Judge's Commentary: The Outstanding Traffic Circle Papers

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Overview of the Problem

Teams who decided to explore the "A" problem in this year's Mathematical Contest in Modeling examined ways to control the movement of vehicles in a traffic circle. A broad overview of the criteria developed by the judges and the experiences of the judges is given.

In the following section, a brief overview of the problem statement is explored. Next, an overview of the judging itself is given. In the subsequent section, a list of some of the common approaches adopted by the teams is given. Finally, a list of some of the common themes and more detailed points that emerged as the judging proceeded is given.

Traffic Circles

The focus on the "A" problem is to control the movement of vehicles in a traffic circle. A number of controls are explicitly given in the problem statement. The teams who submitted papers for this problem mainly focused on the given controls and very few examined other types of controls.

The problem statement includes two requirements. First, the teams were asked to find a way to control the flow of traffic in an optimal way. Second, the teams were asked to write a summary of their findings. These two aspects are explored individually in the subsections that follow.

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The Goal

The goal for this problem is to find a way to move vehicles through a traffic circle in an optimal way. This was stated in the second paragraph of the problem statement :

> The goal of this problem is to use a model to determine how best control traffic flow in, around, and out of a circle.

It is not clear what "best" means. It was left open for the teams to decide what "best" means. The teams were required to make it clear in their report how they interpreted this part of the problem:

> State clearly the objective(s) you use in your model for making the optimal choice as well as the factors that affect this choice.

The judges expected the teams to clearly describe the objectives, and we expected that the subsequent evaluation of the model be consistent with the stated objectives. This can be difficult for the teams to achieve given the dynamic of writing as a team, the nature of how approaches evolve as the problem is explored, and the intense time pressure. Teams that managed to maintain a high level of consistency tended to elicit a more-positive response from the judges.

Technical Summary

An essential requirement was to write a technical summary. The requirements for the technical summary were given in the problem statement. This was a difficult aspect to the problem. The teams were expected to provide a broad set of guidelines for a traffic engineer in a brief note.

The traffic engineer should be able to read the summary and have a strong sense of the different methods available. Additionally, the different circumstances that impact the decision should also be included. Examples of important parameters are the radius or geometry of the circle, the rate of flow of traffic coming into the circle, and the density of traffic coming into the circle. Very few teams considered the traffic capacity of roads leaving the circle, and most assumed that the incoming traffic was a primary limiting factor.

The traffic engineer is also expected to obtain a broad understanding of the conditions for which the model is applicable. This implies that the engineer should be able to read the summary and obtain a basic understanding of how the model was developed and an understanding of the potential pitfalls.

Writing the summary was a difficult task for the teams. The teams had a diverse amount of information to convey in two pages. The teams that managed to convey a sense of the basic models, the underlying assumptions, and the limitations of their models tended to make a stronger impression.

Grading Process

First, a brief overview of the evaluation process is given. The papers are evaluated in three stages. There is an initial round where the focus is on which papers to remove from the pool. The second, or screening round, focuses on which papers meet the minimal requirements for an advanced score. In the final round, the judges focus on which papers meet the highest standards.

Initial Grading

The initial round is designed to remove papers from the pool that are not likely to meet the standards in the following round. Each paper is read by at least two people. Papers that receive consistent low scores are not passed on to the next round. Papers with mixed reviews are read by more people. When the reviewers are unsure, they try to err on the side of caution and pass the paper on to the next round.

It is absolutely essential that a paper be well-written and have a clear, concise summary to make it past the initial round. A paper that does not provide a clear overview including results and a synopsis of the techniques used will not make a strong impression on the judges. The summary and the rest of the paper must also be consistent. Differences between the summary and the following pages can be immediately apparent and do not make a positive impression of the paper.

Screening Rounds

As the judges examine papers in the next set of rounds, they try to decide if the paper meets the minimal requirements to do well in the following rounds. The number of times that a paper is read in these rounds varies from year to year. Again the judges try to err on the side of caution; but as the rounds proceed, the criteria for doing well becomes increasingly stringent.

It is still important to have a strong summary, but the need for consistency across the whole paper is more important. The need for proper citations and correct grammar is also important. This year, a large body of literature was available for the teams. It was even more important than usual to include proper citations and make clear what work was done by the team and what work was found in the literature search.

Final Rounds

In the final rounds of judging, the focus is on finding the best submission. At this point, each paper is read many times, and more time is available for each reading. The judges are able to focus more on each individual step and focus on consistency across the whole paper. The papers that remain in these final stages must maintain high scores to move forward.

Approaches

The flow of traffic in roundabouts is an active research area. The available literature influenced many of the teams. Most teams used either a deterministic approach or a stochastic approach. Here we examine each of these approaches separately.

Deterministic

The teams that adopted a deterministic approach tended to make greater use of models based on partial differential equations. There are a variety of different conservation laws that have been derived to model traffic flow. Such models tend to focus on relatively simple traffic geometries and require considerable adaptations to model a traffic circle.

At first glance, a conservation law for a traffic circle seems to avoid the issues associated with boundary conditions because it is a periodic geometry. Unfortunately, the exits and entrances of the feeder roads create other difficulties. Adapting models to include the exits and entrances occupied the majority of the modeling efforts.

The second difficulty with this approach is to find an approximation to the solution. The equilibrium solutions to the equations are piecewise-constant functions, and the conversation law gives rise to shocks. Given the complex boundaries, the method of characteristics is complicated, and the numerical approximations can be daunting since the techniques must account for upwinding.

