SOLUTIONS

MARKS IN RED AND CIRCLED

BPhO 2005 PAPER 2

Q1 (a) Let T be the original temperature

Heat gained by thermometer = Heat lost by water 1

$$20(50-18) = (T-50)\,4200\,(0.250) \quad 1$$

$$640 = (T-50)\,1050$$

$$T = 50.0 + \frac{640}{1050} \quad 1$$

$$T = 50.6 \quad 1$$

[4]

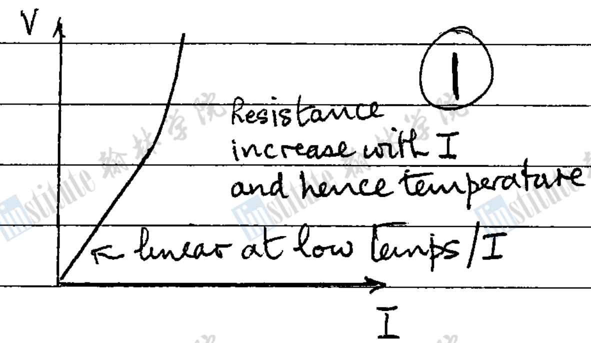
(b) $V = (6I - 3I^2 + 2I^3)/6$

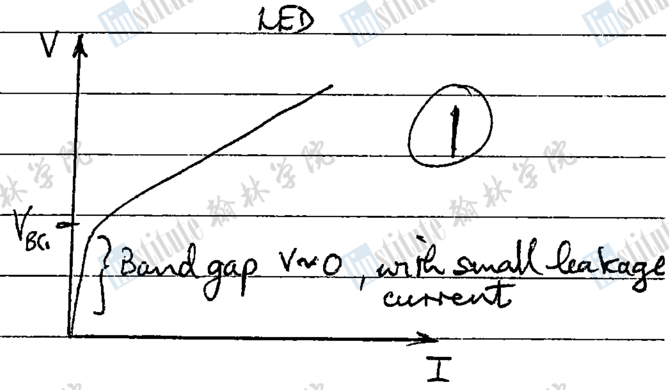
$R = \frac{V}{I} = 1 - \frac{1}{2}I + \frac{1}{3}I^2$

(i) $I = 2$, $R = \frac{4}{3}\ \Omega$ 1

(ii) $I = 0$, $R = 1\ \Omega$ 1

(iii) GRAPHS: LIGHT BULB LED





(iv) EXPLANATIONS:

Resistance increases with temp., and hence current. Linear for small I / low temps. 1

Little current for $V < V_{Band\,gap}$ (V_{BG}), subsequently increases with I. 1

[6]

(c)

(i) The charge repels similar charges in the copper sphere and attracts opposite charges causing a net attraction 1

When earthed the like charges in the sphere are repelled to earth, leaving only opposite charges in the sphere. This produces an increased attraction 1

(ii) On a rough floor friction prevents the base of the ladder from sliding. 1
On a smooth floor there is no force preventing the ladder from sliding; 1
this force is necessary to balance the normal reaction from the wall.

(iii) If the camera hole is too small there is insufficient light to produce a contrasted 1
image (if exceedingly small diffraction effects present). If the hole is too large it
no longer acts as a 'point' hole and produces blurring due to overlapping 1
images from the finite hole.

(iv) The blue cloth only reflects blue light. Sodium light is yellow so
no light is reflected — so cloth appears dark. 1

[7]

(d) (i) Light is a transverse wave and sound is a longitudinal wave. Only
transverse waves can be polarized. 1
Rotate a polaroid sheet in the beam. The intensity will vary from a
maximum to a minimum for a partially polarized beam. A fully
polarized beam causes extinction at the minimum. 1

(ii)

