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论文题目: The Frequency Analysis and Resonance
Simulation of Khoomei Singing

The Frequency Analysis and Resonance Simulation of Khoomei Singing

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Abstract

Khoomei is a Mongolian style of throat singing, originated in the inner Mongolia, Mongolia, Tuva, and Siberia. It is a crucial competent to the life and culture of the certain central Asian populations. One of the major features of Khoomei is that listeners can detect a high pitch sound and a low pitch simultaneously. The first goal of this article is to verify the weak middle frequency observation through physical means. Once observation is confirmed, we can proceed to find out a reason for phenomena through building models and testing on them. One of our hypothesis is that the position of the tongue is responsible for the weak middle frequency because it would shape the oral cavity and alter the resonance structure. To verify, we build 2-Dimensional and 3-Dimensional models of oral and nasal cavities with different positions of the tongue and apply wave equations to solve for the frequencies and their relative amplitudes at a certain point on the models. Using the above procedures, we were able to conclude that the position of the tongue is an important reason for the weak middle frequency observation.

Keywords: Khoomei; frequency spectrum; bionic simulation;

1. Introduction

1.1 Background



<http://www.truthsayer.info/wp-content/uploads/2014/10/6.jpg>

1.1.1 History

Khoomei is a Mongolian style of throat singing, originated in the inner Mongolia, Mongolia, Tuva, and Siberia. Mongolian nomads practice Khoomei at various occasions, such as tribal ceremonies and important household events. The history of Khoomei can be dated back to the ancestry of Mongolic people and their pastoral animism. Khoomei may have been developed from sound mimicry of nature's objects, which was seen as a way to connect with spiritual world. Khoomei is inherited orally from the bearer to the learner. Traditionally, Khoomei singers are mostly men. Besides from some biological disadvantages, women were discouraged by beliefs that singing Khoomei would give them difficulties during childbirths. The situation started to change around 1970s, when several female Khoomei singers started to perform publicly.

1.1.2 Styles

There had been many attempts trying to divide Khoomei into categories. A universally accepted classification is not available due to the various styles and groups that practice Khoomei. Here we introduce some of the most common and widely accepted classification of Khoomei.

In terms of acoustic effect, Khoomei can be generally grouped into two major categories: kharkhiraa and isgereee. In kharkhiraa, the vocalist sings the drone in normal voice while emphasizing the undertone that is one octave below. Kharkhiraa is further divided into two styles: Dag and Xovu. The Dag style features a deeper voice compared to Xovu. In isgereee, the vocalist emphasizes the overtone that is above the drone, adding a whistle-like sound.

In some areas, Khoomei is identified by the part of body that is responsible the most for the production of certain sound. Some of the most identified body parts include nasal cavity, pharynx, chest, palate, lips and tongue.

1.1.3 Artistic Values and Current Situations

The major characteristic of Khoomei is that one vocalist sounds more than one pitch simultaneously. Singing Khoomei involves the movements of multiple organs and structures: larynx, pharynx, oral cavity, nasal cavity, lips, etc. The differences in styles also arise from the distinct usage of these organs and structures. The word “Khoomei” literally means “pharynx” in Mongolian.

The Mongolian traditional art of Khoomei was inscribed in 2010 on the Representative List of the Intangible Cultural Heritage of Humanity of UNESCO. However, the art of Khoomei has been endangered due to its gradually decreasing popularity and inheritance. One of the major motivations of this research is to better understand the art of Khoomei and to provide some basic guidance to learners.

1.2 Music Theory and Physics

In this research, instead of focusing on Khoomei’s artistic values, I want to explore this vocal phenomenon in a physical approach. First, I will briefly summarize the how sound is described in physics.

1.2.1 Sound

Sound is the result of vibration at certain frequency in transmission media, such as gas, liquid and solid. Sound travels in longitudinal wave in air, which means the direction of displacement of air particles is the same as the direction of propagation of the wave. Sound wave is characterized by its wavelength, frequency and amplitude. The wavelength of a longitudinal wave is the distance of one compression and one rarefaction. The amplitude of the sound wave is the pressure between the undisturbed air and the maximum pressure caused by the wave. The frequency is the number of units of wavelength passing through during one second.

The relationship between wavelength(λ), frequency(f), and speed(v) is described in the formula: $v = \lambda f$.

1.2.2 Amplitude and Loudness

Greater amplitude produces louder sound, and smaller amplitude produces quieter sound. The loudness of sound is measured by decibel. Human ear can hear sound with decibel ranging from 0 dB to 194 dB. The normal speech voice has a loudness of about 60 to 70 dB.

1.2.3 Pitch and Frequency

Higher frequency produces high pitch sound, while lower frequency produces low pitch sound. Human ears are capable of detecting sound waves ranging from 20 Hz to 20,000 Hz, which can be broken down into seven frequency bands.

Frequency Range	Frequency Band (kHz)
Sub-Bass	0.02 – 0.06
Bass	0.06 – 0.25
Low Mid-range	0.25 – 0.50
Mid-range	0.50 – 2.00
Upper Mid-range	2.00 – 4.00
Presence	4.00 – 6.00
Brilliance	6.00 – 20.00

1.2.4 Music Theory

Concepts like frequency, wavelength, period indeed are closely related to concepts in music, for example, frequency is a measure of pitch. In western music theory, 12 notes with fixed frequencies are used. The central note is A4 which has a frequency of 440 Hz. The frequency of every other note follows the formula:

$$f = 2^{\frac{n}{12}} \times 440 \text{ Hz}$$

For example, the note C5 is 3 notes above the A4, so n is 3. From the formula, the frequency of C5 is 523 Hz and the frequency of C4 is, half of C5, 261 Hz.

An octave is the interval between two pitch whose frequencies are twice apart. For example, if a note has a frequency of 440 Hz, the same note one octave above has a frequency of 880 Hz and the same note one octave below has a frequency of 220 Hz.

Sound normally vibrate at more than one frequency. In other words, the sound signal we perceive in daily life is actually a superposition of components of various frequencies. The lowest resonant frequency is the fundamental frequency. An integer multiple of the fundamental frequency is the harmonic of the fundamental frequency. Other frequencies that are not integer multiples of the fundamental frequencies are call overtones.

1.3 Special Interests and Research Status

1.3.1 Observations

When listening to Khoomei singing, we often can detect a high pitch voice and a low pitch voice simultaneously. The difference between the two pitches is so distinguishable that it is reasonable to hypothesize that there is a weak middle frequency between the pitches in Khoomei singing.

While looking through some Khoomei learning materials, we noticed that the position of the tongue was especially emphasized, which is unusual for most singing

styles. Therefore, it is possible that the position of the tongue plays an important role in the weak middle frequency phenomenon.

1.3.2 Research Status Review

Most articles on the mechanics of Khoomei had been done from the perspective of either singing techniques or physiology.

