Judges' Commentary:
The Fusaro Award for the Sudoku Problem

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Introduction

MCM Founding Director Fusaro attributes the competition's popularity in part to the challenge of working on practical problems. "Students generally like a challenge and probably are attracted by the opportunity, for perhaps the first time in their mathematical lives, to work as a team on a realistic applied problem," he says. The most important aspect of the MCM is the impact it has on its participants and, as Fusaro puts it, "the confidence that this experience engenders."

The Ben Fusaro Award for the 2008 Sudoku Problem went to a team from the University of Puget Sound in Tacoma, Washington. This solution paper was among the top Meritorious papers and exemplified some outstanding characteristics:

- it presented a high-quality application of the complete modeling process;
- it demonstrated noteworthy originality and creativity in the modeling effort to solve the problem as given; and
- it was well-written, in a clear expository style, making it a pleasure to read.

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The Problem: Creating Sudoku Puzzles

The teams were asked to develop an algorithm to construct Sudoku puzzles of varying difficulty and to develop metrics to define a difficulty level. The algorithm and metrics were to be extensible to a varying number of difficulty levels, and the teams were asked to illustrate the algorithm with at least four difficulty levels. The teams had to guarantee a unique solution, and they had to analyze the complexity of their algorithm with the objective of minimizing its complexity while meeting the requirements.

The University of Puget Sound Paper

Mathematical Formulation

Modeling a real-world problem begins by making assumptions that allow for the formulation of a mathematical description of the problem. In introducing a metric that would define the difficulty level of a Sudoku puzzle, the University of Puget Sound team limited the formal solution techniques to four standard ones, listed in order of increasing difficulty: Naked Singles (NS), Hidden Singles (HS), Naked Pairs (NP), and Locked Candidate (LC). Although this set of solution techniques was more limited than most, the team's algorithm lent itself to the consideration of more techniques. In the solution process, it was assumed that the "average" person would apply easier techniques before more difficult ones; and for the team's metric, a count would be made for the number of times each technique was applied, thus defining a solution path. The team also assumed that as a solution path increases in length, the difficulty increases at a linear rate. In cases when a point was reached where none of the four solution techniques could be applied, "guess and check" was used, with an altered metric categorized into cases of no guess and check (standard metric), one or two guesses with checks (double the standard metric), and three or more guesses and checks (designated as "Fiendish").

Metric Definition

At this point, the team categorized seven different value categories for their metric, designating them from Easiest to Fiendish. Although the clarity of the reasoning was commendable, there was some inconsistency in the formulations for the metric, replacing HS by NS in the later formulas. Furthermore, the team did not consider the situation where there are no Hidden Singles (or perhaps Naked Singles). This situation would have yielded a metric value of zero, indicating the easiest puzzle, when just the reverse would be true. Although this would be an unlikely occurrence—it apparently never occurred in the team's data samples—in the eyes of the judges, this was a serious omission.

Validating the Metric

Once the metric was defined, the team applied the next step in the mathematical modeling process, validating that the metric measured what it claimed to measure. To do this, the team applied their metric to Sudoku puzzles with published rankings at two popular Web sites. The team showed that the numerical scores computed by their metric corresponded well to the ratings there.

Generating New Puzzles

With the metric had been defined and validated, the team undertook the task of generating new puzzles at specified difficulty levels. To do this, they began by generating solved Sudoku boards, starting with a random seed of initial hints, and then applied the Knuth technique of Dancing Links. Although they cited a Web site where the Dancing Links algorithm could be applied to solve a Sudoku puzzle, it was not clear that the team members understood this technique. After the full board was generated, the team, as many teams did, proceeded by removing hints, making sure after each removal stage that there was a unique solution, using the Dancing Links solution technique.

This team demonstrated the average time to generate random Sudoku puzzles as a function of the number of initial hints and then went on to show the relationship between the proportion of puzzles generated at each level and the number of initial hints. Their puzzle generations and resulting observations about the role of the number of initial hints, coupled with the configuration of those hints, were noteworthy. The team found that by constraining the initial configuration of the Sudoku boards, they could control the proportion of puzzles produced at a given difficulty level.

Recognizing Limitations of the Model

Recognizing the limitations of a model is an important last step in the completion of the modeling process. The team recognized that their metric is less sensitive to changes in difficulty as the difficulty level increases, and also that, in reality, their metric does not perfectly mimic human behavior. As mentioned earlier, they observed how their metric could be extended to encompass additional solution techniques.

Conclusion

Despite a few deficiencies, the paper was an excellent illustration of the modeling process. The judges felt that this was a fine example of the fact that mathematical modeling can be done at many levels. The team is to be congratulated on their thoroughness, their clarity, and using the mathematics they knew to create their own model to define a metric and apply it successfully.

About the Authors

Marie Vanisko has retired from Cal State Stanislaus and moved back to Montana, where she taught for 31 years at Carroll College and was a visiting professor at the U.S. Military Academy at West Point. She chairs a College Board committee for the SAT Subject Tests in Mathematics and serves on a national joint committee of the National Council of Teachers of Mathematics and the Mathematical Association of America (MAA). For each of the past two years, Marie has co-directed an MAA Tensor Foundation grant project for high school girls, entitled Preparing Women for Mathematical Modeling, with the hope of encouraging more young women to select careers that involve mathematics. She serves as a judge for the COMAP MCM and HiMCM has also been active in the MAA PMET (Preparing Mathematicians to Educate Teachers) project.