Q1

SOLUTIONS

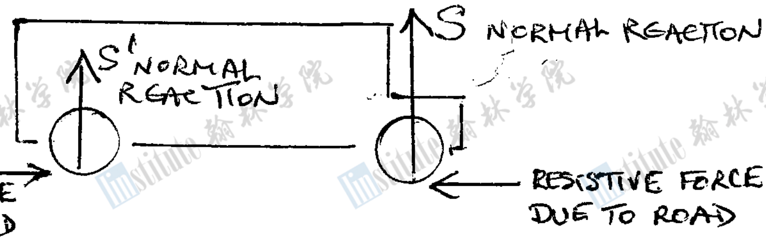
(a) (i) $Z \to Z-2$ $A \to A-4$ $\frac{1}{2}+\frac{1}{2} =$ 1

(ii) $Z \to Z+1$ $A \to A$ $\frac{1}{2}+\frac{1}{2} =$ 1

(iii) $Z \to Z+1$ $A \to A+2$ $\frac{1}{2}+\frac{1}{2} =$ 1

[3]

(b) (i) FORCES ON WHEELS BY ROAD.



$\frac{1}{2}+\frac{1}{2} = 1$

(ii)

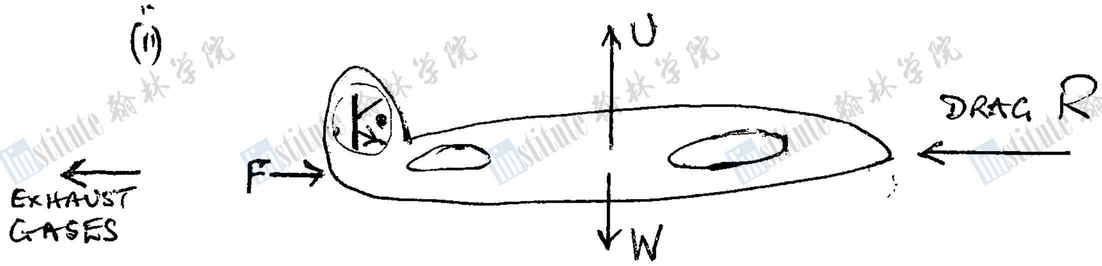


Diagram → 1

Forces

(x) Weight W vertically downwards $\frac{1}{2}$

(xi) Upthrust due to flow of air over wings (Bernoulli effect), U, vertically upwards $\frac{1}{2}$

(xii) Driving force F, reaction resulting from rate of change of momentum of exhaust gases $\frac{1}{2}$

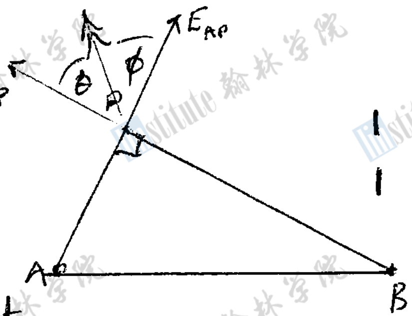
(xiii) Drag, resistance, R due to air resistance $\frac{1}{2}$

$W = U$ and $F = R$ for constant velocity $\frac{1}{2}+\frac{1}{2} = 1$

[5]

(c) $E_{AP} = \frac{1}{4\pi\varepsilon_0}\frac{100\times10^{-9}}{(50\times10^{-3})^2} = \frac{1}{4\pi\varepsilon_0}\left(4\times10^{-5}\right)\ NC^{-1}$ 1

$E_{BP} = \frac{1}{4\pi\varepsilon_0}\frac{576\times10^{-9}}{(120\times10^{-3})^2} = \frac{1}{4\pi\varepsilon_0}\left(4\times10^{-5}\right)\ NC^{-1}$ 1



As these components are at right angle the resultant

$$E = \frac{1}{4\pi\varepsilon_0}\,4\sqrt{2}\times10^{-5}\ NC^{-1}$$ 1

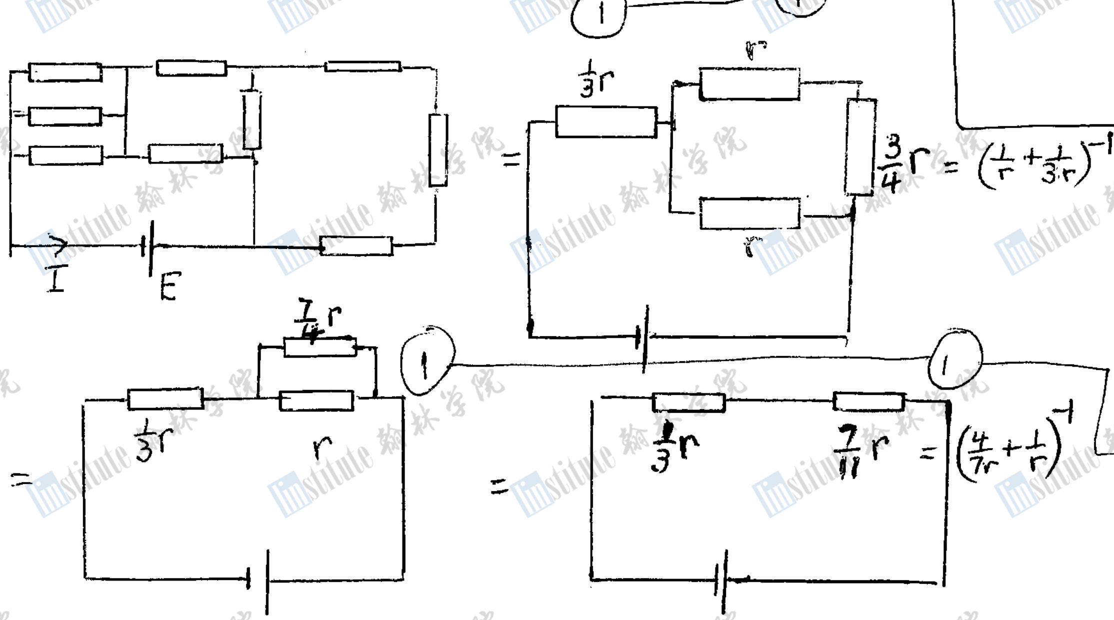
$$= (8.897\times10^{9})(4\sqrt{2})10^{-5}\ NC^{-1}$$

$$E = 5.03\times10^{5}\ NC^{-1}$$ 1

$\theta = \phi = \tan^{-1}1 = 45^\circ$ 1

[5]

