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Investigation on the dynamics of friction induced reciprocation

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ABSTRACT

We study the friction induce reciprocation motion of a ring with rigid body dynamics. When rotate a horizontal oiled shaft around its axis and ring a ring on the shaft, the relative motion between the ring and shaft will lead to friction forces along the axis. Under the action of friction forces, supporting force and gravitational force, the ring will move along the axis and rotate about the contact point between the ring and shaft. The direction of friction along the axis changes as the ring rotate, leading to reciprocation motion. The experiment show that there is a critical thickness of the ring, beyond which the amplitude of reciprocation motion does not decay with time. We employ rigid body dynamics to model the motion of ring. The simulation is in good agreement with experiments.

Keywords:

Rigid body dynamics; friction force; reciprocating motion

Content

1 INTRODUCTION

1.1 Introduction

According to our intuition, when ringing a ring on a horizontal rotating oiled shaft and release ring from rest, the friction force at the contact point is perpendicular to the axis of the shaft. Under the action of friction force, the ring would begin rotating in same direction as the shaft, but would not move along the axis. But in our experiment, the ring moves back and forth along the axis of the shaft, as showed in the video. When the thickness of the ring is beyond a critical value, the amplitude of motion keeps constant, even though the friction force is supposed to lead to dissipation of kinetic energy. The main purpose of the present paper is to explain the mechanism of the reciprocation motion.

1.2 List variables

$\boldsymbol{m}$-The mass of the ring
$\boldsymbol{\rho}$-the density of the ring
$\boldsymbol{R}$-The Outer diameter of the ring
$\mathbf{d}$-The thickness of the ring
$\boldsymbol{r}$-the diameter of the shaft
$\mathbf{k}$-The coefficient of friction
$\boldsymbol{f}$-The friction force
$\mathbf{I}$-The rotation inertia about different axis
$\boldsymbol{\Omega}$-The angular velocity of the rotation frame
$\boldsymbol{\omega}$-The angular velocity of the shaft
$\boldsymbol{M}$-The turning torque
$\boldsymbol{V_z}$-The translational velocity
$\boldsymbol{V_r}$-The rotation velocity
$\Delta\boldsymbol{v}$-The relative velocity between the ring and the shaft
$\boldsymbol{\theta}$-The angle between the z-axis and the projection of rotation axis of the cardboard ring in the y-z plane
$\boldsymbol{\delta}$- the angle between the angular velocity vector and the y-z plane
$\boldsymbol{\varphi}$-The rotation angle about the axis of the ring of the ring
$\boldsymbol{\alpha}$-The angle between normal direction of the Contact surface and the vertical direction
$\boldsymbol{z}$-The displacement along the shaft

1.3 Experimental set up

Figure 1 Experimental setup. 1. Steel horizontal shaft 2. Speed regular module 3. Level gauge 4. Electric motor 5. Mechanical releasing structure 6. Ring

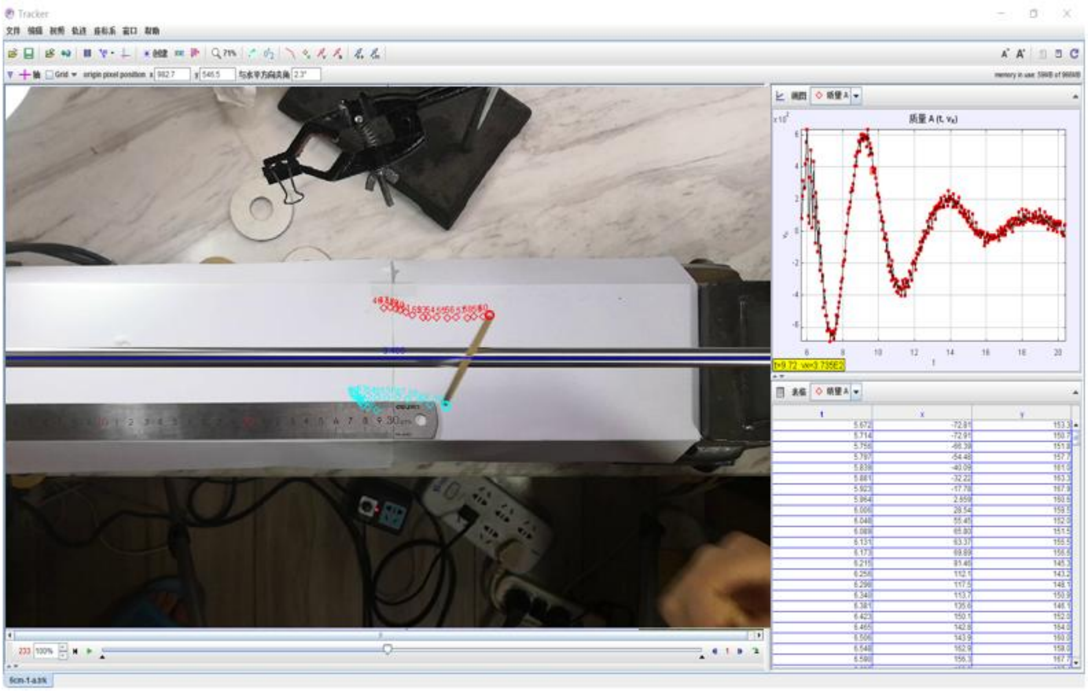

Figure 2 Data analysis method

Figure 1 Experimental shows our experimental devices in research. We use an electric motor with a speed regular module to control the angular velocity, which could increase smoothly from 50 rad/s to 100 rad/s. And a 40cm straight steel horizontal shaft to Minimize unnecessary vibrations as much as possible. We use Acrylic material rings with different diameter and thickness to investigate the effect of geometry on the motion. To minimize initial releasing velocity, we also make a mechanical releasing structure.

We use the Tracker software to trace the upper and lower edge of the ring, which is shown in Figure 2. From these data, we can calculate the velocity of translation, as well as the angular velocity vector.