In regard of singing techniques in general, most articles mainly address two important basic skills for all styles of Khoomei: breathing and the position of lips and tongue. Siqinbilige introduced the oral cavity as a “container” that is responsible for increasing the amplitude of the fundamental frequency sound. The tongue’s change of positions below the palate produce harmonic resonance. [1]

From a more physiological point of view, emphasis has been put on the position of the tongue as well. Theodore Levin and Michael Edgerton, in the article *The Throat Singers of Tuva*, conducted video fluoroscopy and nasoendoscopy on Khoomei singers who performed four different styles of Khoomei. In all the images, they had detected usual movements of the tongue, which they explained with shifting the frequency of a formant and aligning it with a harmonic. [2]

Many researches involve some physics but the extent vary. A few researches had been done using a strictly physical approach.

2. Methodology

2.1 Weak Middle Frequency

Firstly, the most fascinating part of Khoomei is the audience experience that two sounds come from one vocal performer. As I have proposed earlier, there is a weak middle frequency between the pitches in Khoomei singing. Therefore I employ frequency spectrum analysis to verify the observation. Auralization is defined to be a technique of creating audible sound files from numerical data. And the Auralization covers all cognitive aspects of sounds. [3] We took several audio clips that are about 5 seconds long and import them into python, which would then do the Fourier Transform on the audio information. When the audio signals are decomposed into different frequency regions, we plot the graph with x-axis indicating frequency in kHz and the y-axis indicating relative amplitude.

Fourier Transform decompose a time-dependent periodic function into a series of sine and cosine functions. Fast Fourier Transform(FFT) is an algorithm that accelerates this process. To plot the graph, FFT measures the amplitude of each individual sine and cosine function and plot their frequency relative to their amplitudes.

2.2 Pressure Field

Then comes the question, what accounts for such a spectrum? Sound wave fundamentally is the change of pressure field in spacetime. To solve the pressure field, we use the fact that pressure waves propagate according to the wave equation.

We employ the wave equation, along with boundary conditions determined by materials and shapes. Since it is not very likely to solve these equations by hand, we use Matlab to solve the partial differential equations.

Our hypothesis is that the position of the tongue plays an important role in Khoomei singing. To test this hypothesis by simulation, we made 2-Dimensional and 3-Dimensional models of the oral cavity. We will test the wave propagation in the models based on the wave equation.

3. Weak Middle Frequency

The following audio clips were imported into Python and analyzed by Fourier Transform (See Appendix → Python for Fourier Transform). Their graphs after processing are listed below: Khoomei Singing, “A” note, piano music and normal speech. The horizontal axis represents the frequencies in kHz and the vertical axis represents the relative amplitudes of these frequencies, with the greatest amplitude being scaled to 1.0.

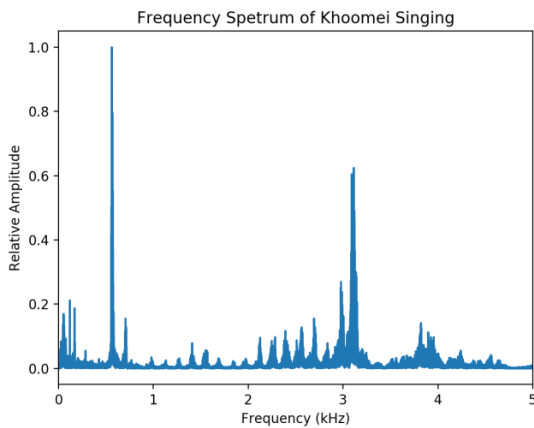


Figure 1

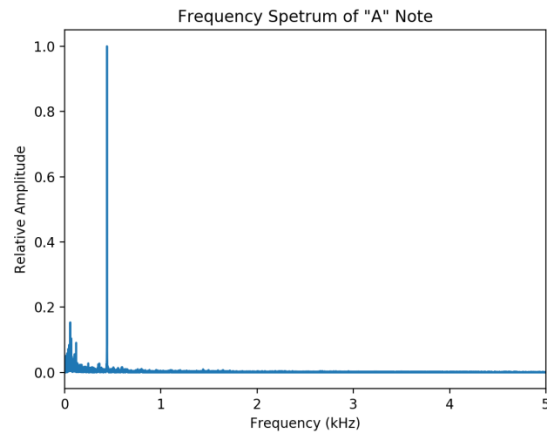


Figure 2

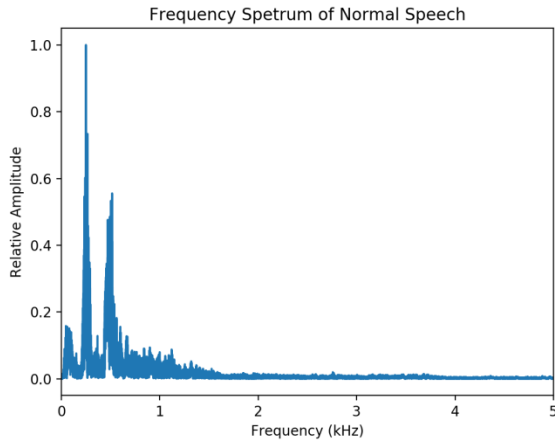


Figure 3

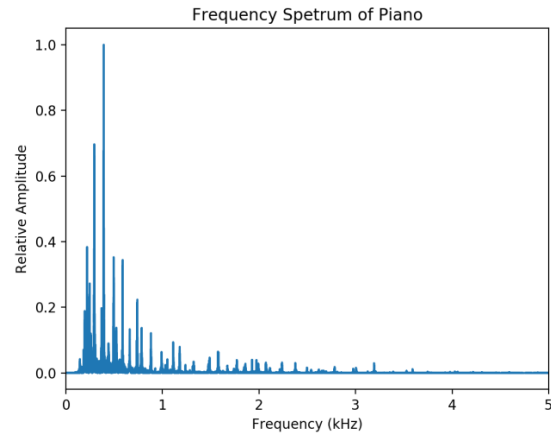


Figure 4

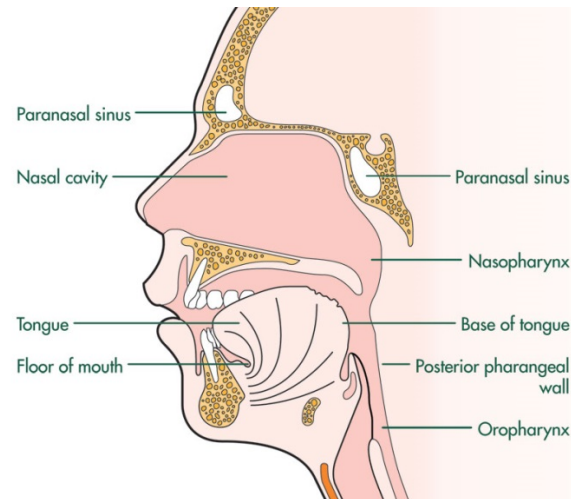
In figure 2, a standard “A” note has a singular frequency of 440Hz (0.44 kHz), which was shown in the graph. Other frequencies with very small amplitude are sources of errors that are negligible. In figure 3, a human voice produces frequencies between around 0.1 to 1 kHz. In figure 4, a piece of piano music has many frequencies ranging from around 0.1 to 1 kHz, which corresponds to a range between C3 to C6 on the piano keyboard. All three graphs show certain cohesion and consistency with the distribution of high-amplitude frequencies. In comparison, Figure 1 has a considerable gap between two high-amplitude frequencies respectively at around 0.6 and 3.1 kHz.