(d)



1 → 2

1 → 2

$$I = \frac{E}{\left(\frac{1}{3}+\frac{7}{11}\right)r} = \frac{33}{32}\frac{E}{r}$$ 1

[5]

(e) $E = h\nu$: $\nu = \frac{50\times10^3 \times 1.602\times10^{-19}}{6.626\times10^{-34}}$ 1

$= \underline{1.29\times10^{19}\ Hz}$ 1

Heat generated $= (50\times10^3)(20\times10^{-3})(0.99)\ Js^{-1}$

$= \underline{990\ W}$ 1

[3]

(f) Let m be mass of bullet and v speed of bullet

1 + 1 + 1 = 3

$$\frac{4}{5}\left(\frac{1}{2}mv^2\right) = (600-320)(0.12\times10^3)m + 21\times10^3 m$$

$$v^2 = \frac{5}{2}\left[\,280\,(0.12\times10^3) + 21\times10^3\right]$$

$$= \frac{5}{2}(10^3)\left[33.60 + 21.00\right] = \frac{5}{2}(10^3)(54.60)$$

$$= 13.65\times10^4$$

$\underline{v = 0.37\ kms^{-1}}$ 1

[4]

(g) Originally 10 uranium atom, presently only 9

Exponential decay for rock of age t:

$9 = 10e^{-\lambda t}$ 1

$\lambda t = -\ln(0.9)$ where 1

$\lambda = -\ln 2 / 4.5\times10^9 = +\frac{0.6931}{4.5\times10^9}$ 2

Substituting for λ, $\underline{t = 6.84\times10^8\ years}$ 1

[5]

(k)(i) Area of cork, radius 1cm, $= \pi(1\times10^{-2})^2\ m^2$

Force on cork, $F = p \times$ Area $= p\pi(1\times10^{-2})^2\ N$ (p = pressure) 1

Energy gained by cork in 2cm $= F(2\times10^{-2}) = p\pi(1\times10^{-2})^2(2\times10^{-2})$

$= p(6.2)10^{-6}\ J$ 1

$= mgh$ m = mass = 10 gms, h = 6 m

$= 10\times10^{-3}(9.81)6$ 1

Thus $p = \dfrac{5.9\times10^{-1}}{6.2\times10^{-6}} \simeq \underline{10^5\ Pa}$ 1

Accept answers in range $10^4 \to 10^6$ Pa.

Alternative solutions acceptable — can use approx $g = 10$.

(ii) Let w be the width of the tyre and t thickness left on road, R radius of tyre

Removed volume $= (5\times10^{-3})2\pi Rw = 50{,}000\times10^3\times t\times w$ 2

Taking $R = 20$ cm, $t = 2\pi(20\times10^{-2})\times10^{-10}$ 1

Giving $\underline{t = 10^{-10}\ m}$ 1

Accept $10^{-9}\to10^{-11}$ m

[8]

Alternative solutions acceptable

(j)(i) Using $\frac{1}{2}mv^2 = mgh$ (conservation of energy) 1

$v^2 = 2g(20\times10^{-2})$ ($h = 20\times10^{-2}$ m)

$\underline{v = 1.98\ ms^{-1}}$ (or $\sqrt{0.4g}$) 1

(ii) Time to fall 0.80 m, t, is given by

$0.80 = \frac{1}{2}gt^2$ 1

$t = \sqrt{\dfrac{1.6}{g}}$ 1

Horizontal distance $s = vt = \sqrt{0.4g}\sqrt{\dfrac{1.6}{g}} = \underline{0.80\ m}$ (A) 1

(iii) For a hole at height h above the floor, conservation of energy requires

$\frac{1}{2}mv'^2 = mg(1-h)$ 1

Time, t', to fall distance h given by

$h = \frac{1}{2}gt'^2$ i.e. $\therefore t' = \sqrt{\dfrac{2h}{g}}$ 1

Horizontal distance s, given by

$s = v't' = \sqrt{\dfrac{2h}{g}}\sqrt{2g(1-h)}$ 1

From (A) $= 0.80$ m

This gives an equation for h,

$h^2 - h + \dfrac{16}{100} = 0$ 1

Thus $h = 0.80$ m or 0.20 m

We ... $h = 0.20$ m (B) ($h = 0.80$ m corresponds to the first hole) 1

(j) [10]

(k) We require

$$\frac{G M_{Moon} M_{Earth}}{R_{ME}^2} = \frac{Q^2}{(4\pi\varepsilon_0) R_{ME}^2}$$ 1

$$Q^2 = (4\pi\varepsilon_0) G\,(0.0123)\,M_E^2$$

$$= 4\pi(8.854\times10^{-12})(6.672\times10^{-11})(0.0123)M_E^2$$ 1

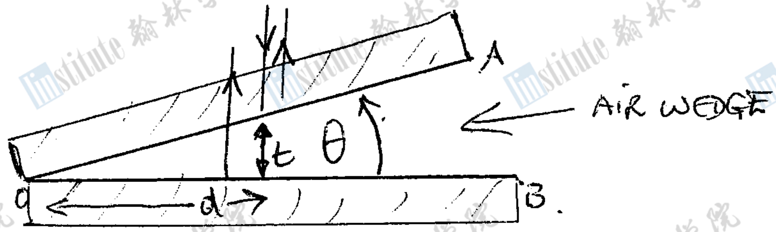
$$Q = 9.555\times10^{-12} M_E$$

Mass of electrons $M_Q = 9.555\times10^{-12}\,\frac{9.109\times10^{-31}}{1.602\times10^{-19}}\,M_E$ 1

$$\underline{M_Q = 5.43\times10^{-23} M_E}$$ 1

[4]

(l)



(i) Light reflected from two faces, OA and OB, upper and lower surfaces of air wedge. Extra phase change of π at OB due to reflection at interface with lower to higher refractive index 1

If thickness of wedge t at reflected region, distance d from O, constructive interference require $2t = (n+\frac{1}{2})\lambda$ Ⓐ (n integer) 1

