British Physics Olympiad 2006

Competition

Paper 2 – 4th November 2005

3 hours plus 15 minutes reading time

There are SEVEN questions in this paper.
The marks for each section are given on the right hand side of the page.

- FOUR questions are to be attempted. Formulae sheets may be used.
- QUESTION 1 IS COMPULSORY. It is expected that students will spend 75 minutes on this question. The total mark allocated to the question is 68. Students can attempt any, or all, of the sections. However the maximum total mark awarded will be 40.
- THREE of the remaining six questions should be attempted. Students are recommended to spend 35 minutes on each of these questions. The maximum mark for each of these questions is 20.

Useful data:

Speed of light	c	3.00×10^{8}	$m\ s^{-1}$
Planck constant	h	6.63×10^{-34}	$J\ s$
Electronic charge	e	1.60×10^{-19}	C
Mass of electron	m_e	9.11×10^{-31}	kg
Acceleration of free fall	g	9.81	$m\ s^{-2}$
Gravitational constant	G	6.67×10^{-11}	$N\ m^{2}\ kg^{-2}$
Gas Constant	R	8.31	$J\ mol^{-1}\ K^{-1}$
Radius of Earth	R_E	6.38×10^{6}	m
Mass of Earth	M_E	5.97×10^{24}	kg
Sun – Earth distance	R_{SE}	1.50×10^{11}	m
Avogadro constant	N_A	6.02×10^{23}	

Q1.

(a) Six identical numbered cubes, each of mass m, lie in a straight line on a smooth horizontal table, Figure 1.a, touching adjacent cubes. A constant force F is applied along the line of cubes.

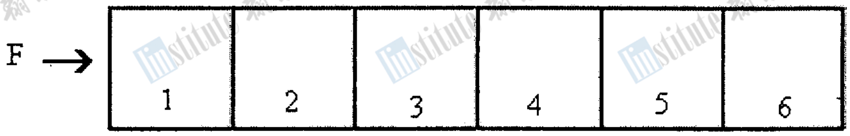

Figure 1.a

Determine:

(i) the acceleration of the system

(ii) the resultant force on each cube.

(iii) the force exerted on the fifth cube by the fourth cube.

[4]

(b) The circuit in Figure 1.b contains a battery, emf E, and resistors all with resistance r. Determine the current I through the battery.

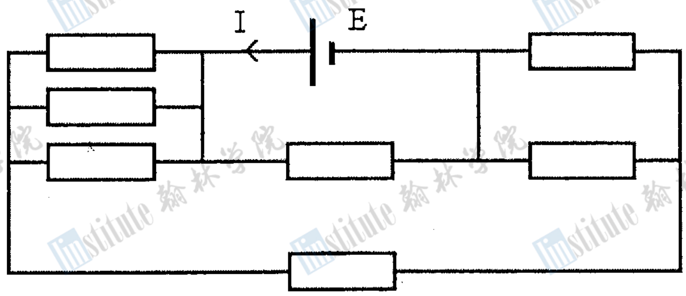

Figure 1.b

[4]

(c) Two identical thin rods are rigidly joined at their ends at right angles. A string is attached to one end and the other end of the string is attached to a support so that the stationary system hangs under gravity.
Determine, using a scale drawing on graph paper:

(i) the position of the centre of gravity G

(ii) the magnitude of the angle between the vertical and the direction of the upper rod using a ruler and calculator only.

[4]

(d) A pond with water of density ρ is covered to a depth b by oil of density $2\rho/3$. A long stick of square cross-section, $4b \times 4b$, with the same density as the oil, floats in the pond. What fraction of the stick is immersed?

[5]

(e) Make the following estimates:

(i) the contribution of the gravitational attraction of the Sun to the acceleration of free fall on Earth.

(ii) the energy required for a man, mass m, to jump 1m on the Earth and the size of the smallest body in the solar system from which a man will be unable to escape by jumping.

(iii) the rate of working of a student's heart if it beats at 72 beats per minute, pumping 7.5×10^{-5} m^3 of blood at each beat against a pressure of 19 kPa.

[12]

(f) A long thin copper strip, width w, is sandwiched between two insulating sheets, each of thickness t and thermal conductivity κ, in an environment at 0°C. The electrical resistance per unit length of the strip, R, at θ°C is given by

$$R = a(1 + b\theta),$$

where a and b are constants.

(i) What is the rate of heat generation in the copper strip, per unit length, by a current I?

(ii) Show that if the current I through the strip is increased, a critical current, I_c, is reached at which the temperature increases indefinitely.

(iii) Calculate I_c using the data in Table 1.f.