1.4 Preliminary Experiment

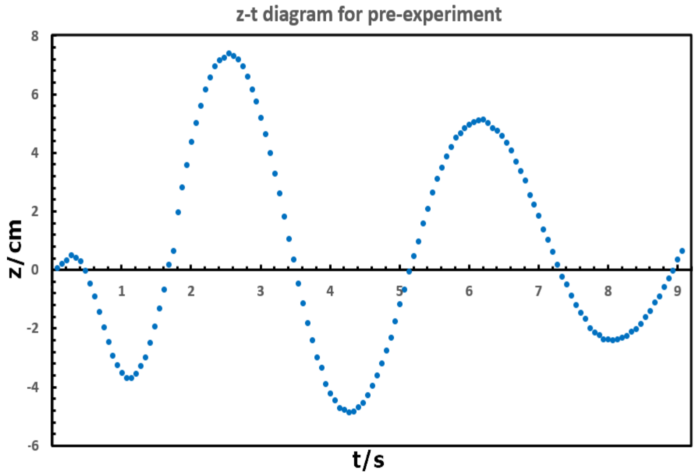

Figure 3

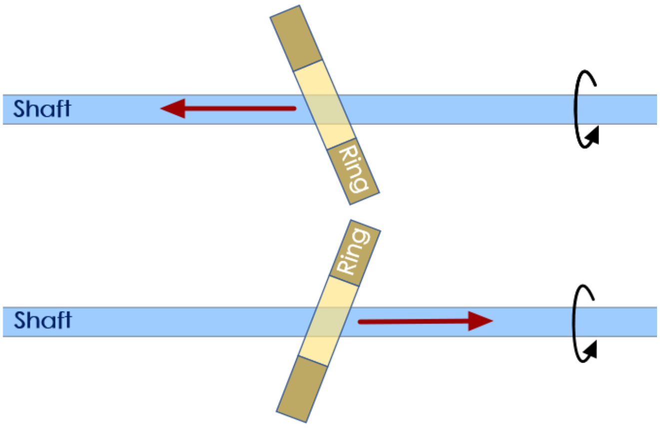

Figure 5 Side view for Pre-Experiment

Figure 4 motion schematic diagram

We can directly observe the basic phenomenon of this problem in Side view of figure 4 and the z-t diagram in figure 2. Here we take the axis of shaft as the z axis, and the release point as the origin. The positive direction of the z axis shows the direction of angular velocity vector of shaft according to the right-hand rule. The direction of angle θ can be defined by taking the cross product between the angular velocity vector of the ring and the positive direction of z axis. Taking the lower panel of Figure 4 for example. Since the ring and shaft rotate in the same direction, the angular velocity vector of the ring point to the lower right corner after projection to the horizontal plane. The cross product between the angular velocity vector of the ring and the positive direction of z axis lead to a vector point outside of the paper. The corresponding tile angle is defined as a positive one. In contrast, the angle θ in the upper panel of figure 4 is a negative one.

We release the ring from rest with a positive tile angle. Figure 2 shows that the ring move in the positive direction in the 0~0.15s, then change its direction of velocity and move along the negative direction of z axis. As time lapse, the ring moves back and forth with a decaying amplitude, and increasing period. The angle θ also shows oscillation. It is positive (negative) when the velocity of ring is along the positive (negative) direction of z axis. When the ring is at the furthest place on the shaft, both the velocity and the angle θ is zero.

After a large number of experiments, we the period the decaying rate of amplitude of reciprocation motion depend on the ratio of the thickness to the diameter of the ring of the shaft. Figure 6 through 8 shows some typical example.

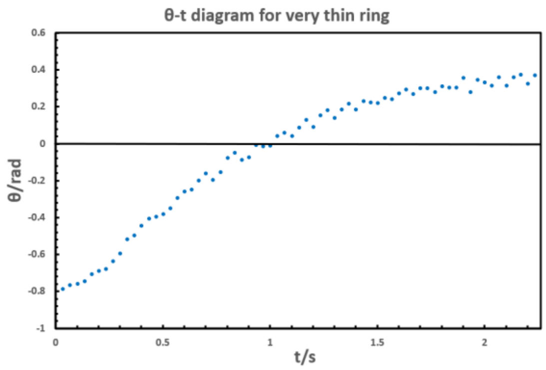

Figure 6 A very thin ring

In this situation, the amplitude of the reciprocation motion is larger than the length of shaft. The ring move in the negative direction at first and then turn to move in the positive direction until it hit the end of the shaft. We do not observe a full circle of reciprocation motion.

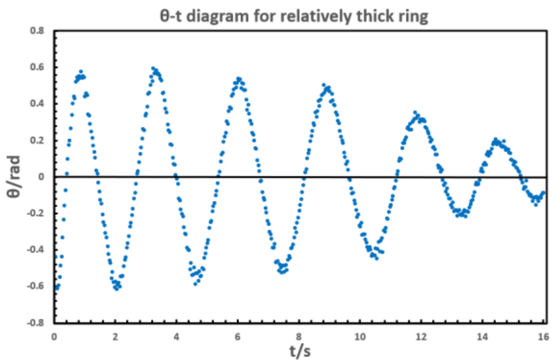

Figure 7 Relatively thick ring

A ring with a larger thickness moves back and forth on the shaft periodically. The amplitude of this motion, including the $\theta - t$ and $z - t$ diagram, is damped over time, and eventually shrink to zero. This means the ring will keep rolling with translation motion in the end.

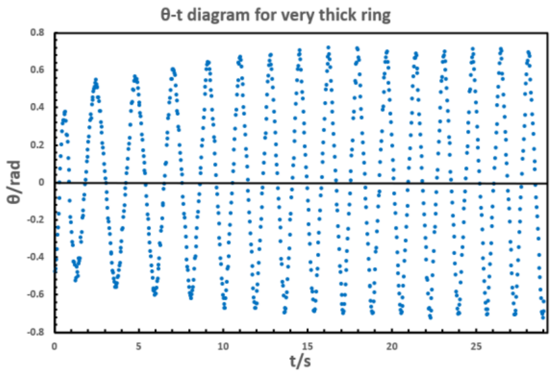

Figure 8 Very thick ring

For a very thick ring, the motion depends on the release angle. When the release angle is smaller than a critical value, the amplitude of this motion shrink to zero. When the release angle is smaller than a critical value, the reciprocating motion gradually turned to be a stable motion with a stable period and amplitude. This mean that energy dissipation and energy input are equal. But the question is during which time the energy is inputted to the ring.

2 THEORY

2.0 Analysis for pre-experiment

There are basically three phenomena if we place the axis horizontally:

1. When the ring is very thin, the ring begins a one-direction movement after a short turn.

2. While the ring is relative thick, the amplitude of the ring is decaying, which means the amplitude of the reciprocating motion and the angle between the axis of the ring decrease continuously.

3. For a thick ring, a stable reciprocating motion mode is possible when we release the ring with a large tilt angle θ.

The first phenomena are essentially the same as the second one. Due to the limited length of the shaft, it is impossible to complete an intact cycle, thus presenting the illusion of a one-way movement.

As a summary of the experimental results, we can conclude that the thickness of the ring plays decisive role in different patterns of motion.

For the experimental result, we have three question to answer

1. Now that the linear velocity of the surface of shaft is perpendicular to the axis, and the friction force is opposite to the relative motion between the ring and shaft, which force lead to the reciprocating motion of ring?
2. Which role dose the thickness of ring played in determine the amplitude and period of reciprocating motion?
3. Why the reciprocating motion become a stable one when the thickness of ring and releasing tilt angle are large than a critical value?

2.1 Degrees of freedom

The system consists of two parts, the ring and the shaft. If we assume that the ring is in contact with the shaft, and neglect the thickness of the ring, this rigid body system has five degrees of freedom. Two of them describes the the position of contact point on the surface of the shaft, which correspond to the translational motion of the ring. The other three describe the angular velocity vector, as shown in the second panel of figure 9.In the following, we give a definition of degrees of freedom.