Destructive interference $2t = n\lambda$

Pattern due to interference fringe system 1

(ii) Now $t = d\tan\theta \approx d\theta$ 1

Substituting into Ⓐ, $2d\theta = (n+\frac{1}{2})\lambda$ for constructive fringes 1

Separation Δd between constructive fringes given by

$$2\Delta d\theta = \lambda \qquad \text{ie } [n+\tfrac{3}{2} - (n+\tfrac{1}{2})]$$

$$\underline{\Delta d = \frac{\lambda}{2\theta}}$$ 1

Δd depends inversely on θ; some marks for a qualitative answer

(iii) For water refractive index $\mu > 1$, path difference becomes $2\mu d\theta$ 1

Giving, fringe separation, $\Delta d = \frac{\lambda}{2\mu\theta}$

Δd becomes smaller than in air. Some marks for a qualitatively correct answer. 1

[8]

(ii)

$(\frac{1}{2}) + (\frac{1}{2})$

(m) (i) Decrease in p.e $V = +(0.20)g(0.16) = +0.3139$ J — 1

(ii) Elastic Energy $E = \frac{1}{2}kx^2 = \frac{1}{2}k(0.16)^2$ — 1

At equilibrium $0.20g = k(0.16)$ or $k = \frac{5}{4}g$ — 1

Thus $E = \frac{1}{2}\left(\frac{0.20g}{0.16}\right)(0.16)^2$

$= \underline{0.157 \text{ J}} = \frac{1}{2}(+V)$ — 1

(iii) Heat generated $V - E = \frac{1}{2}V$

$= \underline{0.157\text{J}}$ — 1

(iv) Loss in p.e. $= \frac{1}{2}k[(0.24)^2 - (0.16)^2] - (0.20)g(0.08)$ — 1

$= \frac{1}{2}\left(\frac{5}{4}g\right)[(0.40)(0.08)] - 0.016g$

$= \frac{1}{2}\left(\frac{5}{4}\right)g\,(0.032) - 0.016g$ — 1

$= 0.016g\left(\frac{5}{4} - 1\right) = 0.016g\left(\frac{1}{4}\right) = 0.004g$

$= 39.2 \times 10^{-3}$ J

$\underline{\text{Gain in KE} = 3.92 \times 10^{-2} \text{ J}}$ — 1

(v) $T = 2\pi\sqrt{\frac{m}{k}} = 2\pi\sqrt{\frac{1/5}{(5/4)g}} = 2\pi\sqrt{\frac{4}{25g}} = \frac{4\pi}{5}\Big/\sqrt{g}$ — 1

$= \underline{0.802\text{s}}$

(vi) Spring is 'compressed' into zero tension state and — ½

spring plus mass falls under gravity — ½

[10]

(n) let n be the number of strokes; working volume of pump V_1

If each stroke produces a volume V_2 with required pressure p_2, then

For ith stroke, $p_1V_1 = p_2V_{2i}$ $(V_2 = V_{2i})$ — 1

Then n strokes will produce

$np_1V_1 = p_2\sum_i V_{2i} = p_2V_2$ where V_2 total vol. of tyre

Giving $n(9.0 \times 10^{-5})(1.0 \times 10^5) = (3.0 \times 10^5)(1.2 \times 10^{-3})$ — 1

thus $\underline{n = 40}$ — 1

In practice the expansion is not slow, so temperature does not remain constant. It is an adiabatic expansion in which heat generated and does not escape; barrel becomes hot — 2

[5]

21 (a) $S_A + S_B = (M+m)g$ (i) 1

Moments about A:

$$mgl + Mg(2l - x) = 2l\,S_B$$ 1

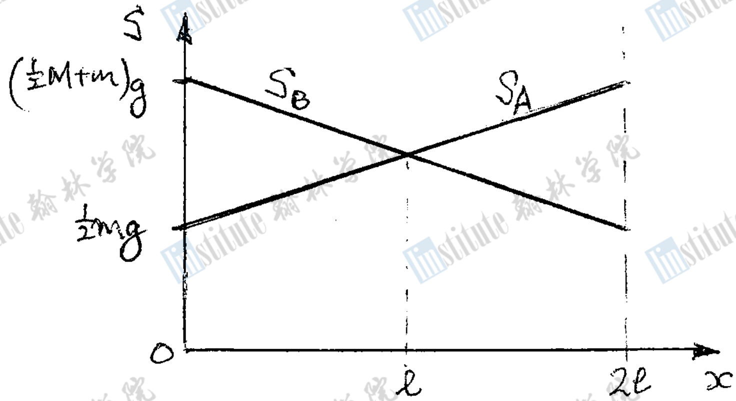
$$\underline{S_B = \left[\tfrac{1}{2}m + \left(1 - \tfrac{x}{2l}\right)M\right]g}$$ (ii) $\frac{1}{2}$

From (i)

$$S_A = (M+m)g - S_B$$ $\frac{1}{2}$

From (ii)

$$\underline{S_A = \left[\tfrac{1}{2}m + \left(\tfrac{x}{2l}\right)M\right]g}$$ 1



S_B graph 1

S_A graph 1

[6]

12 (a) SOLUTION TO Q2

(i)

$i_1 = i_8$ [$\frac{1}{2}$]

$i_2 = i_9$ [$\frac{1}{2}$]

$i_3 = i_{11}$ [$\frac{1}{2}$]

$i_4 = i_{12}$ [$\frac{1}{2}$]

$i_5 = i_{10}$. [$\frac{1}{2}$]

(ii) (x) all current directions are reversed [1]

(xi) the network is the same as in (i) with A and B reversed

Consequently

$i_1 = i_4$ [1]

$i_2 = i_3$ [1]

[Using results in (i)

$i_8 = i_{11} = i_1 = i_4$

$i_9 = i_2 = i_{10} = i_3$]

(iii) No change [1]

(iv) In Figure 2.2. Total resistance of upper five resistors is

$$2R + \left(\frac{1}{R} + \frac{1}{2R}\right)^{-1} = 2R + \frac{2}{3}R = \frac{8}{3}R$$ [1]

Lower 5 resistors also $\frac{8}{3}R$. [$\frac{1}{2}$]

Adding 2R in parallel with $\frac{8}{3}R$ which is in parallel with $\frac{8}{3}R$