$[A] = [y] = [L]$	units: m	$\frac{1}{2}$	2
$[\pi] = [0]$	units: dimensionless	$\frac{1}{2}$	
$[\beta] = [T^{-1}]$	units: s^{-1}	$\frac{1}{2}$	
$[\gamma] = [L]$	units: m	$\frac{1}{2}$	

y will maintain the same value providing

$$2\pi(\beta t - x/\gamma) = \text{const}$$ 1

$$x = -\frac{\text{const}}{2\pi} + \gamma\beta t$$ 1 3

Velocity $u = \gamma\beta$, which is a constant 1 [either $u = \frac{dx}{dt} = \beta\gamma$ or $\frac{\Delta x}{\Delta t} = \gamma\beta$]

Non mathematical explanation OK if correct

[7]

(e) (i) Wind drives water vapour from the clothes. This causes more water to
evaporate, which cools the remaining water in the clothes. Sufficient
cooling will extract sufficient latent heat of fusion from the 2
water to cause freezing of the water

e) (ii) If mass M freezes and mass m evaporates then heat extracted from M $= 333 \times 10^3 M$ J (½)

energy extracted from water $= 2500 \times 10^3 m$ J (½)

Equating

$$333 M = 2500 m \quad (1)$$

$$\frac{M}{m} = \frac{2500}{333}$$

$$\therefore \frac{M}{m+M} = \frac{2500}{2500+333} = \frac{2500}{2833} = 0.882 \quad (1)$$

[5]

f) (i) '$v^2 = u^2 + 2as$'

$(310)^2 = 2a(1.20)$

$a = 4.00 \times 10^4 \text{ ms}^{-2}$ (1)

(ii) '$v = u + at$'

$310 = 4.00 \times 10^4 t$

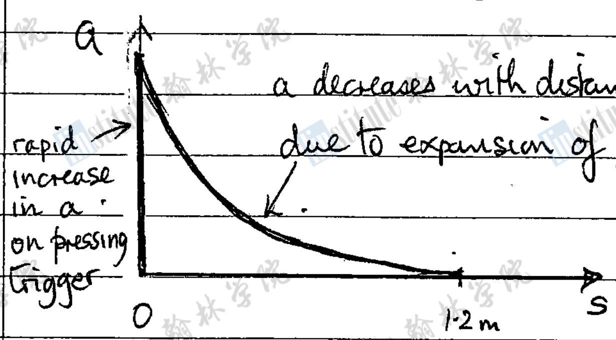
$t = 7.74 \times 10^{-3}$ s (1)

Rotational speed $= 2(2\pi) / 7.74 \times 10^{-3}$ rads/s

$= 1.62 \times 10^3$ rads/s (1)

(iii) Rotation produces stability in the motion of the bullet (1)

iv



a decreases with distance due to expansion of gas. (1)

[5]

g) (i) Upthrust $= \text{Vol.}(\rho_{air} - \rho_{He})g$ (1)

$= 512(1.290 - 0.178)g$

$= 569g$

$= mg$ where m is largest mass raised

$m = 569$ kg (1)

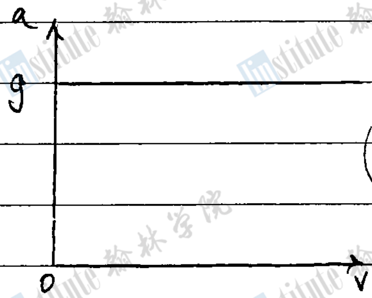
1) (ii) Relative to surrounding air mass, air in balloon has a 'negative' air mass $= V(\rho_{He} - \rho_{air})$, where V is the volume of the balloon. This causes an upthrust when in equilibrium and will behave like a negative mass when applying the equation of motion when in motion.