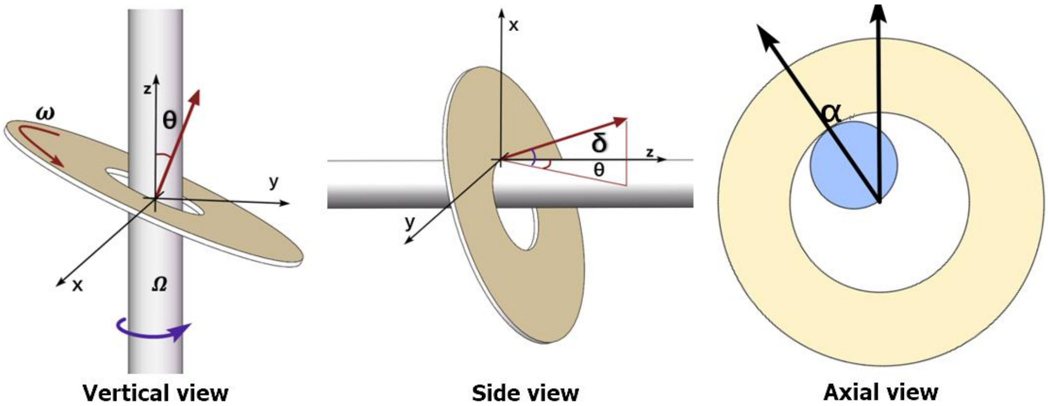

Figure 9

We take the release point as the origin, defined Z axis as the axial direction, the X-axis as upright direction, and the Y-axis is in the horizontal plane. The X, Y and Z axis form a right hand coordinate with Z axis parallel to the angular velocity vector of the shaft. This definition of Z-axis is the same as that in section 1.4. The z coordinate describes the translational motion of ring along the shaft. $\boldsymbol{\alpha}$ is set as the angle between normal direction of the Contact surface and the vertical direction. It describes the height of contact point relative to the axis of shaft. We define the angle $\boldsymbol{\theta}$ and $\boldsymbol{\delta}$ to describe the direction of angular velocity vector of the ring . $\boldsymbol{\theta}$ is the angle between the z-axis and the projection of angular velocity vector of ring in the y-z plane. $\boldsymbol{\delta}$ is the angle between the angular velocity vector and the y-z plane. The rotation angle about the axis of the ring of the ring is set as $\boldsymbol{\varphi}$.

So $\boldsymbol{z}$ and $\boldsymbol{\alpha}$ are the translational degree of freedom which determined the location of the contact point. $\boldsymbol{\theta}$, $\boldsymbol{\delta}$ and $\boldsymbol{\varphi}$ are the rotational degree of freedom here, which describe the angular velocity vector of the ring; (Figure 9). Fisrtly, $\boldsymbol{\theta}$ is exactly the tilt angle we defined in previous section. Secondly, it should be noticed that $\boldsymbol{\theta}$, $\boldsymbol{\delta}$ and $\boldsymbol{\varphi}$ in this problem are not the Euler angles.

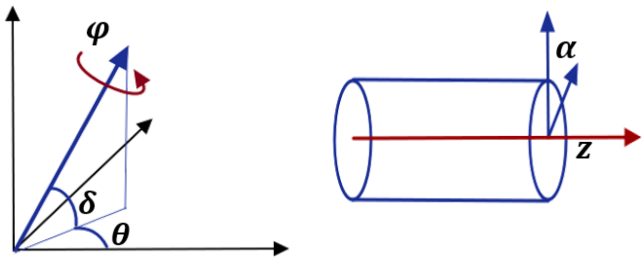

Figure 10 Schematic diagram of the five degrees of freedom

2.2 Effect of the Thickness

In defining the five degree of freedom in section 2.1, we we do not consider the thickness of the ring. After careful examine the dynamic equation, we find that the neither friction or the support force produce a torque along the x axis which can drive the ring to change the value of angle θ. It is in contradiction to our experiment in which tilt angle $\boldsymbol{\theta}$ oscillate with time. Consequently, we have to take thickness into consideration.

On the other hand, when taking thickness into account, the contact between the ring and shaft would change during the motion. Therefore, we have to discuss the possible contact configuration.

2.2.1 Two-point contact& One-point contact

I.

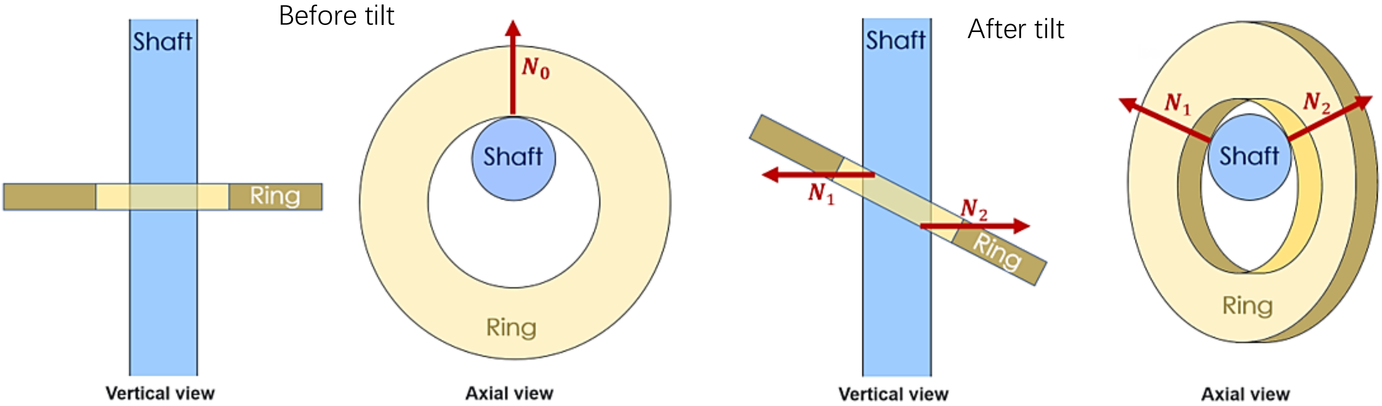

Figure 11 Two Point Contact Situation from vertical and axial view

If the axis of shaft is perpendicular to the ring, namely the tile angle is zero, the surface of shaft and the fillet of the ring are tangent. The tangent points lie on a line segment on the surface of the shaft. The support is vertical upward, and no torque appeared.

When tile angle $\boldsymbol{\theta \neq 0}$ and $\boldsymbol{\delta = 0}$, the fillet of the ring is tangent to the surface of shaft at two points, one on each surface of the ring. The supporting force perpendicular to the surface of the shaft at each contact point generate a restoring torque to drive the tile angle back to equilibrium location (θ = 0), as shown in the vertical view of figure 11.

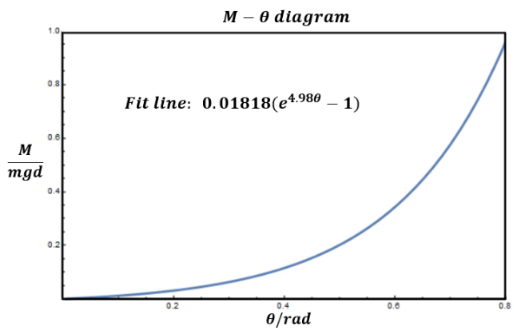

Figure 12relationship between restoring torque and θ

Suppose the resultant force of N_1 and N_2 is in equilibrium with the gravity force, we can calculate N_1 and N_2 as a function of tile angle θ. Further we obtain the relationship between the restoring torque and θ (this time we Ignore the acceleration in vertical direction) This torque is increasing rapidly, which means that the energy required for a larger Angle is increasing exponentially, so the ring is well constrained to a certain range of angles. We get very good results with exponential fitting.(Figure 13)

Another two influence factor is the ratio of the thickness d to inner diameter and the ratio of the diameter of the shaft to inner diameter of the ring. It's obvious that these two ratios are positive related to the restoring torque. The smaller the ring, the thicker the disk, the more significant the geometric constraint is, this lead to bigger restoring torque.

When angle θ is changing, the two contact points move relatively on the surface of shaft, so the friction will produce a resistance torque and lead to the energy dissipation.

II.

When both $\boldsymbol{\theta} \neq \mathbf{0}$ and $\boldsymbol{\delta} \neq \mathbf{0}$, the ring contact with the shaft surface at one point. Due to the finite thickness of ring, the torque of supporting fore and gravity force play the role of restoring force of $\boldsymbol{\theta}$ and $\boldsymbol{\delta}$.