Total Resistance $R_{AB} = \left(\frac{3}{8} + \frac{3}{8} + \frac{1}{2}\right)^{-1} R = \left(\frac{5}{4}\right)^{-1} R$ [1]

$$\underline{R_{AB} = \frac{4}{5}R}$$ [1]

[[10]]

(b) Reversing the p.d. across A.C. requires, by symmetry,

$i_1 = -i_{10}$ [$\frac{1}{2}$]

$i_6 = -i_9$ [$\frac{1}{2}$]

$i_3 = -i_7$ [$\frac{1}{2}$]

$i_5 = -i_{12}$ [$\frac{1}{2}$]

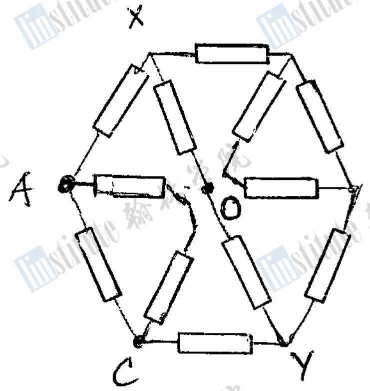
$i_2 = -i_{11}$ [$\frac{1}{2}$]

[If students have correct magnitudes and appreciate the direction of the current give full marks]

Disconnect i_3 and i_7 at junction O
i_6 and i_9 at junction O [1]

This does not alter currents in the arms of network

We now have



Total resistance to right of XY $= 2R + \left(\frac{1}{2R} + \frac{1}{R}\right)^{-1} = \frac{8}{3}R$ [2]

This is in parallel with resistance 2R along XY

Giving total resistance $\left(\frac{1}{2R} + \frac{3}{8R}\right)^{-1} = \frac{8}{7}R$ [1]

Now to right of XY:

$\frac{8}{7}R$ in series with $2R$

Total resistance $\frac{22}{7}R$ [1]

$\frac{22}{7}R$ is in parallel with 2R and R (across AC) [$\frac{1}{2}$]

Giving total resistance

$$R_{AC} = \left(\frac{7}{22} + \frac{1}{2} + \frac{1}{1}\right)^{-1} R$$ [1]

$$R_{AC} = \frac{11}{20}R$$ [1]

[[10]]

3 (a) SOLUTIONS TO Q3

(i) Velocity of waves relative to observer $c_o = c_s + V$ [1]

Wavelength of sound $\lambda = c_s/f_0$ [1]

Frequency detected by observer $f = \dfrac{c_o}{\lambda} = \dfrac{c_s+V}{c_s/f_0}$ [1]

$$\underline{f = \frac{c_s+V}{c_s} f_0} \qquad (A)$$

(ii) Source moving towards stationary observer, separation between successive crests, apparent wavelength,

$$\lambda = \frac{c_s}{f_0} - \frac{u}{f_0} = \frac{c_s - u}{f_0} \qquad [2]$$

Thus

$$f = c_s \Big/ \left(\frac{c_s - u}{f_0}\right)$$

$$\underline{f = \left(\frac{c_s}{c_s - u}\right) f_0} \qquad (B) \qquad [1]$$

(iii) ~~Source and observer moving~~

Velocity of sound relative to observer $c_o = c_s + V$ $\left[\frac{1}{2}\right]$

Wavelength reaching observer

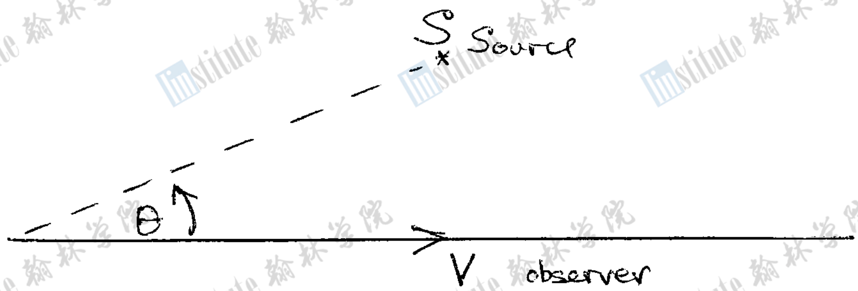
$$\lambda = \frac{c_s - u}{f} \qquad \left[\frac{1}{2}\right]$$

Thus

$$f = \frac{c_o}{\lambda} = \frac{c_s + V}{(c_s - u)/f} = \left(\frac{c_s + V}{c_s - u}\right) f \qquad (C) \qquad [1]$$

[[8]]

(b) (i)



Velocity component in direction of source is $V\cos\theta$. This replaces 'V' in (A). As θ increases the factor '$V\cos\theta$' decreases, reducing the value of f in (A). Once the observer has passed the closest

Q.3

point to source, S, the sign of 'v' changes in (A), as he is moving away from the source, so f continues to decrease.

Correct explanation for approaching and receding from source. CORRECT EXPLANATION [3]

(ii) For small times approaching source $f = 210.4 = \left(\frac{C_s+V}{C_s}\right) f_0$ from (A) (D) [1]

For large times receding from source $f = 181.6 = \left(\frac{C_s-V}{C_s}\right) f_0$ from (A) [1]

Thus $$\frac{210.4}{181.6} = \frac{C_s+V}{C_s-V}$$ [1]

or $$\frac{V}{C_s} = \frac{210.4-181.6}{210.4+181.6}$$

$$V = 330\left(\frac{28.8}{392}\right) = 24.24\ ms^{-1}$$ [1]

Substituting v into (D)

$$f = 210.4 = \frac{330+24.24}{330} f_0$$

$$f_0 = \frac{(210.4)(330)}{354.24} = 196.0\ Hz$$ [1]

(iii) At the point of closest approach to source, time t_0,

$V\cos\theta = 0$ ie $\cos\theta = 0$ $\theta = 90°$ [1]

Giving $f = f_0 = 196.0\ Hz$ [1]

From the graph at $f = 196.0$

$t_0 = 75.5 \pm 1.0\ s$ [1+1]

[[12]]

Q3

SOLUTION Q4

4 (i)

h_2/mm	d/mm	d^3/mm^3
290	1·22	1·816
162	1·01	1·030
41	0·64	0·262
1·3		

Theory correct for small h_2:

$$3mgh_2 = Pd^3 \qquad h_2 = \frac{P}{3mg}d^3.$$

Graph passes through the origin

Table of values h_2 against d^3 [1]

Graph correctly plotted on a reasonable scale making full use of graph paper [1]

Straight line through the origin [1]

Minor deviation from straight line for $h_2 = 290$ [1]

(ii) Gradient $\frac{d^3}{h_2} = \left(\frac{44\cdot3\cdot10}{2\cdot80}\right)^{-1}$ mm^{+2} $= \frac{3mg}{P}$.