The graphs confirmed the observation that Khoomei singing has a relatively weak middle frequency compared to other music forms.

4. Experiment with 2-Dimensional Graph

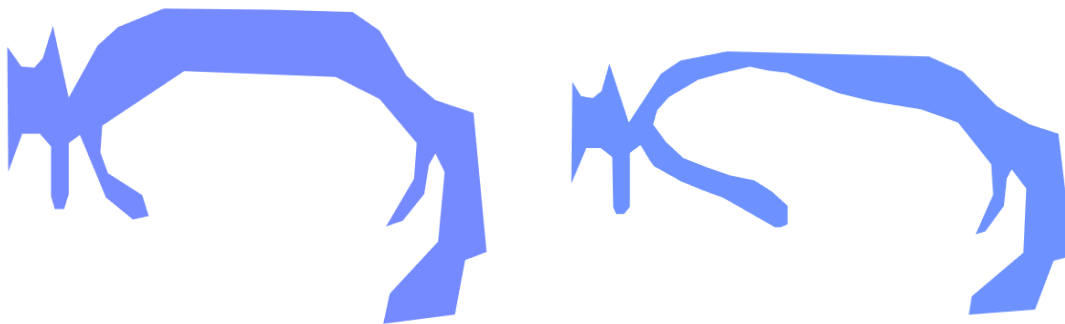
It is to be made clear at the beginning that the subject of interest of this research is the resonance in oral and nasal cavities. We do not concern ourselves with any other parts of the sound production and reception process.

Now we attempt to use 2-Dimensional side view graphs to explain the weak middle frequency observation.



http://www.macmillan.org.uk/Images/Cancerinfo/Longdescriptions/Cancertypes/Headandneckcrosssection_2011_large.jpg

For our particular purposes, we graph two situations in MATLAB pdeModeler (See Appendix → MATLAB → 2D Graph 1): the first one is when the tongue is at its normal position and the second situation is when the tongue touches the upper palate. (For the sake of simplicity, we did not take account of nasal cavity in the 2D phase.)



Sound propagation in pressure field follows the following equation:

$$\nabla^2 p = \frac{\partial^2 p}{\partial x^2} + \frac{\partial^2 p}{\partial y^2} + \frac{\partial^2 p}{\partial z^2} = \frac{1}{v^2} \frac{\partial^2 p}{\partial t^2}$$

where v is the speed of the propagation inside a certain material.

It also follows that:

$$p(x, y, z, t) = A(x, y, z) \cos(\omega t + \phi)$$

(See Mathematical Methods for Physics, H.W. Wyld, page 292) [3]

And then calculate the time derivative of the time part:

$$\frac{1}{v^2} \frac{\partial^2 p}{\partial t^2} = \frac{1}{v^2} \frac{\partial^2 (A(x, y, z) \cos(\omega t + \phi))}{\partial t^2}$$

$$\frac{1}{v^2} \frac{\partial^2 p}{\partial t^2} = \frac{A(x, y, z)}{v^2} \frac{\partial^2 (\cos(\omega t + \phi))}{\partial t^2}$$

Since the term $\cos(\omega t + \phi)$ carries no spatial dependence, the partial derivative is equivalent to the total derivative over time:

$$\frac{1}{v^2} \frac{\partial^2 p}{\partial t^2} = \frac{A(x, y, z)}{v^2} \frac{d^2 (\cos(\omega t + \phi))}{dt^2}$$

$$\frac{1}{v^2} \frac{\partial^2 p}{\partial t^2} = \frac{A(x, y, z)}{v^2} (-\omega \sin(\omega t + \phi))$$

$$\frac{1}{v^2} \frac{\partial^2 p}{\partial t^2} = \frac{A(x, y, z)}{v^2} (-\omega^2 \cos(\omega t + \phi))$$

$$\frac{1}{v^2} \frac{\partial^2 p}{\partial t^2} = \frac{-\omega^2}{v^2} p$$

Therefore the separation of variables gives:

$$\nabla^2 p = -\frac{\omega^2}{v^2} p$$

$$\nabla^2 (A \cos(\omega t + \phi)) = -\frac{\omega^2}{v^2} A \cos(\omega t + \phi)$$

By definition of wave quantities, $\omega = \frac{2\pi}{T}$ and $\lambda = Tv$, where λ is the wavelength of the sound wave and T the period, we have :

$$\frac{\omega^2}{v^2} = \frac{4\pi^2}{\lambda^2}$$

Divide both side by the time term $\cos(\omega t + \phi)$:

$$\nabla^2 A = -\frac{4\pi^2}{\lambda^2} A$$

$$\nabla^2 A + \frac{4\pi^2}{\lambda^2} A = 0$$

This is the equation that tells the amplitude of the pressure field at any point in the region of interest.

Thus, the values of coefficients in PDE Specification in MATLAB follows that:

$$\begin{cases} c = -1 \\ a = 4\pi^2 \\ f = 0 / \lambda^2 \\ m = 0 \\ d = 0 \end{cases}$$

with the relationship $m \frac{\partial^2 u}{\partial t^2} + d \frac{\partial u}{\partial t} - \nabla \cdot (c \nabla u) + au = f$

Human ears are capable of detecting sounds of wavelength ranging from 0.017m to 17m. According to the equation above, the value of a ranges from 136603.52 Hz to 0.1366Hz.

To apply boundary conditions, we consider three cases: glottis, the opening of the mouth, and the surface that encloses the cavity space. For the glottis and the opening of the mouth, the boundary conditions should be set to Dirichlet values for they are relatively fixed. The boundary condition at the opening of the mouth is actually not given a value because it is the value that we observe. The boundary condition of glottis is given a fixed value that is random for all amplitude are relative in our cases.

Glottis: $\begin{cases} h = 1 \\ r = 100 \end{cases}$ satisfies that $h * u = r$

Here we consider glottis as a source of sound waves, and r is a reference amplitude. It is not necessary to consider the oscillation process that took place since we only take account that produced sound wave.