Inertia of air is greater than inertia of He. Therefore air continues forward more than He. When the bus decelerates the passengers will be thrown forward if not restrained, for example, by a seat belt; they have a positive mass. The balloon will be displaced backward with its string at an angle. 2 When the bus comes to rest the string will return to the vertical position. 1

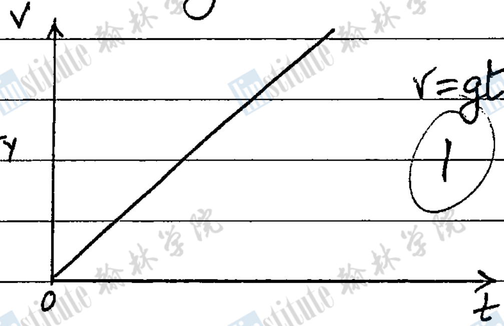
[5]

2) (1) a against v   (2) v against t

(i)

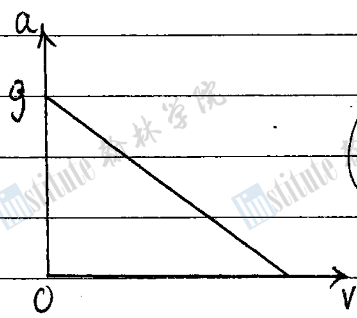


1 MOTION UNDER GRAVITY

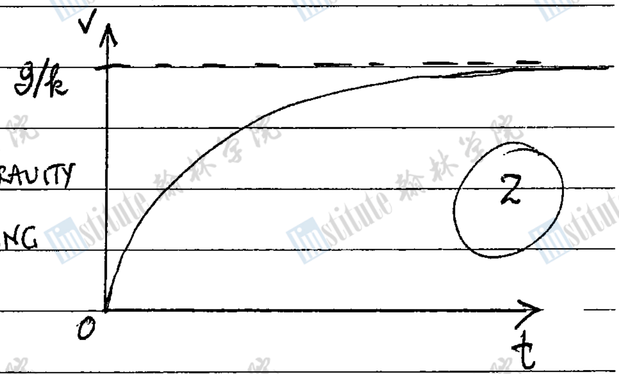


1

(ii)

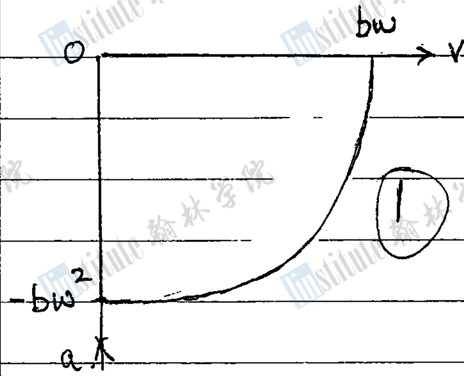


1 MOTION UNDER GRAVITY WITH DAMPING

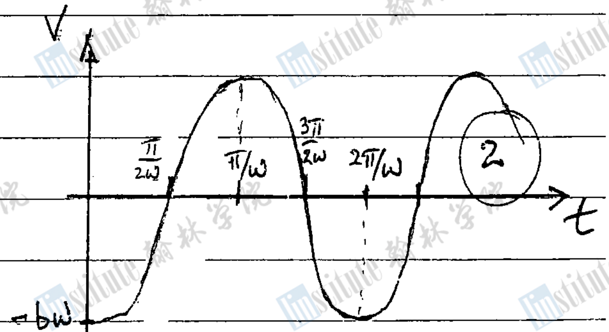


2

(iii)



1 SHM



2

[8]

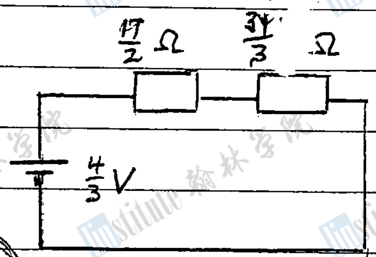
j) (i) 10.0 Ω in parallel with 4.00 Ω : total resistance $\left(\frac{1}{10}+\frac{1}{4}\right)^{-1}$ 1

Total resistance $= \left(\frac{1}{10}+\frac{1}{4}\right)^{-1}+2 = \frac{34}{7}\,\Omega$ 1