$= (6\cdot32 \pm 0\cdot2)10^{-3}$ mm^2 [1+1=2]

$= (6\cdot32 \pm 0\cdot2)10^{-9}$ m^2 ✓

$\frac{P}{3mg} = (1\cdot58 \pm 0\cdot05)10^{+8}$ m^2 $\qquad \frac{P}{3mg} = \frac{h_2}{d^3}$.

$P = (1\cdot85 \pm 0\cdot05)10^7$ Nm^{-2}. [1] result + [1] accurac

(iii) ANALYSIS $\frac{1}{2}mv^2 = mgh$ [½]

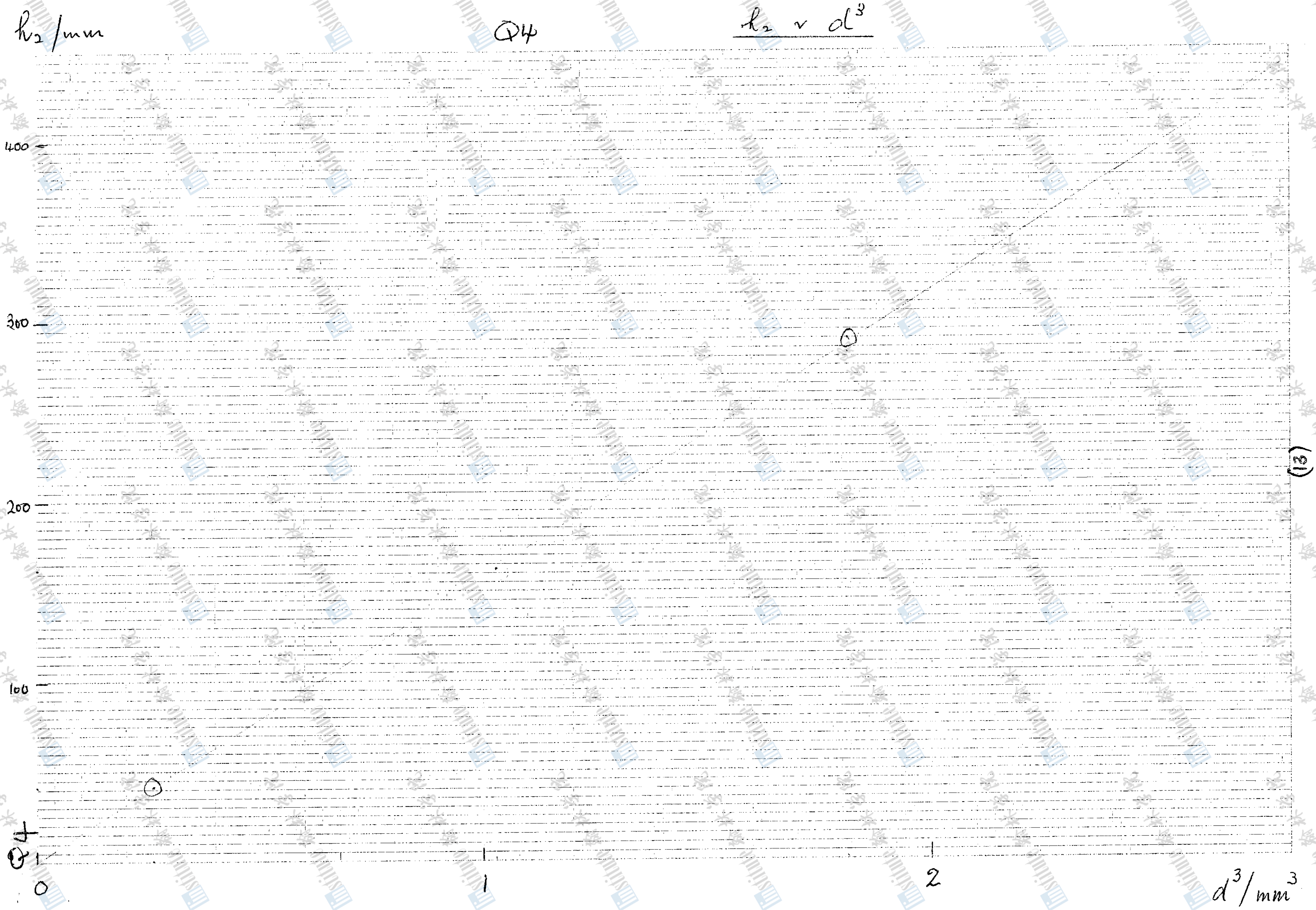
$v^2 = 2gh$

Giving $v_R = \sqrt{2g}\,(h_2)^{\frac{1}{2}}$ [½]

$v_I = \sqrt{2g}\,(h_1)^{\frac{1}{2}}$ [½]

and $\alpha = \left(\frac{h_2}{h_1}\right)^{\frac{1}{2}}$ [½]