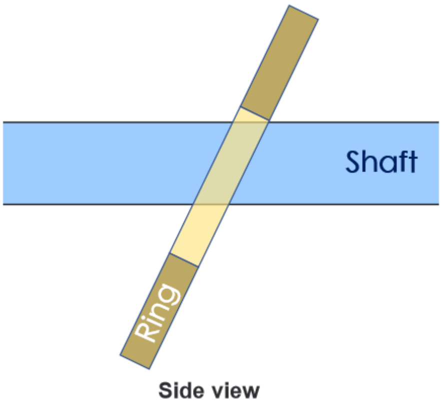

Figure 13

Because of the thickness of the disk, the friction along the Y-axis can also provide a steering torque at this point, and it's obvious that this torque and the thickness d is positive correlated.

III .

To determine whether the ring and shaft contact at one point or two point, we take a frame of reference in which the ring is at rest. Then we assume the system in two-point contact mode and take one of the contact point as the pivot. If the torque of inertial fore in smaller than the torque of gravity, the angle $\boldsymbol{\delta}$ keeps equal 0 and the system keeps in two point contact mode. But If the torque of inertial fore in larger than the torque of gravity, the $\boldsymbol{\delta} \neq \mathbf{0}$ and the ring contact with the shaft at one point.

As a conclusion of this section, the angle between the shaft and the angular velocity vector of the ring changes periodically, due to the restoring torque of supporting force, gravity force and friction.

2.3 Reciprocating motion

The reciprocating motion along the shaft implies restoring force along the shaft. Three force exists in this system, namely the gravity force, the supporting force and the friction. The gravity force and the support which is perpendicular to the shaft axis cannot provide axial force. So the friction force plays a vital role in the reciprocation. In the following, we will prove that the direction of friction changes as the tile angle of ring changes.

2.3.1 Relative velocity at the contact point

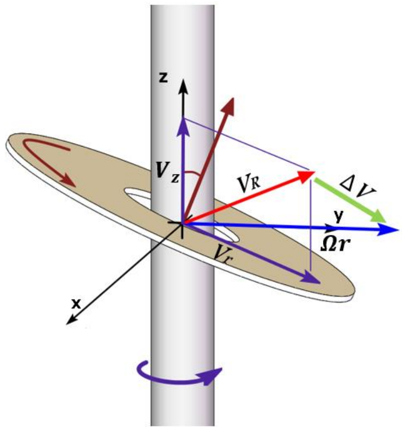

Figure 14 Contact point velocity

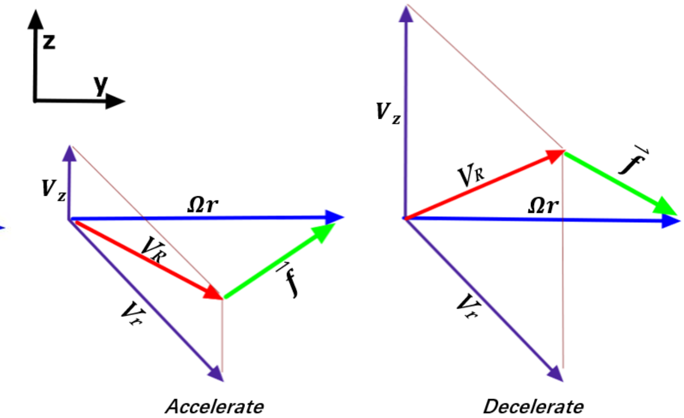

Figure 15 Different conditions with different $\boldsymbol{V_z}$

Since the shaft is oiled, we assumed the friction force is proportional to the support and the relative velocity. Where $\vec{\boldsymbol{f}} = \boldsymbol{k}\overrightarrow{\Delta \boldsymbol{v}}$.

In figure 15 we can find the contact point velocity of the ring during movement can be divided into translational velocity $\boldsymbol{V_z}$ and rotation velocity $\boldsymbol{V_r}$. The velocity of the axis at this point is $\boldsymbol{\Omega r}$.

Then we get the resultant velocity of the ring (the red arrow) and the relative velocity between the axis and the ring at the contact point (the green arrow). As The friction force is proportional to the relative velocity, so the green arrow in the diagram is the direction of friction.

On one hand, the translational velocity $\boldsymbol{V_z}$ of the ring determine the direction of the friction (Figure 16). Suppose the $\boldsymbol{V_r}$ dose not change. When $\boldsymbol{V_z}$ is relatively small, the z component of friction, namely the z component of green arrow, is the same as the direction of $\boldsymbol{V_z}$, as shown in the left panel of figure 16. So ring is accelerating. When $\boldsymbol{V_z}$ is larger, the friction component is reversed with the $\boldsymbol{V_z}$, so the ring decelerate, as shown in the right panel of figure 16. If $\boldsymbol{V_r}$ do not change, the translational velocity $\boldsymbol{V_z}$ will reach a constant value which is equal to the z component of $\boldsymbol{V_r}$.

On the other hand, when the tile angle change, the direction of $\boldsymbol{V_r}$ also change.

Suppose the $\boldsymbol{V_z}$ keeps constant. When the tilt angle is large, z component of V_r is larger that $\boldsymbol{V_z}$, ring is accelerating. When the tilt angle is small, z component of V_r is smaller that $\boldsymbol{V_z}$, ring is decelerating.

As a summary of this section, we show that the direction of friction depends on both translational velocity $\boldsymbol{V_z}$ and tile angle $\boldsymbol{\theta}$. As the tile angle change periodically, the direction of friction also changes. This is the essence of reciprocating motion along the shaft.

2.3.2 Qualitative Explanation

According to the analysis from 2.3.1 and 2.3.2, the reasons for the systematic reciprocating movement can be explained.

On on hand, after being released with a finite tilt angle, the ring will begin to rotate around its symmetry axis due to the friction between ring and shaft. The rotational angular velocity reaches a stable value quickly. At the same time, since $\boldsymbol{V_z}$ is still small, friction is along the positive z axis. This explain why the ring will move along the positive z axis just after being release, as indicate by the motion during 0~0.15s in figure 3.

On the other hand, due to restoring torque of supporting force and friction, the tilt angle quickly decreases from positive value to negative value. The direction of friction also also changes at the same time. This explain why the ring change the direction of motion after 0.15s, as shown in figure 3.

In the following time, the tilt angle will oscillate due to the restoring torque of supporting fore and friction force. As the tilt angle change periodically, the direction of friction also changes periodically, which lead to reciprocation motion along the z axis.

2.3.3 Dimensional analysis

In this problem, there are five basic variables:

density ρ of the ring, rotation moment of inertia J, the inner diameter and the outer diameter R of the ring ,the diameter of the shaft r ,thickness of the ring d, initial releasing angle θ, angular velocity of the shaft $\boldsymbol{\Omega}$,coefficient of the Friction k.

Now we're going to do some dimensional analysis.