At the opening of the mouth: $h = 0, r = 0$ which means u is not set to any value. Even though in real life, there is a distance the sound wave has to travel from the speaker's mouth to the listener's ear, traveling in air doesn't for a short distance does not alter the composition of sound waves. Thus, it also makes sense to acquire data at the opening of the mouth.

The surface that encloses the cavity space is different in that there is no motion normal to the surface. Suppose p represents the pressure inside the cavities and ρ represents air density. If we take partial derivatives normal to the wall, we get $\frac{\partial p}{\partial n} = \frac{\partial \rho}{\partial n} = 0$.

Since equation follows Newton's Second Law, we get

$$\begin{aligned} \rho_0 \frac{\partial v}{\partial t} &= -\nabla p \\ \rho_0 \frac{\partial v}{\partial t} + \nabla p &= 0 \end{aligned}$$

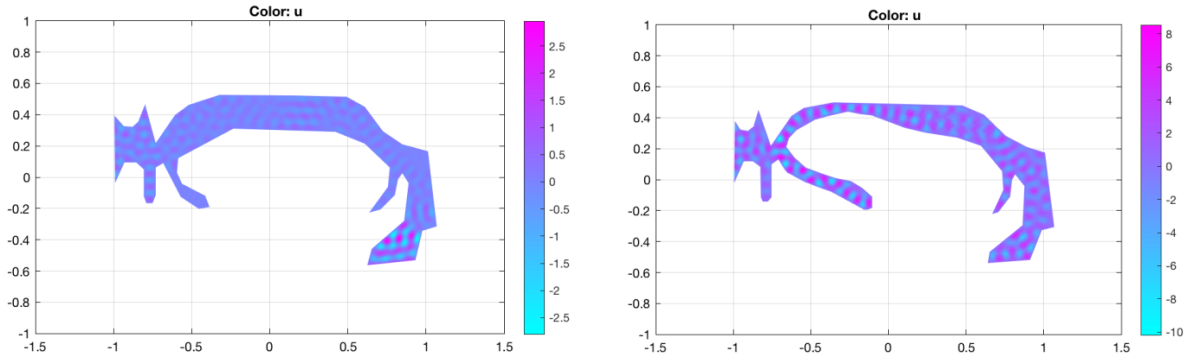
To separate the time dependence,

$$\rho(r, t) = \sum_{n=1}^n u_n(r) [A_n \sin k_n c t + B_n \cos k_n c t]$$

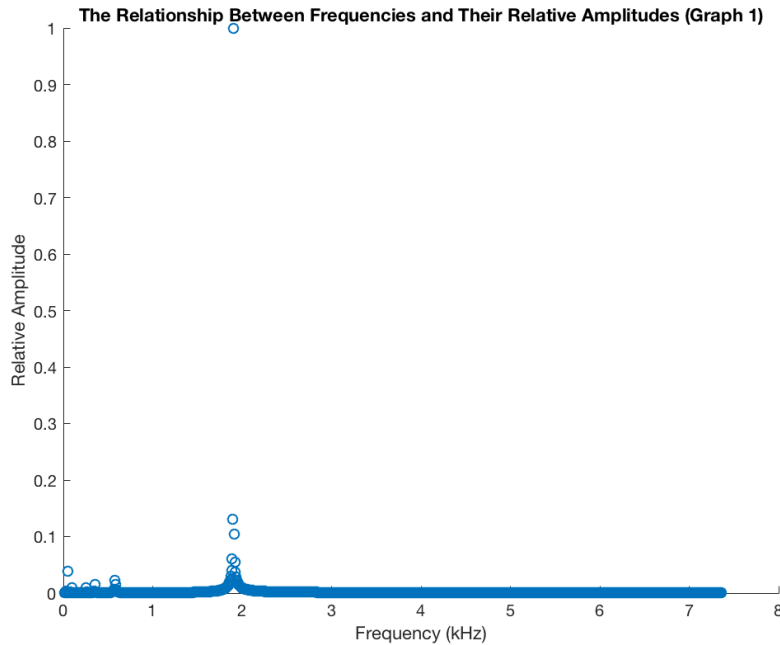
$$\begin{aligned}\nabla^2 u_n(r) + k_n^2 u_n(r) &= 0 \\ \vec{n} \cdot \nabla u_n(r) &= 0\end{aligned}$$

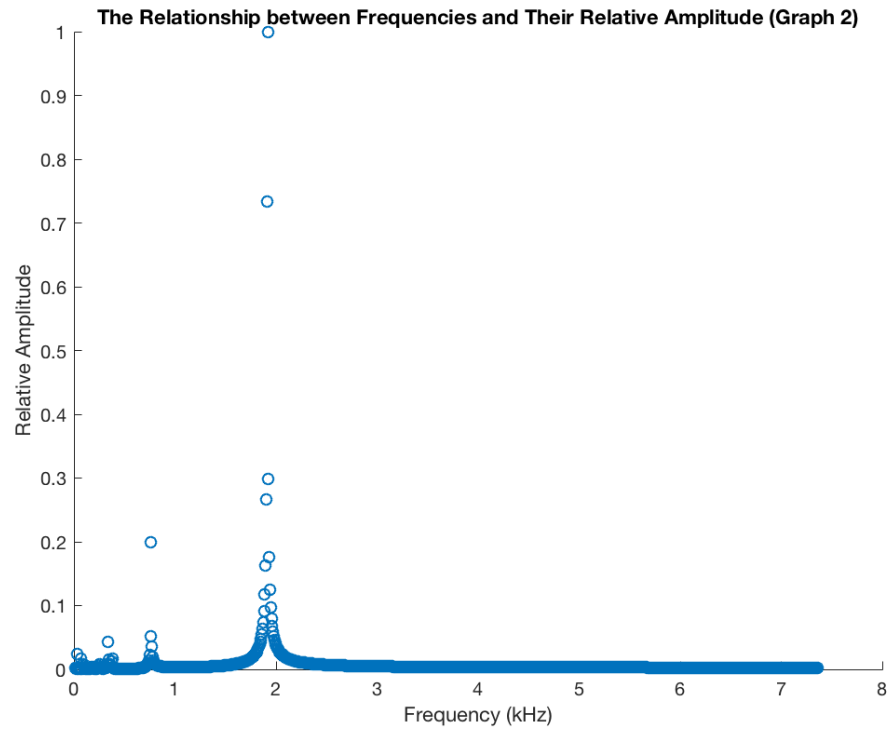
On the surfaces of the cavities, $\begin{cases} g = 0 \\ q = 0 \end{cases}$ satisfies that $n \cdot (\nabla u) + qu = g$

The following graphs demonstrate the pressure field in Graph1 and Graph 2 at $a = 8000$ or $\lambda = 0.07m$