Q4

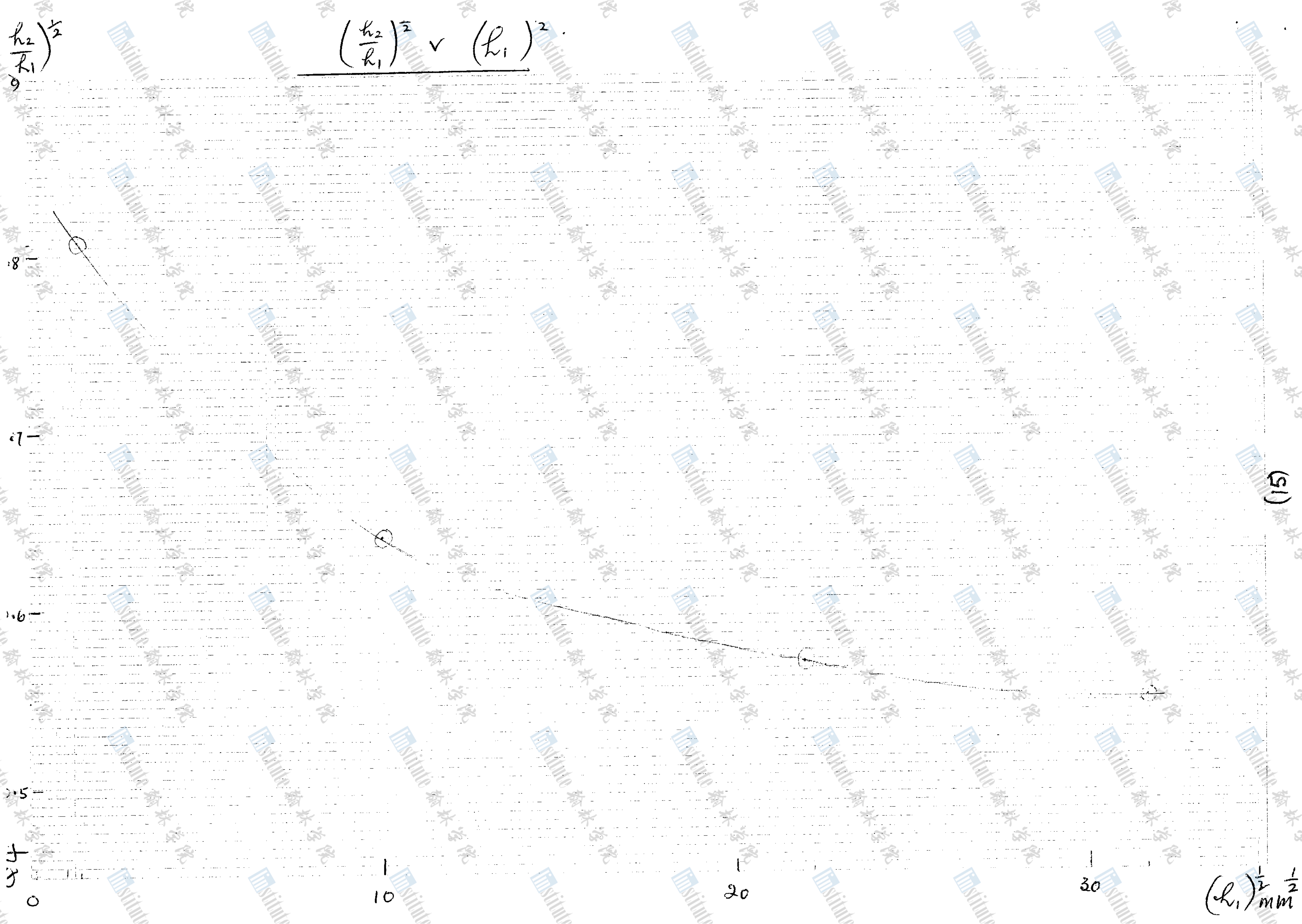


SOLUTION Q4

(iii)

h_2 /mm	h_1 /mm	$(h_2/h_1)^{1/2}$/mm$^{\frac{1}{2}}$	$(h_1)^{\frac{1}{2}}$ /mm$^{\frac{1}{2}}$
290	1000	0.538	31.6
162	500	0.569	22.4
41	100	0.640	10.0
1.3	2.0	0.806	1.41

$$\left(\frac{h_2}{h_1}\right)^{\frac{1}{2}} \text{ v } (h_1)^{\frac{1}{2}}.$$



Q4

Correct table of values [1]

Plot $(h_2/h_1)^{\frac{1}{2}}$ against $(h_1)^{\frac{1}{2}}$ [1]

Correct graph using major portion of graph paper. [1]

Smooth curve through points [1]

(iv) Time to fall 900mm, t_0, given by

$$\frac{900}{1000} = \frac{1}{2} g t_0^2 \quad \text{ie} \quad t_0 = \sqrt{\frac{2}{g}}\sqrt{\frac{900}{1000}} \qquad [\tfrac{1}{2}]$$

Time t_1 to rise to maximum height after first bounce given by

$$h_2 = \frac{1}{2} g t_1^2 \quad \text{ie} \quad t_1 = \sqrt{\frac{2}{g}}\sqrt{h_2}$$

$$2t_1 = 2\sqrt{\frac{2}{g}}\sqrt{h_2} \qquad \text{(A)} \qquad [\tfrac{1}{2}]$$

From $x - (h_1)^{\frac{1}{2}}$ graph

$$\left(\frac{h_2}{h_1}\right)^{\frac{1}{2}} = 0.541 \qquad [1]$$

Giving

$$h_2^{\frac{1}{2}} = 0.541(h_1)^{\frac{1}{2}} = 0.541\sqrt{\frac{900}{1000}}$$

From (A)

$$2t_1 = 2\sqrt{\frac{2}{g}}(0.541)\sqrt{\frac{900}{1000}} \qquad [1]$$

So require time

$$t_0 + 2t_1 = \sqrt{\frac{2}{g}}\sqrt{\frac{900}{1000}}\left[1 + 2(0.541)\right]$$

$$= 0.89 \pm 0.01 \text{ s} \qquad [1]$$

(v) Heat, sound, vibration, deformation of anvil [2]

(half mark for each mechanism up to max. 2)

Q5 (a) SOLUTION TO Q5

(i) $Q_1 = Q_2$ [1]

(ii) $V_1 = \dfrac{Q_1}{C_1}$ $\qquad V_2 = \dfrac{Q_2}{C_2}$ [2]

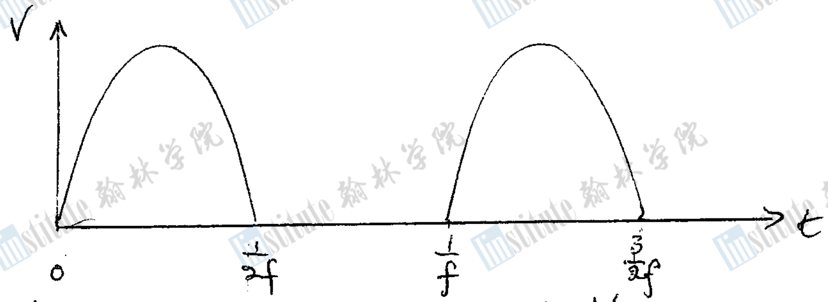
(iii) $E = V_1 + V_2 = Q_1\left(\dfrac{1}{C_1} + \dfrac{1}{C_2}\right)$ [1]
$= \dfrac{Q_1}{C}$