First for the turning torque M cause by the supporting force and the rotation inertia J of the ring:

$$\mathrm{M} \propto \rho g \mathrm{R}^2 d^2 \quad , \quad \mathrm{J} \propto \rho \mathrm{R}^4 d$$

And then we can get the angular momentum of the ring L , we can understand the motion as the torque here drives the ring to do precessional motion, where $\vec{\boldsymbol{M}} = \vec{\boldsymbol{\omega}} \times \vec{\boldsymbol{L}}$,so we can also get the cycle T here.

$$\mathrm{L} \propto \mathrm{J}\Omega \propto \rho \mathrm{R}^4 d\Omega$$
$$\mathrm{T} \propto \theta \mathrm{L} \mathrm{M}^{-1} \propto \theta\ g^{-1} \mathrm{R}^2 d^{-1} \Omega$$

f represents the friction force, and a means the acceleration along the shaft

$$\mathrm{f} \propto \mathrm{kmgv} \propto \mathrm{k}\rho g \mathrm{R}^2 d\Omega \mathrm{r}$$
$$\mathrm{a} \propto \theta \mathrm{f} \mathrm{m}^{-1} \propto \theta \mathrm{kg}\Omega \mathrm{r}$$
$$\mathrm{A} \propto \mathrm{aT}^2 \propto \theta^3 k g^{-1} R^4 d^{-2} \Omega^3 \mathrm{r}$$

Finally, we can get the expression of the amplitude A above.

We can actually find that the cycle and the amplitude is positive related with the initial angle, outer diameter and the angular velocity of the shaft, while it's negative related to the thickness

2.4 Two patterns of motion

As we discussed in the section 1.4, the amplitude of reciprocation motion decay when the ring is not thick enough. But when the thickness is beyond a critical value, to reciprocation motion become stable. If examine this problem in terms of Energy, it means that the energy input is smaller than the energy dissipation when the ring is not thick enough. When we increase the thickness, the energy input increases or energy dissipation decreases. Beside, even for the same ring, different initial releasing angle lead to different patterns of motion. A larger releasing angle may lead to a stable motion, and a smaller one lead to a damped one.u

In this system, the energy is inputted through the work done by friction, and dissipate through the work done by friction. First consider the effect of friction on translational motion along the shaft. The friction tends to increase $\boldsymbol{V_z}$ when $\boldsymbol{V_z}$ is small, while tends to decrease $\boldsymbol{V_z}$ when $\boldsymbol{V_z}$ is large. This mean the friction in translational motion tend to balance the energy input and dissipation, thus is irrelevant to the decay of amplitude of reciprocation motion. Then consider the tilting of ring. As the tilt angle increase, the height of center of mass increases, and so do the gravity potential energy. When the tilt angle decrease, the gravity potential energy. This means some energy could be stored as potential energy. But in either cases, the torque of friction tends to dissipate the energy into heat. So it is an important ingredient.

Through experimental observation, we found that in the stable pattern of motion, the ring mainly contact with the shaft at one point, while in the attenuation motion the ring mainly contact with the shaft at one point. During the one-point contact mode, the ring could roll on the shaft without sliding. That means the contact point could be move on the surface of shaft without any relative displacement. Consequently, no energy dissipates through friction. But during the two-point contact mode, the pure roll of ring is not possible, and the energy dissipation is inevitable.

The next question is why the one-point contact dominate over two-point contact mode with large thickness or large releasing tile angle. As we state in section 2.2, the judging criteria of the one and two-point contact is the axial acceleration of the ring. In fact, the acceleration of the disk increase when we increase the thickness and releasing tilt angle. When we increase the thickness d of the ring, the torque of supporting force increases as $\mathrm{M} \propto d^2$, while the moment of inertial of the ring increases as $\mathrm{J} \propto d$. This means the oscillate frequency of tilt angle will increase as $f \propto d^{0.5}$. Since the oscillation of tilt angle lead to oscillation of friction with the same frequency, both the oscillate frequency and amplitude of friction increases, which favor the large translational acceleration and one-point contact. Beside, the increase in initial releasing tilt angle lead to increase in the amplitude of tilt angle, friction and translational acceleration, which also favor one-point contact mode.

2.5 Quantitative Theory

2.5.1 Basic theory

Because we didn't use the Euler angle or the Euler function. So in our theory we use the angular momentum theorem and the Newton second Law.

$$\vec{M} = \frac{D\vec{L}}{Dt} = \frac{d\vec{L}}{dt} + \vec{\Omega} \times \vec{L}$$

Equation 1

This equations of the angular momentum theorem are descried the evolution of angular velocity vector. Where $\frac{D\vec{L}}{Dt}$ represents the derivative of angular momentum in the x y z coordinate; $\frac{d\vec{L}}{dt}$ represents the derivative of angular momentum in the rotational coordinate n_1,n_2,n_3; $\vec{\Omega}$ *is the* angular velocity of the frame. $\vec{L}$ is the angular momentum in the rotational coordinate? So we can actually obtain three equations in the n_1,n_2,n_3 directions, as it shown in Figure 17.

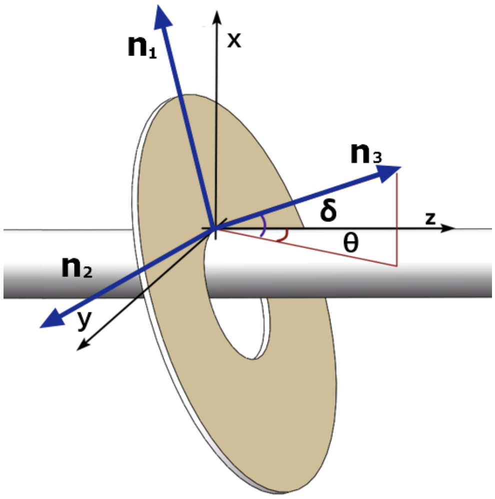

Figure 16 Defines the direction of the principal axis and the rotation coordinate frame

$$\vec{M} = \overrightarrow{M_N} + \overrightarrow{M_f} + \overrightarrow{M_d}$$

$\vec{M}$ is the torque in x y z coordinate.

$\overrightarrow{M_N}$ represents the torque cause by the support of two contact point

$\overrightarrow{M_f}$is cause by the friction and the distance between the contact point and the central of mas when the ring is being tossed up.

$\overrightarrow{M_d}$is the damping torque caused by the moving of the contact point on the shaft when angleθ is changing

$$\vec{F} = m\vec{a}$$

Equation 2

We also employ the Newton second law to describe how center of mass of the ring move under the action of friction. We can also get three equations in x y z directions. There are already 6 equations, exactly the five degrees of freedom plus the supporting force. Which means our theory is self-consistent.

$$\vec{F} = \overrightarrow{F_G} + \overrightarrow{F_N} + \overrightarrow{F_f}$$

$\overrightarrow{F_G}$ is the gravity force, $\overrightarrow{F_N}$ is the supporting force, $\overrightarrow{F_f}$ is the frictional force.

2.5.2 Simplification

Since the simulation of exact function is time consuming, we simplify our equation by two assumptions.

Firstly, according to our experiment, α and δ are small quantities oscillating around zero. So during the solution, we set $\alpha = 0$ and $\delta = 0$. The comparison between the approximate solution and the exact solution shows that our approximations do not lead to observable errors.

Secondly, both experiment and exact solution show that the increase in the amplitude of angular velocity ω_r with time can be describe by exponential growth plus a very small amplitude damped vibration. Therefore we neglect the equation of ω_r, fit the function of it, and substitute the fitted function to the equation of z and θ to simplify the equations.