To view the difference among all the frequencies at a single point near the opening of the mouth, we made the following graphs whose x-axis represents a scaled wavelength and y-axis represents amplitude at that frequency. The scale of the x-axis is 1:5cm. (See Appendix → MATLAB → 2D Graph 1 PDE Solution)

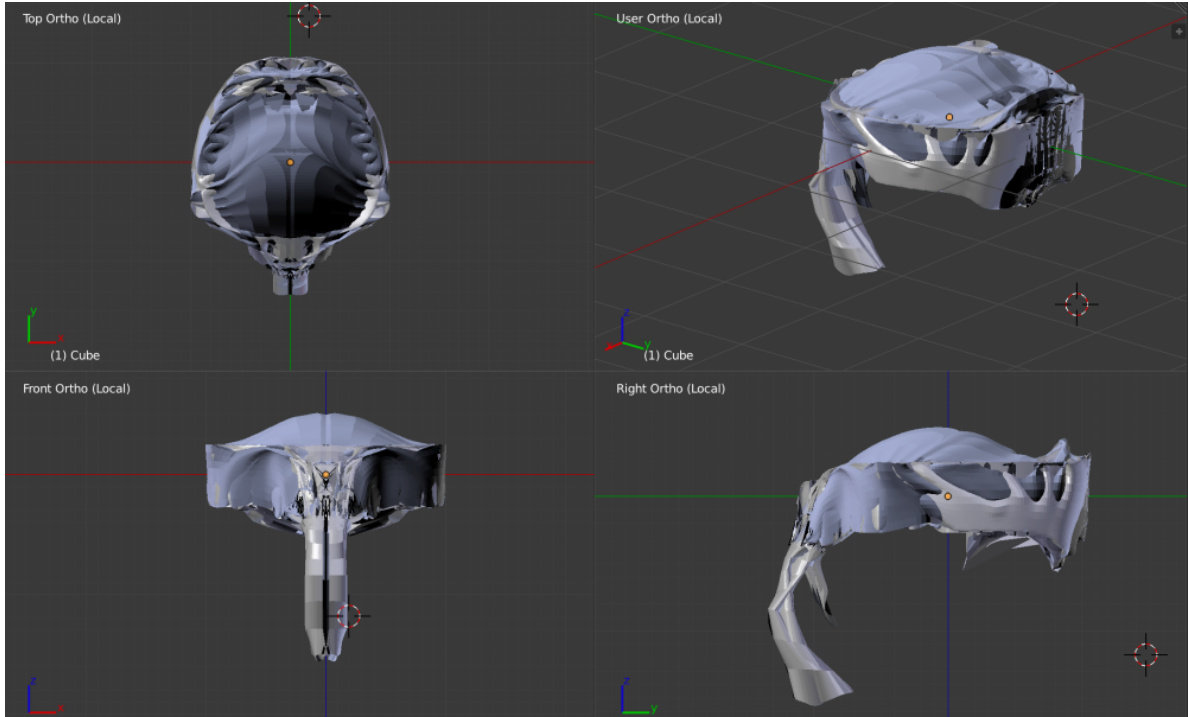




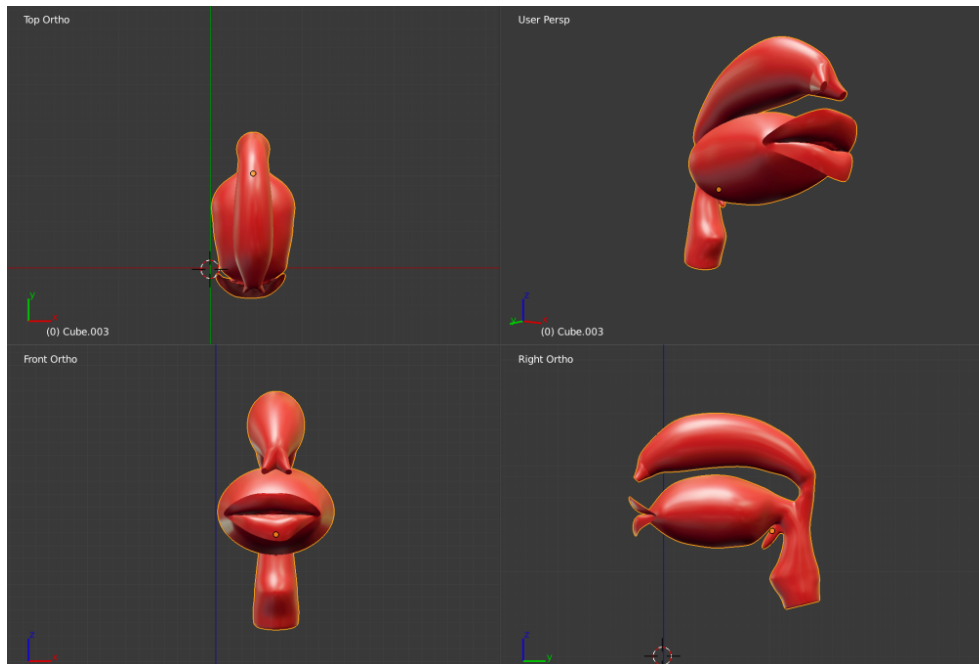
5. Experiment with 3-Dimensional Graphs

There is not any existing 3-Dimensional template for oral and nasal cavities. To achieve such a graph, we construct the shape in the app Blender based on a few 2-D graphs. There are certain difficulties in constructing a 3-D model since most of our information lacks a dimension. To verify the accuracy of the model, an otolaryngologist had been consulted.

We have two versions of models. The first one is strictly based on our real oral and nasal cavity structures. However, the high accuracy of the detailed model seems to make it very difficult for the equation solving process. Thus, a second simplified model was made to achieve the same purpose. The two models are displayed below.