$\therefore \underline{\dfrac{1}{C} = \dfrac{1}{C_1} + \dfrac{1}{C_2}}$ [1]

(iv) Energy $\mathcal{E} = \frac{1}{2} C E^2 = \frac{1}{2}\left(\frac{1}{C_1} + \frac{1}{C_2}\right)^{-1} E^2$ [1]
$= \frac{1}{2} C \left(\frac{Q_1}{C}\right)^2$
$= \underline{\frac{1}{2} \frac{Q_1^2}{C}}$ [1]

[[7]]

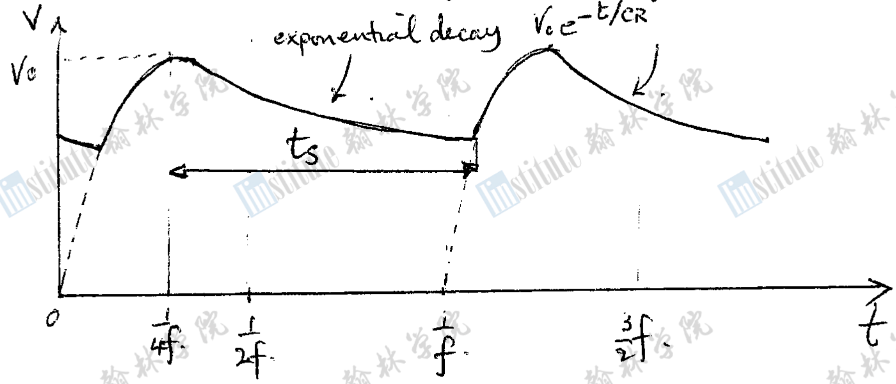
(b) (i)



[1]

Period $= \frac{1}{f}$ [1]

(ii)



Graph → [2]

Explanation → [1]

Exponential decay of voltage across capacitor when diode terminates current

(iii) $7.99049 = 10.0000\, e^{-t_s/cR}$ [1]

$t_s = -cR \ln(0.799049) = -(400\times10^{-6})(100)\ln(0.799049)$
$= -4\times10^{-2}\ln(0.799049)$

$\underline{t_s = 0.00897332 \text{ s}}$ [1]

15(b) (iv) Input voltage $V_I = 10\sin 2\pi(100)[t_s - \frac{3}{4}\frac{1}{f}]$ see diagram in (ii) [1]

$= 10\sin(200\pi)[0.00897332 - 0.0075]$ [1]

$= 10\sin(200\pi)[0.00147332]$

$\underline{= 7.990..}$ to 4 sig-figs [1]

(v) ESTIMATES

$V_d \simeq \frac{1}{2}(10 + 7.990) = 8.995$ [1]

$V_a \simeq \frac{1}{2}(10 - 7.990) = 1.005$ [1]

$f_a \simeq f = 100\ Hz.$ [1]

[[13]]

SOLUTION TO Q6

(a) E is the induced emf in volts due to a conductor cutting the magnetic flux field. It is equal to the rate of change of the flux linkage, Φ, in units of webers per sec.
The minus sign indicates that the induced current produce by the conductor cutting the magnetic field creates a magnetic flux in the opposite direction to the external magnetic flux; the current flows in such a direction as to oppose the change that is taking place. [3]

(b) (i) $E = \dfrac{(6\times10^{5})(80)(720)10^{3}}{60\times60}$ [1]

$\underline{E = 0{\cdot}96\,V}$ [1]

(ii) ZERO — no flux cut [1]

(iii) $E = \dfrac{(3\times10^{-5})(8)(720)\,10^{3}}{60\times60}$ [1]

$\underline{E = 48\ mV}$ [1]

[½]

(iv) Horiz. wing compt. $E_w = \dfrac{(\sin 66^\circ)(80)(720)\times10^{3}(5\times10^{-5})}{60\times60}$ [1]

$E_w = 0{\cdot}72\,V$ [1]

[½]

Vertical component $E_v = \dfrac{(\cos 66^\circ)(8)(720)\times10^{3}(5\times10^{-5})(\cos 45^\circ)}{60\times60}$ [1 [1]

$\underline{E_v = 23\,mV}$ [1]

[[14]]

Q6

(c)

(i) $$\underline{V = \frac{1}{2}B\omega\left(\frac{L}{2}\right)^2 = \frac{1}{8}B\omega L^2}$$ [2]

(ii) ZERO [1]

(iii) $$\frac{1}{2}B\omega(L-x)^2 - \frac{1}{2}x^2 B\omega$$ [1+1]

$$= \underline{\frac{1}{2}B\omega L(L-2x)}$$ [1]

[[6]]

Q7 (a)

SOLUTION TO Q7

(i)

$$T = kR^{\alpha}$$
$$\ln T = \alpha \ln R + \ln k \qquad [1]$$

Plot $\ln T$ against $\ln R$

gradient α, intercept $\ln k$

PLANET	$R/10^8$ km	$\ln R$	T/days	$\ln T$
EARTH	1.49	0.3920	365	5.900
MARS	2.28	0.8242	687	6.532
JUPITER	7.78	2.0516	4333	8.37
URANUS	28.7	3.357	30690	10.33

TABLE OF VALUES [2]

CORRECT GRAPH [2]

GRADIENT $\alpha = \frac{7.60}{5.00} = 1.52$ [1]

approx 1.5

ACCURACY ± 0.07 [1]

(ii) When $\ln R = 0$, $\ln T = \ln k$. i.e. $k = (T)_{R=0}$

From graph $\ln k = 5.4 \pm 0.1$ [1]