Stochastic

The majority of teams used a stochastic approach. In general, they examined either queues or networks, and a common approach was to use a hybrid model combining the two. A typical paper included an overview of the model, some theoretical results for a simple situation, and results for a computational model.

Teams adopting this approach were expected to use proper citations because of the wide body of work available. The judges also paid more attention to the consistency across the whole paper. The summary, model, results, and discussion had to be consistent.

Another issue that emerged with some papers is the disconnect between the section in the paper discussing the theory and that with the numerical simulations. Many of the top-rated papers provided some theoretical results for simplistic geometries or simulations. The majority of these went on to include the results of numerical simulations for the more complicated cases. The few teams that provided a confirmation of the numerical model on a simple geometry made an immediate positive impression.

The other issue is how to report the results of simulations in a coherent manner. The development of the model requires a probabilistic approach. The

analysis of the numerical trials requires a shift to a statistical approach. The majority of teams simply reported means and sometimes standard deviations. Few teams reported results using qualitative methods such as boxplots or histograms, and even fewer teams made use of appropriate quantitative statistical methods.

Finally, when designing the numerical trials, few teams examined a range of values for the parameters in their models. Every year, the judges rate this aspect of the problem as a crucial part of the problem. We expect to see an exploration of the results given small changes in parameters or assumptions. The few teams that did examine this aspect immediately caught the judges attention.

Common Themes

In the previous section, some observations specific to this year's competition are given. Some general observations that come up every year are explored here.

Summary

The summary is an important part of the team's entry. It is the first thing that a judge will read. The summary is the first impression. It is vital that a paper have a complete and well-written summary to make it past the initial rounds. It is also vital that the details in the summary be consistent with the rest of the paper.

Writing a one-page summary of the team's efforts is a difficult task. The teams are expected to provide a brief overview of the problem. They are then expected to let the reader know their specific conclusions and recommendations. Finally, the teams are expected to provide the reader with an overview of the approach that they used.

It is difficult to include all three of these parts within the one-page summary. Many teams find it tempting to include a large amount of background information or provide clever narratives motivating the problem. Unfortunately, such material in the summary can drastically reduce the amount of space available to discuss the team's results and discussion of the approach that they adopted.

Grammar, Punctuation, and Equations

The presentation of the team's model and results cannot be separated from the model itself. A team must have a reasonable model including a basic analysis of the model. The teams are expected to then share their results in a clear and concise discussion.

Teams that do not make use of proper grammar and punctuation are not likely to make it past the initial rounds of the competition. Teams must know

how to include equations in their writing and use proper punctuation. Advisers should not take it for granted that their students know how to do these things.

Proper Citations

The judges expect every entry to include proper citations. Many teams are comfortable exploring the resources available to them, and it is unusual to come across an entry with a unique approach. The different types of approaches can be easily categorized, and the judges quickly figure out the sources available for each approach.

Sensitivity and Stability

Sensitivity and stability are always important. The few teams that make a concerted effort to explore this aspect of their model will almost always stand out. Exploration of the sensitivity of a model can be as simple as testing what happens for a different range of values in a parameter, and it can include the use of more sophisticated methodologies such as an exploration of a sensitivity matrix.

Every year, teams are able to implement nontrivial numerical simulations. The teams must make decisions about what numerical trials to examine. It is extremely rare for teams to scale a problem as a way to decide the combination of parameters that are important.

Figures and Tables

The integration of graphs and tables into a paper is a challenge for many teams. It is not uncommon to see entries in which figures and tables are included with no detailed discussion of them. The teams need to integrate the figures and tables into their discussion.

Given the increased use of simulations and numerical results it is vital that the teams find a way to include descriptions of their figures and tables into their narrative. The teams need to make sure to let their readers know the key aspects of their figures and tables and inform their readers how to look at the figures and tables.

Consistency Across the Paper

The teams have a limited time to understand the problem, derive a mathematical description of the problem, perform the requisite analysis of their model, and then come back and interpret their work with respect to the original context. Over the course of the weekend, teams make decisions and explore a variety of different approaches. The time constraints make it extremely difficult to complete a paper in which the wide array of assumptions and analyses are consistent across the whole paper.

Conclusions

A team's submission must satisfy a wide array of criteria to be successful and proceed through each stage of the judging. The presentation and grammar are vital aspects of a submission. The team's results are given through the filter of the team's writing.

The team must provide a strong analysis. The teams only have four days, and the judges do not expect extensive and sophisticated models. A careful analysis of the resulting model is required, though.

Each year, the expectations are different, but there are a few constants. For example, a clear discussion of the basic assumptions—with some justification, citations, and a discussion of the implications—is necessary. Additionally, judges always expect a focused discussion on stability and sensitivity.

In this year's competition, the use of simulation was a part of the majority of entries. Incorporating an analysis of simulations is a difficult task, and the top entries did a remarkable job of integrating the development and analysis of their model with the discussion of the results of their numerical trials.

Teams that were able to tie together the theoretical analysis of their model along with their numerical trials received immediate positive recognition. The best entries were able to develop multiple models of varying complexity and verify their numerical models with the theoretical results of the simpler models.

About the Author

Kelly Black is a faculty member in the Dept. of Mathematics and Computer Science at Clarkson University. He received his undergraduate degree in Mathematics and Computer Science from Rose-Hulman Institute of Technology and his Master's and a Ph.D. from the Applied Mathematics program at Brown University. He has wide-ranging research interests, including laser simulations, ecology, and spectral methods for the approximation of partial differential equations.