Judge's Commentary: The Outstanding Irrigation Problem Papers

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Introduction

Irrigation planning is a real-life activity with many complexities; good system design can demonstrate profound water savings. For the contest problem, an entire region must be minimally watered but not overwatered, and trade-offs between fixed and periodically moved equipment must be made.

As in any real-life modeling activity, the approaches, metrics, and results of others can be obtained with little effort—when applicable, this earlier work should be used, or improved upon. For example, the most widely used measure of irrigation uniformity in the turf industry is Christiansen's uniformity coefficient. Also, manufacturers' specifications of sprinkler characteristics are easily obtained.

The components appearing in a solution must be identified. For this problem, the judges were looking for the following components:

- Defined constraints on the problem, such as the needed water flow rate.
- Subjective constraints on the problem, such as what "optimal" is.
- Created constraints on the problem, such as the water distribution pattern from a single sprinkler.
- One or more metrics by which a solution can be evaluated.
- A procedure for obtaining an optimal solution.
- A description of the optimal solution.

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Problem Specifics

One of the first considerations is the meaning of "optimal" for this problem. It could be the number of times that the pipes must be moved, or it could be related to the distance that the pipes must be moved. Either of these, or other similar metrics, are reasonable. Minimizing the actual time of watering—selected as a metric by some teams—does not seem to be as useful; it does not obviously correlate to cost.

The contest problem was stated without much detail. In fact, the problem didn't state exactly where the water outlet was in the field. For this reason, a high-level model, appropriate for simple problem descriptions, is warranted. The judging focused on resolving the difficulties—for this problem, the difficulty was in determining the pipe layout. Excessive detail in, say, the distribution of water from a sprinkler head, is not warranted. Use of either a realistic model (easily available from manufacturers) or a simple model is appropriate.

In a real situation, the actual sprinkler head water distribution, wind, and other secondary considerations could be important. A description of how they affect the high-level model, and its solution, is warranted—even for a high-level model. However, only if the high-level model solution is complete is it appropriate to incorporate their effects.

Problem Areas

Most of the teams approached the problem well and identified most of the components noted in the **Introduction**. There were two areas, however, that confused several teams.

- The maximal soaking rate of 0.75 cm/h was intended to be (in the words of the Colorado team) an "average, not instantaneous overwatering" constraint. While mathematically equivalent to 0.0125 cm/min, it was not the problem's intent to prohibit a solution that watered at the rate of 0.025 cm/min, if this watering occurred for less than 30 min in an hour.

- Care is needed to determine the flow rate and pressure from the sprinkler heads when there is more than one. The Duke team had a very clean derivation of this result (although atmospheric pressure of approximately 100 kPa is missing in their computations). In summary: When the flow is *pressure-limited* (i.e., few sprinkler heads), then energy balance (Bernoulli's equation) can be used to determine the output speed. When the flow is *volume-limited* (i.e., several sprinkler heads), then mass conservation can be used to determine the output speed.

What Made Them Outstanding

The Outstanding papers obtained solutions that could be shown to a customer. These papers obtained schedules by using analytical thinking and by numerical optimizations (using, for example, simulated annealing, genetic algorithms, or methodical searches); some did both. The length of the Outstanding papers, as submitted, varied from 17 to 60 pages, with an average of 29 pages.

Further Comments and Advice

Some overall comments on the submissions and the judging process:

- The summary should include:
  - problem synopsis,
  - description of analysis, and
  - results.

  This section is worth writing, and then rewriting; it gets much attention.

- An ideal paper concluded with an explicit watering/movement schedule and a statement about the effectiveness of the schedule. While a result may be exact for a given model, the model is only an approximation to reality. As such, it is unrealistic to report many decimal places in the results. In practice, pipe placement may be accurate to a foot or so; a placement schedule should not require centimeter accuracy.

- Many submissions created very detailed models. It is always an advantage to start with a simple, perhaps idealized, version of the problem. Even an approximate solution to this idealized problem, perhaps obtained by hand, can be used as a bound when comparing the results from more detailed models. Such back-of-the-envelope checks can be vital in checking the reasonableness of a solution.

- When there are different ways to attack a problem, try using several techniques. If they lead to the same answer, then the answer is probably close to correct. And when computer models are used, sensitivity analysis is especially important (and it should be relatively easy to carry out). For example, what happens to the field watering if the pipes are not placed in the exact right positions?

- "Dead ends" are typically useful only if they lead to an insight or constrain a model in some way. Details on such "dead ends" rarely contribute to a paper's overall ranking; at some point, more is less.

- Graphics indicating the pipe layout and the resulting water distribution were created by most teams. These graphics reveal much information about a pipe layout and its solution.

- As is usually the case, the judges wanted to see justifications for the assumptions made. Reusing standard results (say, those obtained from a book or from the Web) is appropriate; but justifying their applicability is an important aspect of reuse. Note that re-deriving standard results adds little value. Also, the only assumptions that should appear are those used in the problem analysis.

- Finally, details must appear somewhere; if they appear in two places, then error-checking can occur. For example, when a mathematical statement appears suspect, the judges will often locate the computation in the code to see exactly what was implemented.

About the Author

Daniel Zwillinger attended MIT and Caltech, where he obtained a Ph.D. in applied mathematics. He taught at Rensselaer Polytechnic Institute, worked in industry (Sandia Labs, Jet Propulsion Lab, Exxon, IDA, Mitre, BBN, Ratheon), and has been managing a consulting group for the last dozen years. He has worked in many areas of applied mathematics (signal processing, image processing, communications, and statistics) and is the author of several reference books.