$k = 221 \pm 25$ days $/ (10^8 \text{km})^{3/2}$

In SI units

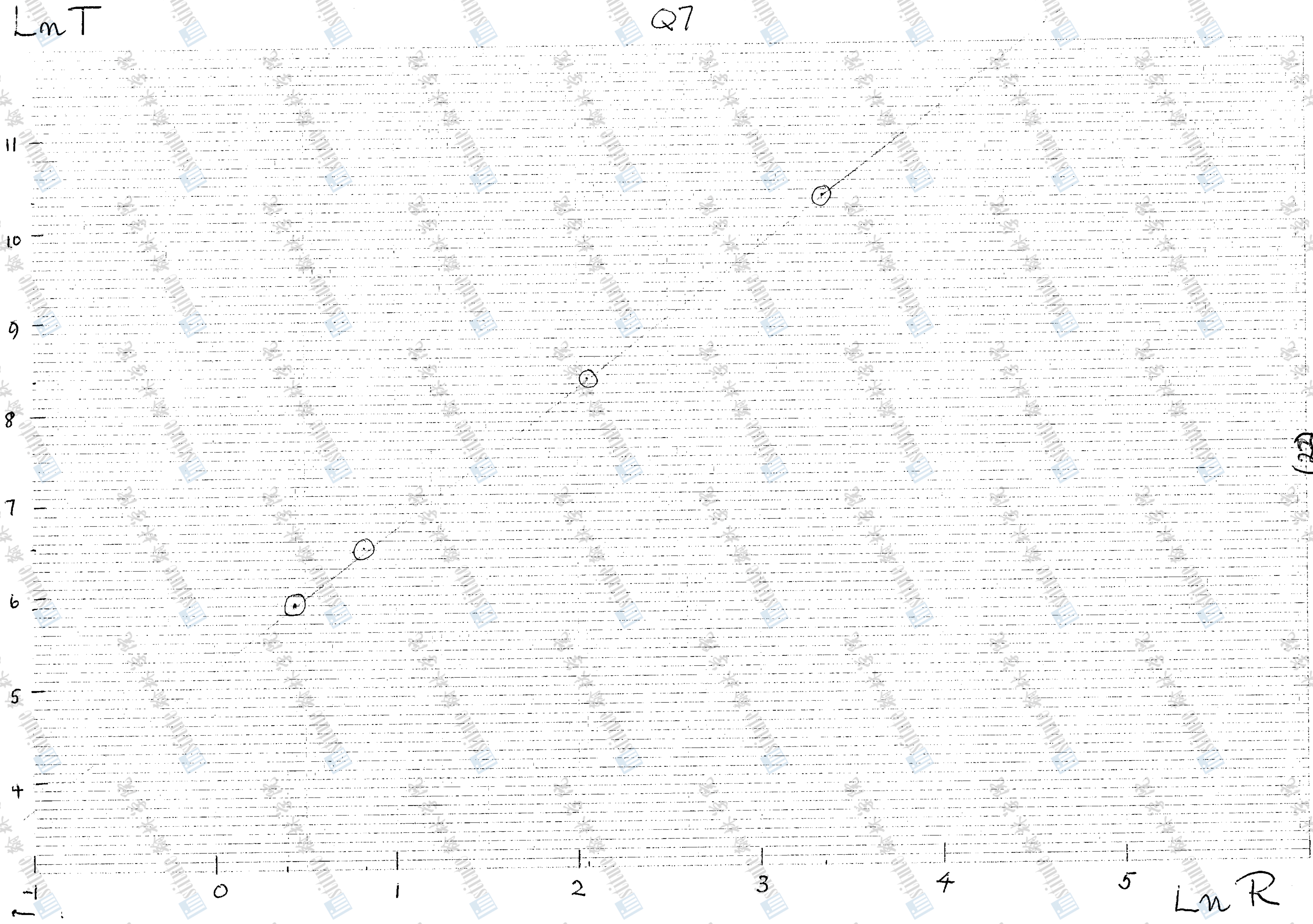
$$k = (221 \pm 25)\, 24 \times 60 \times 60 / (10^{11})^{3/2} \quad s\, m^{-3/2}$$
$$= (1.90 \pm .2)\sqrt{10}\; 10^{-10} \quad s\, m^{-3/2}$$
$$k = (6.0 \pm 0.6)\, 10^{-10}\; s\, m^{-3/2}$$

Magnitude [1]

Dimensions [1]

[[10]]

Q7



(22)

Q7 (b)

(i) Equation for circular motion for planet of mass m ang. vel. ω

$$mR\omega^2 = \frac{GM_s m}{R^2} \qquad [1]$$

$$R^3 = GM_s\left(\frac{T}{2\pi}\right)^2 \qquad T = 2\pi/\omega \; [1]$$

$$T^2 = \frac{(2\pi)^2}{GM_s}R^3$$

$$T = \frac{2\pi}{\sqrt{GM_s}}R^{3/2} \qquad [1]$$

(ii) Determination of M_s

As points all lie on a straight line to within accuracy of the graph, one can use any set of data to determine M_s.

MARS DATA GIVES USING

$$T^2 = \frac{(2\pi)^2}{GM_s}R^3.$$

$$M_s = \frac{(2\pi)^2}{G}\frac{R^3}{T^2} \qquad [1]$$

$$= \frac{(2\pi)^2(2.28\times10^{11})^3}{6.67\times10^{-11}\,(6.87\times24\times60\times60)^2} \qquad [1]$$

$$M_s = (1.98 \pm 0.08)\,10^{30}\,\text{kg} \qquad [1]$$

All the other planets' data give same result

ALTERNATIVELY USING $k = \frac{2\pi}{\sqrt{GM_s}} = 6.0\times10^{-10}$ from part (a) [3]

GIVES SAME RESULT BUT LESS ACCURATE

(iii) Now
$$T_{Moon}^2 = \frac{(2\pi)^2}{GM_E}R_{Moon}^3$$

and
$$T_E^2 = \frac{(2\pi)^2}{GM_s}R_E^3 \qquad [1]$$

Thus

$$\frac{M_s}{M_E} = \left(\frac{R_E}{R_{Moon}}\right)^3\left(\frac{T_{Moon}}{T_E}\right)^2 \qquad [1]$$

$$\frac{M_S}{M_E} = \left(\frac{1.49\times10^{8}}{3.8\times10^{5}}\right)^{3}\left(\frac{27.3}{365}\right)^{2}$$ [1]

$$= 3.4\times10^{5} \pm 0.2\times10^{5}.$$ [1]