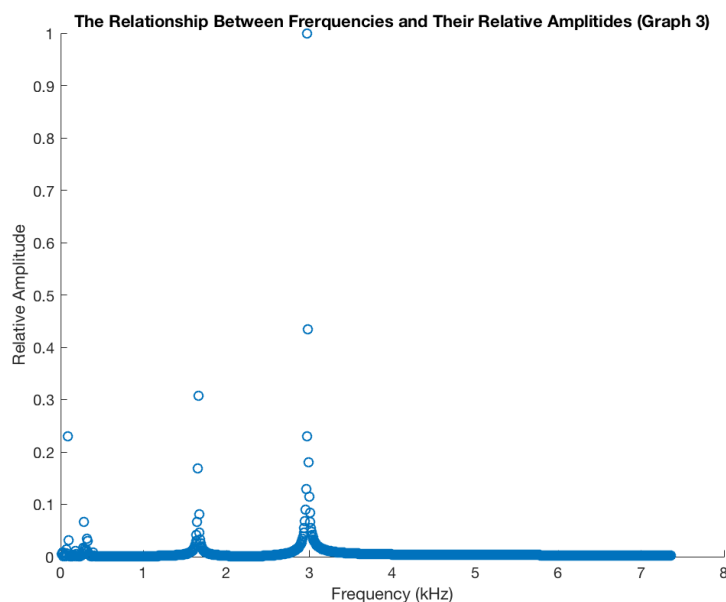
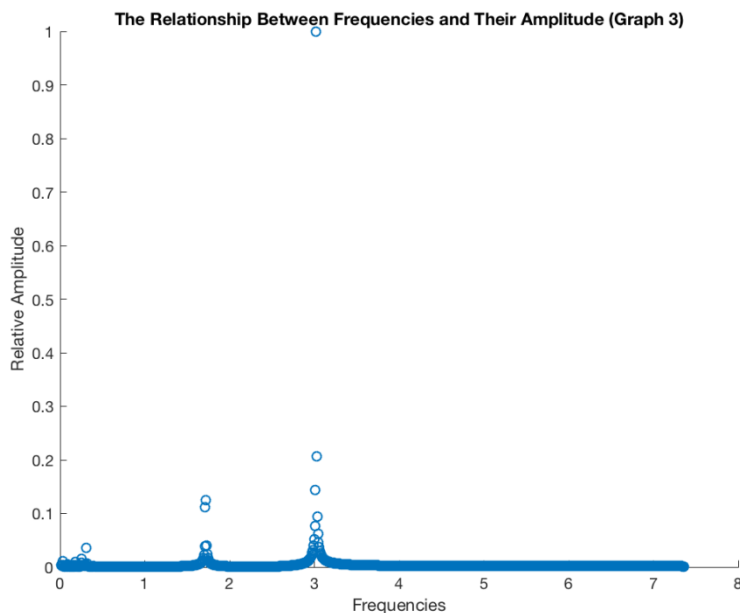
3-Dimensional Oral and Nasal Cavities (Original Version) Software: Blender



3-Dimensional Oral and Nasal Cavities (Simplified Version) Software: Blender

We use the open source 3D creation software Blender to complete the above model. Since wave equations are appropriate for modeling, we are able to use a tool for developing acoustic scenes such as Blender. [5] The model is exported as a stl. file and imported into MATLAB, where we

repeat the process explained in the previous section. (See Appendix → MATLAB → 3D Graph 3 PDE Solution) To view the difference among all frequencies at a single point near the opening of the mouth, we made the following graphs whose x-axis represents a scaled wavelength and y-axis represents amplitude at that frequency. The scale of the x-axis is 1:10cm.



6. Results and Conclusion

6.1 Analysis of Results

In the 2-Dimensional Graph 1, we can see that there is only one single apparent high-amplitude frequency at around 2 kHz. In the 2-Dimensional Graph 2, we can see that there are two relatively apparent amplitude: a relative amplitude of 1 at around 2 kHz and a relative amplitude of 0.2 at around 0.8 kHz. In the 3-Dimensional Graph 3, we can observe high amplitude frequencies at around 1.7 KHz with a relative amplitude of 0.16 and a at 3 kHz with a relative amplitude of 1. In the 3-Dimensional Graph 4, there are both visible high amplitude frequencies at around 1.7 KHz with a relative amplitude of 0.35 and at 3 kHz with a relative amplitude of 1.

Both sets of graphs, to certain extent, demonstrated that the observation that there exists a weak middle frequency in Khoomei Singing. However, comparatively speaking, the 3-D model better confirms this observation.

Through the simulations, we are able to confirm that the uniqueness of Khoomei lies in the different positions of the tongue. The tongue changes the resonance cavity of the sound wave, thus effecting the frequencies.

6.2 Conclusion

This research has let us come to several important conclusions. The tools that have helped us to achieve the confirmation of our hypothesis could also be used in many other occasions, either for future researches or for educational purposes. First, we came to see the importance of frequency spectrums, which is the scientific way to collect audio information. For ancient and less known singing styles such as Khoomei, the appliance of science and modern technology to analyze and to record seem highly beneficial. The application side does not restrict itself to music forms; frequency spectrum applies to all audio information such as languages and musical instruments.

One of our initial motivation was to build a model that can help Khoomei learners to better understand the singing mechanics. Our attempt at first to find existing oral and nasal cavity templates for simulation failed as we realized that no one has done such a model before. The originality of the model provides a new approach to the existing literatures and other researches, which may come across similar problems in the future. The same process of frequency spectrum, model building, and simulation to analyze and record has broad applications, such as preserving endangered languages, fostering the learning of foreign languages, and understanding the body structures and living habits of animals that make special sounds.

Most importantly, we have confirmed our observation and hypothesis from the very beginning. Though there had been other attempts to do so, none of them includes a strictly physical approach. Our take on the subject adopts the physical approach, which also contributes to the existing research.

6.3 Discussion for Improvements

There are several places that can be improved in our experiment. The first possible improvement concerns the accuracy of the model. As we discussed earlier, the accuracy of the 3-Dimensional model is compromised by our limited means of solving partial differential equations. We would receive better results if there are means to solve the equations while retaining the accuracy of the model.

Another flaw with our current simulation is that we did not take into account the characteristics of different biological materials of the surfaces. For the sake of simplicity, we treat all surfaces as if they would only experience pure reflection, ignoring the fact that depending on the material, surfaces can selectively absorb, reflect or deflect.

Last but not least, the fact that we treated the source of sound waves as a boundary condition also deserves some thought for improvements in the future. Our choice was limited by the knowledge on the mechanics of glottis, vocal folds and other complicated speech organs. If we can get hold of more accurate input for the wave propagation equation, the accuracy of the overall result can have a significant improvement.

Reference

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- [3] Vorlander, Michael. *Auralization Fundamentals of Acoustics, Modeling, Simulation, Algorithms and Acoustics, and Acoustic Virtual Reality*. Springer-Verlag Berlin Heidelberg, 2008.
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- [5] van Mourik, Jelle; Murphy, Damian. *Geometric and Wave-Based Acoustic Modelling Using Blender*. February 2013