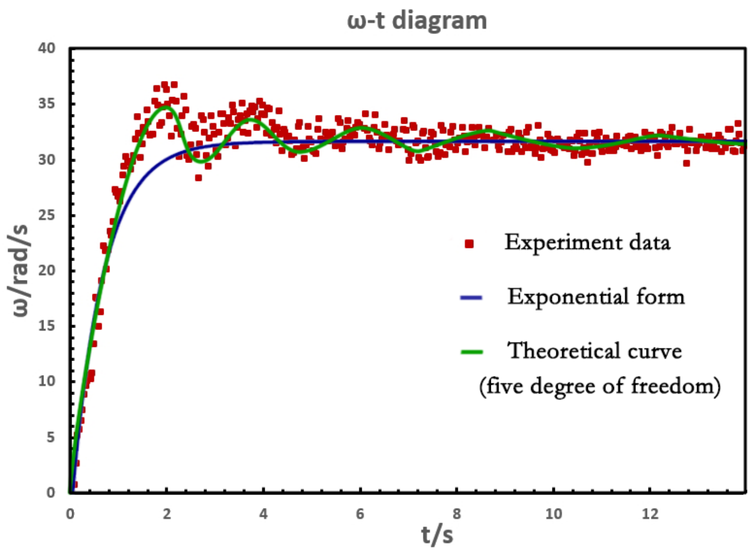

Figure 17

3. Experimental theoretical comparison

We again focus on the effect of thickness on the motion of ring. The experiments were done using rings with inner diameter of about 1cm and outer diameter of about 3cm, and a shaft with diameter of 0.5cm.

We can see that for different cases, whether a thin \relatively thick or a very thick ring, the experiment is basically consist with our theory prediction.

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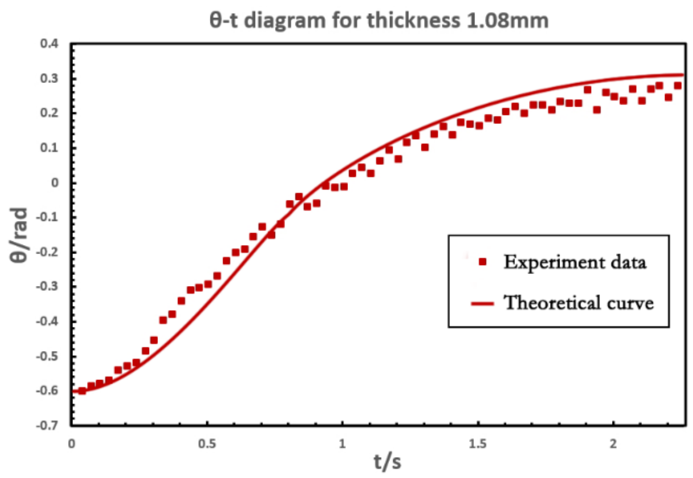

Figure 21

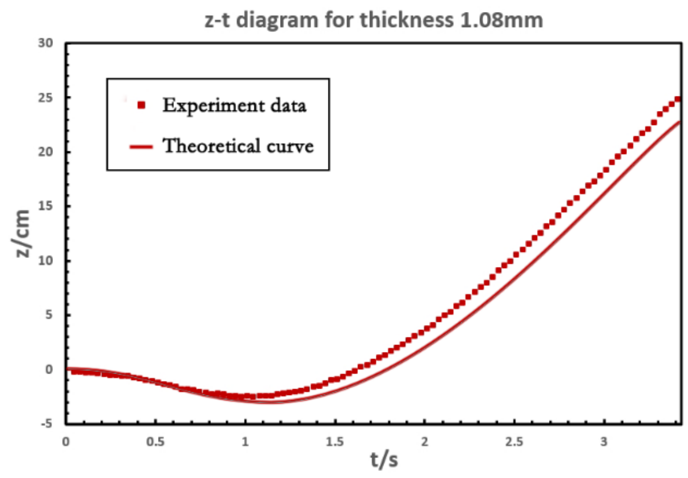

Figure 20

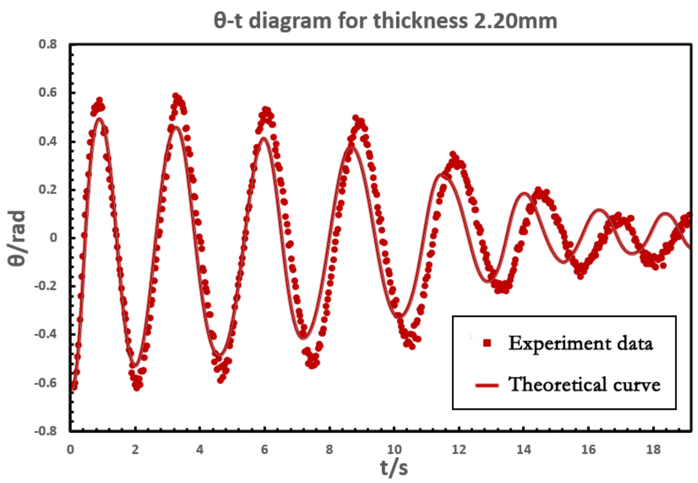

Figure 18

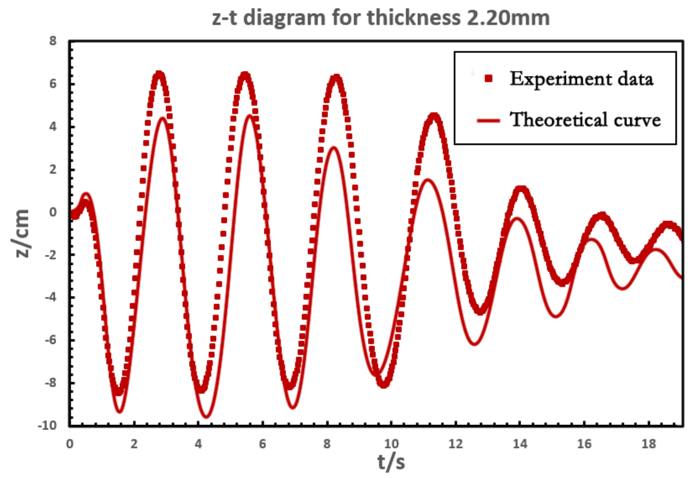

Figure 19

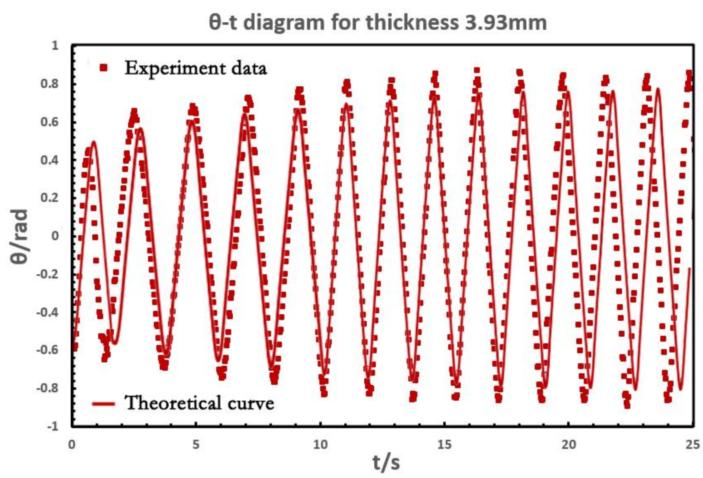

Figure 23

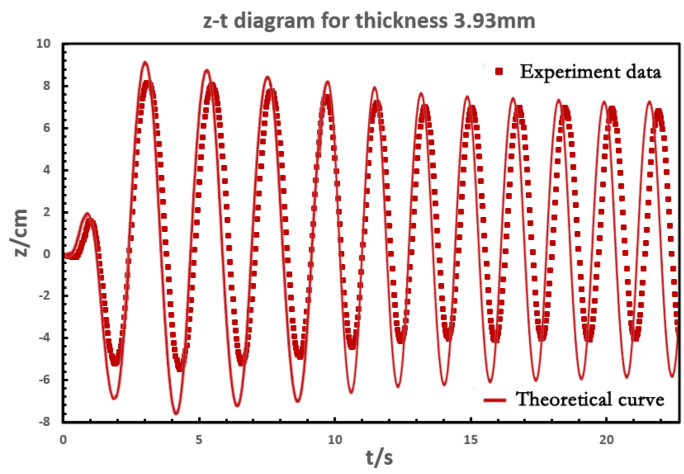

Figure 22

4.Conclusion and Prospect

In this work, we firstly investigate the reciprocation motion of ring on a rotating shaft. We found that for rings with finite thickness, the restoring torque of supporting force leads to the oscillation of tilt angle. Consequently, the relative velocity and friction fore between the ring and shaft also show oscillation with the same frequency, which finally leads to reciprocation motion along the shaft.