Quantity	Value	Quantity	Value
w	5.00 mm	a	2.20×10^{-2} Ωm^{-1}
t	1.00 mm	b	4.30×10^{-3} K^{-1}
κ	1.30×10^{-1} Wm^{-1}K^{-1}		

Table 1.f

[5]

(g) Use graph paper and a ruler to draw *accurate* ray diagrams for the following two optical situations:

(i) A point object is reflected in a plane mirror. Draw two rays of light from the object that are reflected in the mirror.

(ii) A point object is situated between two plane mirrors that are joined to form a right angle. Draw a ray of light from the object that is reflected in both mirrors.

(iii) If, in (ii), the ray is incident on the first mirror at an angle of incidence θ, determine the angle through which the ray has been rotated after two reflections.

[7]

(h)

(i) A sound wave source transmits energy radially in all directions. It can just be detected at 0.50 km from the source, where the intensity is 1.00 pWm^{-2}. What is the power of the source?

(ii) What is understood by the superposition principle of two travelling waves? The two wave forms in Figure 1.h are travelling in opposite directions. Draw three diagrams to show the resultant wave forms when point O reaches A, B and C.

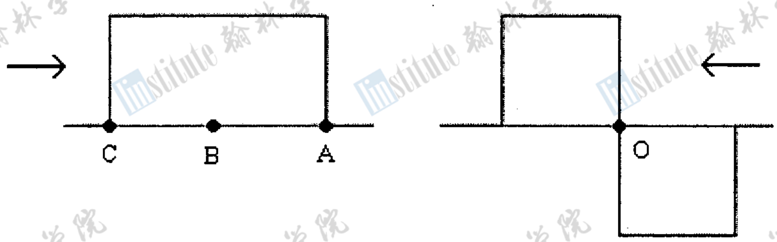

Figure 1.h

[6]

(j)

(i) Rain is falling vertically at 8.0 ms^{-1}. The rain drops make tracks on the side of a car window at an angle of 30° below the horizontal. Calculate the speed of the car, giving a full explanation.

(ii) Calculate the maximum speed at which a car could travel across a hump-backed bridge, having a radius of curvature of 40.0 m, without leaving the road at the top of the bridge.

(iii) A 45 m length of rope, mass 15 kg, hangs over a smooth horizontal peg. One side of the rope is 5 m longer than the other. It is released from rest. When it is initially no longer in contact with the peg, and has not reached the ground, calculate the change in potential energy and the speed of the chain.
What effect does doubling the mass of the rope have on the speed?

[10]

(k) Using the data in Table 1.k, calculate the mass change and the energy released when 10.0 kg of ${}^{235}_{92}U$ undergoes the fission reaction (1u = 931 MeV):

$${}^{235}_{92}U + {}^{1}_{0}n \rightarrow {}^{141}_{56}Ba + {}^{92}_{36}Kr + 3{}^{1}_{0}n$$

Nucleus	Mass / u
${}^{235}_{92}U$	235.04
${}^{141}_{56}Ba$	140.91
${}^{92}_{36}Kr$	91.91
${}^{1}_{0}n$	1.01

Table 1.k

[4]

(l) A fibre optic cable, surrounded by air of refractive index n_a, consists of a glass core of refractive index n_g enclosed in cladding of refractive index n_c.

(i) Explain, with a diagram, how light is transmitted down the cable with little loss of intensity.
(ii) What are the limitations on the light paths?

[4]

(m) A car starts from rest at time $t = 0$ and travels with constant acceleration a_1 for time t_1. From time t_1 to t_2 it travels with constant speed u. After t_2 a retarding acceleration is applied, of initial magnitude a_2, which decreases linearly to zero at time t_3 when the car comes to rest.

Sketch the following graphs:

(i) acceleration – time
(ii) velocity – time
(iii) distance – time

[3]

Q2.

A small sound transmitter T radiates uniformly in all directions and at four times the power of each of two similar transmitters, S_1 and S_2, which are situated a distance $l = 0.25$ m on either side of T, along a north-south line. T is wired to be at 180° out of phase with S_1 and S_2. All three transmitters emit a 200 kHz signal. A small receiver, R, is placed a distance $d = 10.0$ m due east of T and slowly moved eastwards; being, in general, a distance x from T, $x \geq 10.0$ m. The speed of sound is 330 $\mathrm{ms^{-1}}$.

(a)

(i) How does the intensity, I, of a simple point source of sound vary with distance r from the source?