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Appendix

1. Python for Fourier Transform

```
import wave
import pylab as pl
import numpy as np
from numpy import fft as fft
import matplotlib

import subprocess

# convert to wav
command = "ffmpeg -i ANote.m4a -ab 160k -ac 2 -ar 44100 -vn ANote.wav"
subprocess.call(command, shell=True)

# read the file
f = wave.open("testpiano.wav")
nframes = f.getnframes()
framerate = f.getframerate()
str_data = f.readframes(nframes)
f.close

wave_data = np.fromstring(str_data, dtype = np.short) # returns an array of data type short
if (f.getnchannels() == 2):
    wave_data.shape = -1, 2
    wave_data = wave_data.T
time = np.arange(0, nframes) * (1.0 / framerate)

#Fourior Transformation
if(f.getnchannels()==2):
    fourior = np.fft.fft(wave_data[0])
    n = len(wave_data[0])
else:
    fourior = np.fft.fft(wave_data)
    n = len(wave_data)

# plotting
fourior = fourior[0:(n/2)]
fourior = fourior / float(n)
freqArray = np.arange(0, (n/2), 1.0) * (44100*1.0/n)

a=abs(10*fourior)
aMax = max(a)
b=a/aMax

pl.plot(freqArray/1000, b)
axes = pl.gca()
axes.set_xlim([0,5])
pl.xlabel('Frequency (kHz)')
pl.ylabel('Relative Amplitude')
pl.title('Frequency Spetrum of Piano')
pl.show()
```

2. MATLAB

2D Graph 1

```

3. function pdemodel
4. [pde_fig,ax]=pdeinit;
5. pdetool('appl_cb',1);
6. set(ax,'DataAspectRatio',[1 1 1]);
7. set(ax,'PlotBoxAspectRatio',[1.5 1 1]);
8. set(ax,'XLim',[-1.5 1.5]);
9. set(ax,'YLim',[-1 1]);
10. set(ax,'XTickMode','auto');
11. set(ax,'YTickMode','auto');
12. pdetool('gridon','on');
13.
14. % Geometry description:
15. pdepoly([-0.9943257676902536,...
16. -0.93424566088117489,...
17. -0.8781708945260347,...
18. -0.84212283044058744,...
19. -0.79806408544726293,...
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22. -0.51769025367156196,...
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25. 0.49165554072096107,...
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27. 0.72396528704939955,...
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29. 1.0123497997329776,...
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31. 0.97630173564753031,...
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36. 0.88818424566088172,...
37. 0.84813084112149584,...
38. 0.82009345794392541,...
39. 0.80006675567423269,...
40. 0.70794392523364502,...
41. 0.63584779706275052,...
42. 0.75600801068090817,...
43. 0.76802403204272407,...
44. 0.60781041388518053,...
45. 0.41955941255006679,...
46. -0.23331108144192259,...
47. -0.58578104138851805,...
48. -0.5937917222963951,...
49. -0.56174899866488648,...
50. -0.41355140186915884,...
51. -0.38551401869158863,...
52. -0.45360480640854473,...
53. -0.56975967957276374,...
54. -0.68190921228304391,...
55. -0.72997329773030706,...
56. -0.72997329773030706,...
57. -0.75,...
58. -0.79005340453938588,...
59. -0.80607476635514019,...
60. -0.80607476635514019,...
61. -0.85413885180240312,...
62. -0.93024032042723626,...
63. -0.99032042723631508,...
64. ],...
65. [ 0.39452603471295045,...
66. 0.3264352469959948,...
67. 0.32242990654205617,...
68. 0.35447263017356501,...
69. 0.46662216288384517,...
70. 0.21829105473965305,...
71. 0.3985313751668893,...
72. 0.46261682242990654,...
73. 0.526702269692924,...
74. 0.52269692923898559,...
75. 0.51468624833110832,...
76. 0.45060080106809108,...

