pmatrix}$, $\mathbf{A}^{-1} = \frac{1}{\det \mathbf{A}} \begin{pmatrix} d & -b \\ -c & a \end{pmatrix}$	$\det \mathbf{A} \neq 0$	2, 3
$\mathbf{AB}$ is equivalent to $\mathbf{B}$ then $\mathbf{A}$	for transformations represented by these matrices	2, 3
$(\mathbf{AB})^{-1} = \mathbf{B}^{-1}\mathbf{A}^{-1}$	$\det \mathbf{AB} \neq 0$	2, 3
$\begin{pmatrix} 0 & \pm 1 \\ \pm 1 & 0 \end{pmatrix}$	reflection in the line $y = \pm x$	2, 3 •
$\begin{pmatrix} \cos\theta & -\sin\theta & 0 \\ \sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{pmatrix}$	rotation by θ about the z-axis; the direction of positive rotation is taken to be anticlockwise when looking towards the origin from the positive side of the axis of rotation	2, 3 •
$\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & -1 \end{pmatrix}$	reflection in the plane $z = 0$	2, 3 •

Mechanics		
Formula	**Comment**	**Papers**
mg	weight	1, 2, 3
$F \leqslant \mu R$	frictional force related to normal reaction R	1, 2, 3
$F = ma$	scalar version of Newton's second law; constant mass	1, 2, 3
$\mathbf{F} = m\mathbf{a}$	vector version of Newton's second law; constant mass	1, 2, 3
$\frac{1}{2}mv^2$	kinetic energy	2, 3 •
mgh	change in gravitational potential energy; h is vertical height	2, 3 •
mv	momentum	2, 3 •
$mv - mu$	impulse	2, 3 •
$T = \dfrac{\lambda x}{l} = kx$	Hooke's law	2, 3 •
$E = \dfrac{\lambda x^2}{2l} = \dfrac{1}{2}kx^2$	elastic potential energy	2, 3 •
$v = \dfrac{\mathrm{d}r}{\mathrm{d}t}, \quad a = \dfrac{\mathrm{d}v}{\mathrm{d}t} = \dfrac{\mathrm{d}^2r}{\mathrm{d}t^2}$	motion in a straight line (where acceleration, a, may not be constant)	1, 2, 3 •
$v = u + at, \quad s = ut + \frac{1}{2}at^2, \quad s = \frac{1}{2}(u+v)t, \quad v^2 - u^2 = 2as$	motion in a straight line with constant acceleration, a	1, 2, 3 •
$\mathbf{v} = \dfrac{\mathrm{d}\mathbf{r}}{\mathrm{d}t}\ \mathbf{a} = \dfrac{\mathrm{d}\mathbf{v}}{\mathrm{d}t}$	motion in two (STEP 1) or three (STEP 3) dimensions where acceleration, $\mathbf{a}$, may not be constant)	1, 2, 3 •
$\mathbf{v} = \mathbf{u} + \mathbf{a}t, \quad \mathbf{s} = \mathbf{u}t + \frac{1}{2}\mathbf{a}t^2, \quad \mathbf{s} = \frac{1}{2}(\mathbf{u}+\mathbf{v})t, \quad \mathbf{v}.\mathbf{v} - \mathbf{u}.\mathbf{u} = 2\mathbf{a}.\mathbf{s}$	motion in two (STEP 1) or three (STEP 2) dimensions with constant acceleration, $\mathbf{a}$	1, 2, 3 •
$v_1 - v_2 = -e(u_1 - u_2)$ or relative speed of separation $= e \times$ relative speed of approach	Newton's experimental law	2, 3 •
speed $= r\dot{\theta}$, radial acceleration $= \dfrac{v^2}{r} = r\dot{\theta}^2$ towards the centre, tangential acceleration $= r\ddot{\theta}$	motion in a circle of radius r	3 •

Probability/Statistics		
Formula	**Comment**	**Papers**
$\mathrm{P}(A \cup B) = \mathrm{P}(A) + \mathrm{P}(B) - \mathrm{P}(A \cap B)$	probability of the union of two events	1, 2, 3 •
$\mathrm{P}(A \cap B) = \mathrm{P}(A \mid B)\,\mathrm{P}(B)$	probability of the intersection of two events	1, 2, 3 •
$\mathrm{E}(aX + bY + c) = a\mathrm{E}(X) + b\mathrm{E}(Y) + c$	algebra of expectation	3 •
$\mathrm{Var}\,(aX + bY + c) = a^2\,\mathrm{Var}\,(X) + b^2\,\mathrm{Var}\,(Y)$	algebra of variances for independent variables	3 •
$\mu = \mathrm{E}(X) = \sum_i x_i \mathrm{P}(X = x_i)$	expectation of a discrete random variable X	1, 2, 3 •
$\mu = \mathrm{E}(X) = \int x\mathrm{f}(x)\,\mathrm{d}x$	expectation of a continuous random variable X with p.d.f. f	2, 3 •
$\sigma^2 = \mathrm{Var}\,(X) = \sum_i (x_i - \mu)^2 \mathrm{P}(X = x_i)$ $= \sum_i x_i^2\,\mathrm{P}(X = x_i) - \mu^2$	variance of a discrete random variable X	1, 2, 3 •
$\sigma^2 = \mathrm{Var}\,(X) = \int (x - \mu)^2 \mathrm{f}(x)\,\mathrm{d}x = \int x^2 \mathrm{f}(x)\,\mathrm{d}x - \mu^2$	variance of a continuous random variable X with p.d.f. f	2, 3 •
$\mathrm{F}(x) = \mathrm{P}(X \leqslant x) = \int_{-\infty}^{x} \mathrm{f}(x)\,\mathrm{d}x$	cumulative distribution function (c.d.f.)	2, 3 •

Random variables				
Distribution	$\mathrm{P}(X = x)$	$\mathrm{E}(X)$	$\mathrm{Var}\,(X)$	**Papers**
Binomial $\mathrm{B}(n, p)$	$\binom{n}{x} p^x (1-p)^{n-x}$	np	$np(1-p)$	1, 2, 3 •
Uniform distribution over 1, 2, …, n	$\frac{1}{n}$	$\frac{1}{2}(n+1)$	$\frac{1}{12}(n^2-1)$ (included for completeness; memorisation not required)	1, 2, 3 •
Poisson $\mathrm{Po}(\lambda)$	$\frac{\lambda^x \mathrm{e}^{-x}}{x!}$	λ	λ	2, 3 •

Distribution	p.d.f.	$\mathrm{E}(X)$	$\mathrm{Var}\,(X)$	**Papers**
Uniform distribution over $[a, b]$	$\frac{1}{b-a}$	$\frac{1}{2}(a+b)$	$\frac{1}{12}(b-a)^2$ (included for completeness; memorisation not required)	2, 3 •
Normal $\mathrm{N}(\mu, \sigma^2)$	$\frac{1}{\sqrt{2\pi\sigma^2}}\, \mathrm{e}^{-\frac{1}{2}\left(\frac{x-\mu}{\sigma}\right)^2}$ (included for completeness; memorisation not required)	μ	σ^2	1, 2, 3 •