Secondly, by examining the motion of ring in terms of energy, we discover two kinds of contact mode between the ring and shaft, namely one-point contact and two-point contact. Although these two modes of contact can both produce a restoring torque, the extra energy dissipation is inevitable when the ring and shaft contact at two different point. Therefore, when the two-point contact mode dominate during the motion of ring, the energy and amplitude of reciprocation motion would decay with time.

Finally, to test our model, we employ rigid body dynamics to describe the evolution of motion. The simulation result is in good agreement with the experiment.

In the further research, we plan to determine the critical thickness and releasing tilt angle. A corresponding phase diagram is also under construction.

Reference:

[1] Л Д. Landau, E. M. Chestnut, seats, theoretical physics tutorial. The first volume, mechanics [M]. Higher education press, 2007

Appendix1：

Specific solving process and equation for five degree of freedom rigid body problem

We first list down the Transition matrix from rotation coordinate(*1 2 3* coordinate) to ground coordinate(*x y z* coordinate) T_1, and it's inverse matrix T_2 (*x y z* to *1 2 3*)

$$T_1 = \begin{pmatrix} Cos\delta & 0 & Sin\delta \\ Sin\delta\ \ Sin\theta & Cos\theta & -Cos\delta\ \ Sin\theta \\ -Sin\delta\ \ Cos\theta & Sin\theta & Cos\delta\ \ Cos\theta \end{pmatrix}$$

$$T_2 = T_1^{-1}$$

Then we calculate the normal vector of the ring surface $\overrightarrow{n_1}$,and the normal vector of contact point $\overrightarrow{n_2}$

$$\overrightarrow{n_1} = \begin{pmatrix} Sin\delta \\ -Cos\delta\ \ Sin\theta \\ Cos\delta\ \ Cos\ \ \theta \end{pmatrix}, \overrightarrow{n_2} = \begin{pmatrix} Cos\alpha \\ Sin\alpha \\ 0 \end{pmatrix}$$

$\overrightarrow{R_c}$ is the relative position of the center of mass to the contact point in ground coordinate. "$\overrightarrow{n_1} \times (\overrightarrow{n_1} \times \overrightarrow{n_2})$" means the fixed orientation ring is tangent to the shaft.

$\overrightarrow{S_p}$ represents the position of the contact point in the x y z coordinate

So we can get the position of the central of mass $\overrightarrow{S_c}$ below

$$\overrightarrow{R_c} = 2\ \ r\ \ \ \ \overrightarrow{n_1} \times (\overrightarrow{n_1} \times \overrightarrow{n_2})$$

$$\overrightarrow{S_p} = \begin{pmatrix} r\ \ Cos\alpha \\ r\ \ Sin\alpha \\ z \end{pmatrix}$$

$$\overrightarrow{S_c} = \overrightarrow{R_c} + \overrightarrow{S_p}$$

Now we consider about the rotation of the ring. $\overrightarrow{\Omega_r}$ is defined to be the angular velocity in the rotational coordinate or the 1,2,3 coordinate. And $\omega_1\omega_2\omega_3$ are the components of Angular velocity on each principal axis of inertia of the ring. $\overrightarrow{\Omega_c}$ represents the angular velocity in the x,y,z coordinate

$$\overrightarrow{\Omega_r} = \begin{pmatrix} \omega_1 \\ \omega_2 \\ \omega_3 \end{pmatrix}, \overrightarrow{\Omega_c} = T_1 \cdot \overrightarrow{\Omega_r}$$

$$\begin{cases} \omega_1 = \dot{\theta}\ \ Cos\delta \\ \omega_2 = \dot{\delta} \\ \omega_3 = \dot{\theta}\ \ Sin\delta + \omega_r \end{cases}$$

Knowing the angular velocity and the displacement, we can calculate the relative velocity between the ring and the shaft and further ger the magnitude of the friction force. $\overrightarrow{V_s}$ is the velocity of the shaft at the contact point, $\overrightarrow{V_r}$ represents the relative velocity cause by the rotation between the central of mass and the contact. $\overrightarrow{V_c}$ is the velocity of the central of mass.

$$\overrightarrow{V_s} = \begin{pmatrix} Cos\alpha \ \ \Omega r \\ Sin\alpha \ \ \Omega r \\ 0 \end{pmatrix}$$

$$\overrightarrow{V_r} = \overrightarrow{\Omega_c} \times \overrightarrow{R_c}$$

$$\overrightarrow{V_c} = \frac{d\overrightarrow{\mathrm{S_c}}}{dt}, \overrightarrow{A_c} = \frac{d^2\overrightarrow{\mathrm{S_c}}}{dt^2}$$

$$\overrightarrow{F_f} = -k\left(\overrightarrow{V_s} - \overrightarrow{V_r} + \overrightarrow{V_c}\right) = \begin{pmatrix} -kr\alpha(\ \ \Omega - 2\mathrm{Cos}\theta\omega0) + O(2) \\ kr(\ \ \Omega - 2\mathrm{Cos}\theta \ \ \omega + \mathrm{Cos}2\theta\alpha' - \mathrm{Sin}2\theta \ \ \alpha \ \ \theta') + O(2) \\ kr(2\mathrm{Sin}\theta \ \ \omega + z' + 2\mathrm{Sin}\theta(-\mathrm{Cos}\theta\alpha' + \mathrm{Sin}\theta\alpha\theta')) + O(2) \end{pmatrix}$$

The expression of the friction force $\overrightarrow{F_f}$ is shown above, because it's very complex we do linear process and consider α, δ as small quantities.

$$\overrightarrow{F_N} = \left(mg \ \ Cos\alpha + \left|\overrightarrow{n_1} \cdot \overrightarrow{A_c}\right|\right)\overrightarrow{n_1}$$

$$\vec{F} = \overrightarrow{F_G} + \overrightarrow{F_N} + \overrightarrow{F_f}$$

$$\vec{F} = m\overrightarrow{A_c}$$

Then we use Newton second law to get the expression of the support and finally get the two differential equations in y and z direction.