```

```
77. 0.29439252336448618,...
78. 0.210280373831776,...
79. 0.16622162883845149,...
80. -0.31441922563417868,...
81. -0.34245660881174866,...
82. -0.53070761014686219,...
83. -0.5627503337783708,...
84. -0.45861148197596768,...
85. -0.27837116154873143,...
86. -0.038050734312416346,...
87. 0.026034712950601113,...
88. -0.014018691588784771,...
89. -0.11415220293724948,...
90. -0.20627503337783692,...
91. -0.22630173564752987,...
92. -0.06208277703604792,...
93. 0.062082777036048364,...
94. 0.21428571428571441,...
95. 0.29038718291054755,...
96. 0.31041388518024049,...
97. 0.12216288384512697,...
98. 0.030040053404539524,...
99. -0.042056074766354978,...
100. -0.11815754339118811,...
101. -0.19025367156208262,...
102. -0.20226969292389829,...
103. -0.12616822429906516,...
104. 0.09012016021361835,...
105. 0.062082777036048364,...
106. -0.11415220293724948,...
107. -0.16622162883845104,...
108. -0.16622162883845104,...
109. -0.12216288384512652,...
110. 0.050066755674232466,...
111. 0.094125500667556983,...
112. 0.094125500667556983,...
113. -0.038050734312416346,...
114. ],...
115. 'P1');
116. set(findobj(get(pde_fig,'Children'),'Tag','PDEEval'),'String','P1')
117.
118. % Boundary conditions:
119. pdetool('changemode',0)
120. pdesetbd(49,...
121. 'dir',...
122. 1,...
123. '0',...
124. '0')
125. pdesetbd(48,...
126. 'dir',...
127. 1,...
128. '1',...
129. '0')
130. pdesetbd(47,...
131. 'dir',...
132. 1,...
133. '1',...
134. '0')
135. pdesetbd(46,...
136. 'dir',...
137. 1,...
138. '1',...
139. '0')
140. pdesetbd(45,...
141. 'dir',...
142. 1,...
143. '1',...
144. '0')
145. pdesetbd(44,...
146. 'dir',...
147. 1,...
148. '1',...
149. '0')
150. pdesetbd(43,...
151. 'dir',...
152. 1,...
153. '1',...
154. '0')
```

```
155. pdesetbd(42,...
156. 'dir',...
157. 1,...
158. '1',...
159. '0')
160. pdesetbd(41,...
161. 'dir',...
162. 1,...
163. '1',...
164. '0')
165. pdesetbd(40,...
166. 'dir',...
167. 1,...
168. '1',...
169. '0')
170. pdesetbd(39,...
171. 'dir',...
172. 1,...
173. '1',...
174. '0')
175. pdesetbd(38,...
176. 'dir',...
177. 1,...
178. '1',...
179. '0')
180. pdesetbd(37,...
181. 'dir',...
182. 1,...
183. '1',...
184. '0')
185. pdesetbd(36,...
186. 'dir',...
187. 1,...
188. '1',...
189. '0')
190. pdesetbd(35,...
191. 'dir',...
192. 1,...
193. '1',...
194. '0')
195. pdesetbd(34,...
196. 'dir',...
197. 1,...
198. '1',...
199. '0')
200. pdesetbd(33,...
201. 'dir',...
202. 1,...
203. '1',...
204. '0')
205. pdesetbd(32,...
206. 'dir',...
207. 1,...
208. '1',...
209. '0')
210. pdesetbd(31,...
211. 'dir',...
212. 1,...
213. '1',...
214. '0')
215. pdesetbd(30,...
216. 'dir',...
217. 1,...
218. '1',...
219. '0')
220. pdesetbd(29,...
221. 'dir',...
222. 1,...
223. '1',...
224. '0')
225. pdesetbd(28,...
226. 'dir',...
227. 1,...
228. '1',...
229. '0')
230. pdesetbd(27,...
231. 'dir',...
232. 1,...
```

```
233.         '1',...
234.         '0')
235.     pdesetbd(26,...
236.         'dir',...
237.         1,...
238.         '1',...
239.         '0')
240.     pdesetbd(25,...
241.         'dir',...
242.         1,...
243.         '1',...
244.         '0')
245.     pdesetbd(24,...
246.         'dir',...
247.         1,...
248.         '1',...
249.         '0')
250.     pdesetbd(23,...
251.         'dir',...
252.         1,...
253.         '1',...
254.         '0')
255.     pdesetbd(22,...
256.         'dir',...
257.         1,...
258.         '1',...
259.         '0')
260.     pdesetbd(21,...
261.         'dir',...
262.         1,...
263.         '1',...
264.         '0')
265.     pdesetbd(20,...
266.         'dir',...
267.         1,...
268.         '1',...
269.         '0')
270.     pdesetbd(19,...
271.         'dir',...
272.         1,...
273.         '1',...
274.         '0')
275.     pdesetbd(18,...
276.         'dir',...
277.         1,...
278.         '1',...
279.         '1')
280.     pdesetbd(17,...
281.         'dir',...
282.         1,...
283.         '1',...
284.         '0')
285.     pdesetbd(16,...
286.         'dir',...
287.         1,...
288.         '1',...
289.         '0')
290.     pdesetbd(15,...
291.         'dir',...
292.         1,...
293.         '1',...
294.         '0')
295.     pdesetbd(14,...
296.         'dir',...
297.         1,...
298.         '1',...
299.         '0')
300.     pdesetbd(13,...
301.         'dir',...
302.         1,...
303.         '1',...
304.         '0')
305.     pdesetbd(12,...
306.         'dir',...
307.         1,...
308.         '1',...
309.         '0')
310.     pdesetbd(11,...
```



```
311.     'dir',...
312.     1,...
313.     '1',...
314.     '0')
315. pdesetbd(10,...
316.     'dir',...
317.     1,...
318.     '1',...
319.     '0')
320. pdesetbd(9,...
321.     'dir',...
322.     1,...
323.     '1',...
324.     '0')
325. pdesetbd(8,...
326.     'dir',...
327.     1,...
328.     '1',...
329.     '0')
330. pdesetbd(7,...
331.     'dir',...
332.     1,...
333.     '1',...
334.     '0')
335. pdesetbd(6,...
336.     'dir',...
337.     1,...
338.     '1',...
339.     '0')
340. pdesetbd(5,...
341.     'dir',...
342.     1,...
343.     '1',...
344.     '0')
345. pdesetbd(4,...
346.     'dir',...
347.     1,...
348.     '1',...
349.     '0')
350. pdesetbd(3,...
351.     'dir',...
352.     1,...
353.     '1',...
354.     '0')
355. pdesetbd(2,...
356.     'dir',...
357.     1,...
358.     '1',...
359.     '0')
360. pdesetbd(1,...
361.     'dir',...
362.     1,...
363.     '1',...
364.     '0')
365.
366. % Mesh generation:
367. setappdata(pde_fig,'Hgrad',1.3);
368. setappdata(pde_fig,'refinemethod','regular');
369. setappdata(pde_fig,'jiggle',char('on','mean',''));
370. setappdata(pde_fig,'MesherVersion','preR2013a');
371. pdetool('initmesh')
372. pdetool('refine')
373. pdetool('refine')
374.
375. % PDE coefficients:
376. pdeseteq(1,...
377.     '-1',...
378.     '0',...
379.     '0',...
380.     '1.0',...
381.     '0:10',...
382.     '0.0',...
383.     '0.0',...
384.     '[0 100]')
385.
386. setappdata(pde_fig,'currparam',...
387.     ['-1';...
388.     '5000';...
```

```

389.     '0';...
390.     '1.0 '])
391.     display(a)
392.
393.
394.
395.
396.     % Solve parameters:
397.     setappdata(pde_fig,'solveparam',...
398.     char('0','5616','10','pdeadworst',...
399.     '0.5','longest','0','1E-4','','fixed','Inf'))
400.
401.     % Plotflags and user data strings:
402.     setappdata(pde_fig,'plotflags',[1 1 1 1 1 1 1 1 0 0 0 1 1 0 0 0 0 1]);
403.     setappdata(pde_fig,'colstring','');
404.     setappdata(pde_fig,'arrowstring','');
405.     setappdata(pde_fig,'deformstring','');
406.     setappdata(pde_fig,'heightstring','');
407.
408.
409.     % Solve PDE:
410.     pdetool('solve')
411.
412.     pdetool('exportsolution')

```

2D Graph 1 PDE Solution

```

413.     for i=1:1:N;
414.         j=0.05+0.05*i;
415.         output(i,1)=j;
416.         specifyCoefficients(model,'m',0,...
417.             'd',0,...
418.             'c',-1,...
419.             'a',4*(pi/j)^2,... %this is the wavelength part
420.             'f',0);
421.         result = solvepde(model);
422.         querypoints = [-0.95,0.2]';
423.         output(i,2)=abs(interpolateSolution(result,querypoints));
424.
425.
426.     end
427.     outputMax = max(output(:,2));
428.     scatter(output(:,1)*0.147,output(:,2)/outputMax);

```

3D Graph 1 PDE Solution

```

429.     model = createpde();
430.     importGeometry(model,'cavi_bend.stl')
431.
432.     pdegplot(model,'FaceLabels','on','FaceAlpha',0.1)
433.     applyBoundaryCondition(model,'dirichlet','Face',9,'r',100,'h',1);
434.     applyBoundaryCondition(model,'dirichlet','Face',10,'r',0,'h',0); % 100% reflection; you
can make it a mix b.condition
435.     applyBoundaryCondition(model,'neumann','Face',1:8,'g',0,'q',0);
436.     applyBoundaryCondition(model,'neumann','Face',11:14,'g',0,'q',0);
437.
438.     generateMesh(model);
439.
440.     N=1000;
441.     output=zeros(N,2);
442.
443.     for i=1:1:N;
444.         j=0.05+0.05*i;
445.         output(i,1)=j;
446.         specifyCoefficients(model,'m',0,...
447.             'd',0,...
448.             'c',-1,...
449.             'a',4*(pi/j)^2,... %this is the wavelength part
450.             'f',0);
451.         result = solvepde(model);
452.         querypoints = [0.5,-0.5,14.5]';
453.         output(i,2)=abs(interpolateSolution(result,querypoints));
454.     end
455.     outputMax = max(output(:,2));
456.     scatter(output(:,1)*0.147,output(:,2)/outputMax);

```

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