(ii) Show that the path difference $p = S_1R - TR$ is given approximately by

$$p = \frac{1}{32\,x}, \qquad x \geq 10$$

[3]

(b) When all transmitters are on, determine the condition for the intensity of the signal at R to be:

(i) a maximum
(ii) a minimum
(iii) Where do these maxima and minima occur?
(iv) Sketch the variation of the intensity, I, against x, for $x \geq 10$, when all transmitters are on.

[9]

(c) At the position/s of maximum signal intensity the following transmitters are switched off:

(i) S_1 and S_2
(ii) T.

Determine, with an explanation, by what factor the power received falls in each case.

[4]

(d) Indicate graphically how the intensity, I, of the signal received at R varies with x when:

(i) T is switched off
(ii) S_2 is switched off

[4]

Note: $(1+y)^{1/2} \approx 1 + \frac{1}{2}y$ for $y << 1$

Q3.

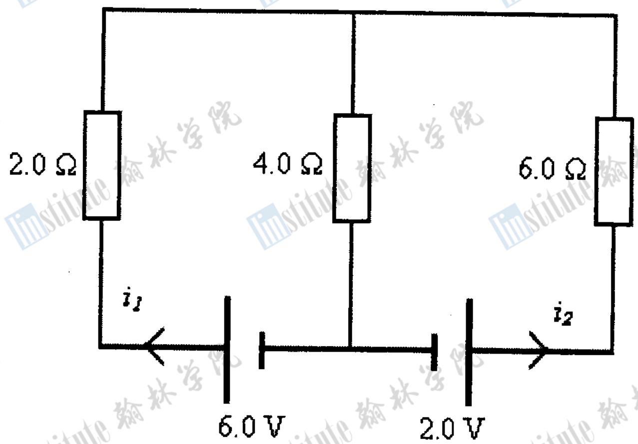

Figure 3.1

(a) In the circuit, Figure 3.1, calculate the currents when a conductor replaces:

(i) the 6.0 V source; i_{16} and i_{26} replacing i_1 and i_2
(ii) the 2.0 V source; i_{12} and i_{22} replacing i_1 and i_2

[4]

(b) It can be shown that the currents in Figure 3.1 are given by $i_1 = i_{16} + i_{12}$ *and* $i_2 = i_{26} + i_{22}$.

(i) Determine, using this result, i_1 and i_2.
(ii) Deduce the current through the 4.0 Ω resistor.
(iii) Verify that i_1 and i_2 satisfy Kirchoff's equations for the circuit.

[7]

(c)

(i) Calculate the power dissipated in the 4.0 Ω resistor.
(ii) Determine the rate of energy conversion for the 6.0 V cell.

[3]

(d)

(i) What modifications, if any, are required to the solutions to (b)(i), if the batteries are replaced by a.c. sources with, respectively, amplitudes of 6.0 V and 2.0 V and a common phase and angular frequency ω?
(ii) Comment on the case in which the a.c. sources of voltage in (i) differ in phase by π.

[6]

Q4.

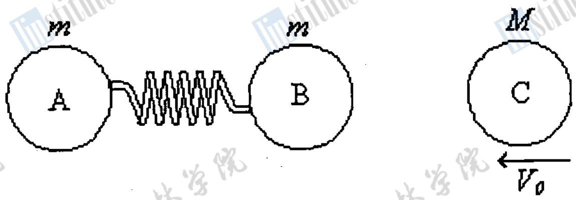

Figure 4.1

Two identical balls, A and B, each of mass m are joined by a massless spring, spring constant k, Figure 4.1. They are at rest on a smooth horizontal surface. A ball C, mass M and velocity V_o, strikes B. All balls are constrained to move along a straight line.

(a)

(i) Write down the energy and momentum equations for the collision in which, immediately after impact, the velocities of the balls are V_A, V_B and V_C in the direction of V_o.

(ii) Verify, by substituting into the equations, that there are two solutions:

$$V_C = V_o, \qquad V_B = 0, \qquad V_A = 0$$

and

$$V_C = \frac{M-m}{M+m}V_0, \qquad V_B = \frac{2M}{M+m}V_0, \qquad V_A = 0$$

(iii) Why must the first solution be rejected? Explain why $V_A=0$

(iv) Deduce the velocity, after the collision, of the centre of mass of the system consisting of A and B.

[10]

(b) Consider the subsequent motion of A and B in their centre of mass system. Assume that their displacements, in their centre of mass system, about their initial positions following the impact are given by:

$$x = A\sin\omega t,$$

where x is measured in the direction of the centre of mass, A is the amplitude, ω is the angular frequency and t is the time measured from the instant of collision.
Determine:

(i) ω
(ii) A
(iii) Write down the position of B, x_B, in the laboratory system of coordinates, measured from the instant of collision.