Now we consider about the angular momentum theorem part. $\vec{M}$ represents the torque in x y z coordinate. "$F_\delta \frac{d}{2}\overrightarrow{n_1}$" means different contact point of the two surface of the ring.

$$\vec{M} = (\overrightarrow{R_c} + F_\delta \frac{d}{2}\overrightarrow{n_1}) \times \vec{\mathrm{F}}$$

$$F_\delta = \begin{cases} 1, \delta > \dfrac{d}{2r} \\ \delta \dfrac{2r}{d}, -\dfrac{d}{2r} \le \delta \le \dfrac{d}{2r} \\ -1, \delta < -\dfrac{d}{2r} \end{cases}$$

$\vec{\mathrm{L}}$ represents the angular momentum in 1,2,3 coordinate, $\overrightarrow{\omega_r}$ represents the angular velocity between the rotational coordinate and the ground coordinate.

$$\vec{\mathrm{L}} = \begin{pmatrix} I_1\omega_1 \\ I_2\omega_2 \\ I_3\omega_3 \end{pmatrix}, \overrightarrow{\omega_r} = \begin{pmatrix} \omega_1 \\ \omega_2 \\ \omega_1 Tan\delta \end{pmatrix}$$

$$\begin{cases} I_1 = I_2 = I_0 = m(4r^2 + R^2) \\ I_3 = 2I_0 \end{cases}$$

Finally we use the angular momentum theorem and get three equations in each direction.

$$\vec{M} \;=\; \frac{d\vec{L}}{dt} + \overrightarrow{\omega_r} \times \vec{L}$$

The five differential equations for the five degree of freedom is shown below

$$2krCos\theta\alpha(r\omega Sin\theta + Cos\theta\ \ z') + I_0(2\omega_0\delta' + \theta'') + 0.01818\left(e^{4.98(\theta^2)^{0.5}} - 1\right)mgd + \frac{1}{5}d^2k\theta' = F_\delta\frac{d}{2}\left(k\left(-2r\omega_0 - Sin\theta\ \ z' + rCos\theta(\omega + \alpha')\right) + \alpha(gmCos\theta - 2krSin\theta\ \ \theta')\right),$$

Equation 3

$$-2kr^2\omega Sin\theta - 2gmr\ \ Sin\theta\ \ \alpha - 2kr\ \ Cos\theta\ \ z' + 2kr^2\ \ Sin\theta\ \ \alpha' - 2\ \ I_0\ \ \omega\ \ \theta' + \delta\ \ \left(2gmr - I_0\theta'^2\right) + I_0\delta'' = F_\delta\frac{d}{2}\left(-gm - \delta(kr\omega Sin\theta + kCos\theta z' + 2mr\delta'') + r\alpha(k\omega - 2kCos\theta\omega_0[t] + 2mSin\theta\delta'')\right),$$

Equation 4

$$2(-kr^2\omega Cos\theta + 2kr^2\omega 0 + krSin\theta z' - kr^2Cos\theta\alpha' + 2kr^2Sin\theta\alpha\theta' + I_0\delta'\theta' + I_0\omega 0' + I_0\delta\theta'') = 0,$$

Equation 5

$$2krSin\theta\omega 0 + kz' - krSin2\theta\alpha' + kr\alpha\theta' - krCos2\theta\alpha\ \ \theta' - 4mrCos2\theta\alpha'\theta' - 4mrSin\theta\delta'\theta' + 4mrSin2\theta\ \ \alpha\ \ \theta'^2 - 2mr\ \ Cos\theta\ \ \delta\ \ \theta'^2 + mz'' - mrSin2\theta\alpha'' + 2mrCos\theta\delta'' - 2mrCos2\theta\ \ \alpha\theta'' - 2mrSin\theta\delta\theta'' = 0,$$

Equation 6

$$\alpha(-gm + krSin2\theta\ \ \theta' + 4mrCos2\theta\theta'^2 + 2mr\ \ Sin2\theta\ \ \theta'') - r(k\omega - 2kCos\theta\omega 0 + 4mCos\theta\ \ \delta'\theta' - 2mSin\theta\ \ \delta\ \ \theta'^2 + \alpha'(kCos2\theta - 4mSin2\theta\ \ \theta') + mCos2\theta\ \ \alpha'' + 2m\ \ Sin\theta\ \ \delta'' + 2mCos\theta\ \ \delta\ \ \theta'') = 0$$

Equation 7

Appendix 2:

The process of simplification from five to three degree of freedom

This time we ignore the value of α, and it's approximately zero. Then the We think that the rotation speed of a ring is basically exponential, which is affected very little by other factors.

$$\omega_{r(t)} = (1 - e^{-pt})2\Omega$$
$$p = \frac{4r^2 k}{I_0}$$

We bring in the expression of $\omega_{r(t)}$, and use Newton second law in z direction, Angular momentum theorem in 1, 2 direction ,totally three differential equations here

The simplified equation of three degrees of freedom is shown below

$$e^{-pt}I_0\left(p\Omega\mathrm{Sin}\delta + \mathrm{Cos}\delta\ \delta'((-1+e^{pt})\Omega + 2e^{pt}\mathrm{Sin}\delta\ \theta') - e^{pt}\left(-3 + \frac{1}{2}\ \mathrm{Cos}2\delta\right)\theta''[t]\right) + 0.01818\left(e^{4.98(\theta^2)^{0.5}} - 1\right)mgd + \frac{1}{5}d^2k\theta' = F_\delta\frac{d}{2}\left(k(r\Omega(-1+e^{-pt} + \mathrm{Cos}\theta) - \mathrm{Sin}\theta\ z')\right),$$

Equation 8

$$-2r(-gm\mathrm{Sin}\delta + kr\Omega\mathrm{Cos}\delta\mathrm{Sin}\theta + k\mathrm{Cos}\delta\mathrm{Cos}\theta z') - e^{-pt}I_0\left(p\Omega\mathrm{Cos}\delta\mathrm{Sin}\theta + (-1 + e^{pt})\ \Omega\ \mathrm{Cos}\delta\mathrm{Cos}\theta\theta' + e^{pt}\mathrm{Cos}\delta\mathrm{Cos}\theta\mathrm{Sin}\delta\theta'^2 - \mathrm{Sin}\theta\delta'\left((-1+e^{pt})\Omega\mathrm{Sin}\delta - 2e^{pt}\mathrm{Cos}\delta^2\theta'\right) - e^{pt}\mathrm{Cos}\theta\delta'' + e^{pt}\ \mathrm{Cos}\delta\ \mathrm{Sin}\delta\ \mathrm{Sin}\theta\ \theta''\right) = F_\delta\frac{d}{2}(-gm\ \mathrm{Cos}\delta - kr\Omega\ \mathrm{Sin}\delta\ \mathrm{Sin}\theta - k\mathrm{Cos}\theta\ \mathrm{Sin}\delta\ z'),$$

Equation 9

$$k((1-e^{-pt})r\Omega\mathrm{Sin}\theta + z') + m(-2r\mathrm{Cos}\theta\ \mathrm{Sin}\delta\ \delta'^2 - 4r\mathrm{Cos}\delta\ \mathrm{Sin}\theta\ \delta'\theta' - 2r\mathrm{Cos}\theta\ \mathrm{Sin}\delta\ \theta'^2 + z'' + 2r\mathrm{Cos}\delta\ \mathrm{Cos}\theta\ \delta'' - 2r\mathrm{Sin}\delta\ \mathrm{Sin}\theta\ \theta'') = 0$$

Equation 10

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Academic Research

In the physics department of Tsinghua University, I learned from Tutor Wen Bo Sun and studied SPDC (spontaneous parametric down-to-conversion) effect in the field of quantum optics, during which I conducted a lot of experiments, processed data and learned relevant knowledge. I finally won the first prize of the project in the university.	2016-2018

Honors and Awards

The First Prize and Gold Medal, as one of the China Team Members in International Young Physicists' Tournament (IYPT) 2018	2018
First prize of Beijing competition district, third prize of China in 34th Chinese Physics Olympiad (CPHO)	2017
First prize of Beijing in 36th Beijing Adolescents Science & Technology Innovation Contest	2016