[10]

Q5.

I / mA	V / μV
2	2201
3	3302
4	4428
5	5514
6	6624

(a)

t / ms	R / m
0.24	19.9
0.66	31.9
1.22	41.0
4.61	67.3
15.00	106.5

(b)

Table 5.1

(a) In an experiment to determine the self heating of a platinum resistance thermometer the values of current, I, and voltage, V, Table 5.1 (a), were transcribed from a laboratory notebook. The uncertainties in V are of the order 1 μV and the values of I were of a much greater accuracy than those of V.

(i) Identify and explain any anomalous result/s in Table 5.1 (a)
(ii) What should be done about it/them?
(iii) Show graphically that the resistance, R, of the thermometer increases linearly with I^2 due to a self heating.
(iv) Determine the parameter/s, and their accuracy, that determine the relation between R and I^2. Give explicitly the equation relating R and I^2.

[11]

(b) The explosion of an atomic bomb in the atmosphere produces a spherical fireball of radius $R(t)$ at time t following detonation. The constant energy E released at this instant depends on R, t and ρ, where ρ is the density of the atmosphere. The relation between these parameters is given by

$$E = \rho R^{\alpha} t^{-2},$$

where α and ρ are constants.

Explain how, from a graph, you could:

(i) verify that the data in Table 5.1 (b), taken following an explosion, satisfies this relation.
(ii) deduce the relation between E and ρ.
(iii) obtain the value of α from the graph.

Determine the theoretical value of α obtained by equating units or dimensions in the equation.

[9]

Q6.

(a)

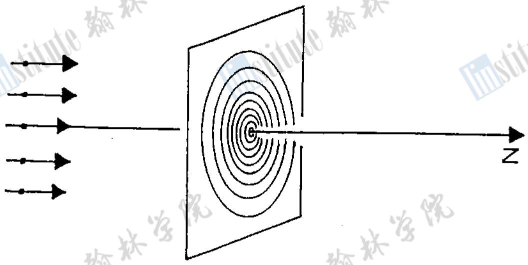

Figure 6.1

A uniform parallel beam, of alpha particles, number per unit volume ρ and velocity u, each consisting of two protons and two neutrons, travels along the z-axis. Consider a plane perpendicular to the z-axis consisting of concentric circles, centres on the z-axis, with radii nt, where the constant t is the radial distance between adjacent circles and n is an interger (0, 1, 2, 3…), Figure 6.1.

(i) Determine the number N of alpha particles per second that pass through an annulus between the *nth* and *(n+1)th* circle.

(ii) Sketch the graph of N against n.

[3]

(b) The beam of alpha particles encounters a **fixed** gold nucleus, along the z-axis, consisting of 79 protons and 118 neutrons, Figure 6.2.

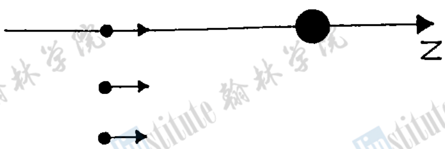

Figure 6.2.

(i) Sketch the trajectories of the three alpha particles indicated in Figure 6.2.

(ii) Why are relatively few particles scattered through large angles?

(iii) What is the largest angle through which a particle is scattered?

(iv) Why do bound electrons, in orbit around the gold nucleus, have a negligible influence on the alpha particle trajectories?

[5]

(c)

(i) Determine the distance of closest approach, r_1, of an alpha particle to the nucleus.

(ii) If the gold nucleus is free to move and initially at rest, what is the velocity of an alpha particle *relative* to the gold nucleus at the distance of closest approach, r_2? Determine r_2 in this case.

Assume the masses of the neutron and proton are both equal to m.

[12]

Q7.

(a) A projectile, mass m, is thrown vertically upwards with velocity v. Calculate:

(i) the maximum height, H, reached.
(ii) the time taken, T, to return to the ground.

[2]

(b) A horizontal wind exerts a horizontal force on the projectile of αmg, where α is a constant. The projectile has initial vertical and horizontal components of velocity v and u respectively; u being taken in the same direction as the wind.

(i) What is the resultant force on the mass?
(ii) Determine the horizontal range, R, of the projectile.
(iii) Sketch the trajectory, indicating any axis of symmetry and giving its direction.
(iv) Under what initial condition, with the wind present, will the trajectory be a straight line? Give the elevation of this trajectory.

[11]

(c) Determine the work done *on* the mass during its motion:

(i) from its initial position to the maximum height in the case of a straight line trajectory.
(ii) for the general trajectory, from launch to its return to the ground.

[7]