Current through 2.00 Ω $= 2/\left(\frac{34}{7}\right)$

∴ $I_0 = \frac{2}{17} = 0.118$ amps 1 ✓

(ii) $V_{AB} = 2.00\left(\frac{4}{6}\right) = \frac{4}{3} = 1.333$ volts 1



(iii) $\frac{V_{AB}}{I_0} = \frac{4}{3}\left(\frac{17}{2}\right) = \quad \Omega$ 1

$I = \frac{4}{3}/\left(\frac{17}{2}+\frac{34}{3}\right) = \frac{4}{3}\left(\frac{6}{119}\right) = \frac{8}{119} =$ 67.2 mA 1 = 67.2 mA

[6]

k) Escape velocity for planet Mars, mass M and radius R, is V_{ME}

For body mass m escaping

$$\frac{1}{2} m V_{ME}^2 = \frac{GMm}{R}$$

$$V_{ME} = \sqrt{\frac{2GM}{R}}$$ 1

On Earth escape velocity

$$V_E = \sqrt{\frac{2GM_E}{R_E}}$$ 1

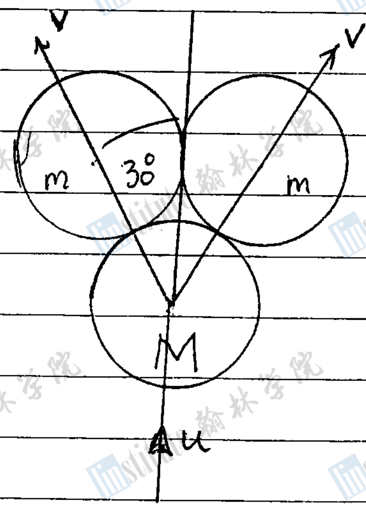
Ratio

$$\frac{V_{ME}}{V_E} = \sqrt{\frac{M}{M_E}\frac{R_E}{R}} = \sqrt{(0.1074)/(0.5326)}$$

$$V_{ME} = 0.4491\ V_E$$ 1

[3]

l) Let velocities of the m masses be v at 30° to incident velocity u.



Conservation of momentum: 1

$$Mu = 2mv\cos 30° = 2mv\frac{\sqrt{3}}{2} = \sqrt{3}\,mv$$ 1 ①

Conservation of energy:

$$\tfrac{1}{2}Mu^2 = 2\left(\tfrac{1}{2}mv^2\right)$$

$$Mu^2 = 2mv^2$$ 1 ②

Substituting from ① for v into ②

$$Mu^2 = 2m\left(\frac{Mu}{m\sqrt{3}}\right)^2$$

$$M = \frac{3}{2}m$$ 1

[4]

ii) Let current in R_1 be I_1 and that in R_2, I_2, the forces will be pointing out of the plane of the diagram and perpendicular to it. 1

The resultant torque τ = moment of the forces

$$\tau = \tfrac{1}{2}d\,[BI_2L - BI_1L]\ 1$$

$$= \tfrac{1}{2}BLd\,[I_2 - I_1] \quad (1)$$ anticlockwise around I as viewed from above

For the circuit of parallel resistors

$$I_1 = \frac{R_2}{R_1+R_2}I \quad \& \quad I_2 = \frac{R_1}{R_1+R_2}I \quad 1$$

Substituting in (1)

$$\tau = \frac{(R_1-R_2)BLdI}{2(R_1+R_2)} \quad 1$$

Anticlockwise 1 around I as viewed from above

[6]

2 (i) Applying Newton's gravitational law of attraction to mass m at h

$$m g_h = \frac{G M_E m}{(R_E+h)^2} \quad 1$$

$$g_h = \frac{G M_E}{(R_E+h)^2}$$

But $$g_0 = \frac{G M_E}{R_E^2}$$

Thus $$g_h = g_0 \frac{R_E^2}{(R_E+h)^2} \quad 1$$

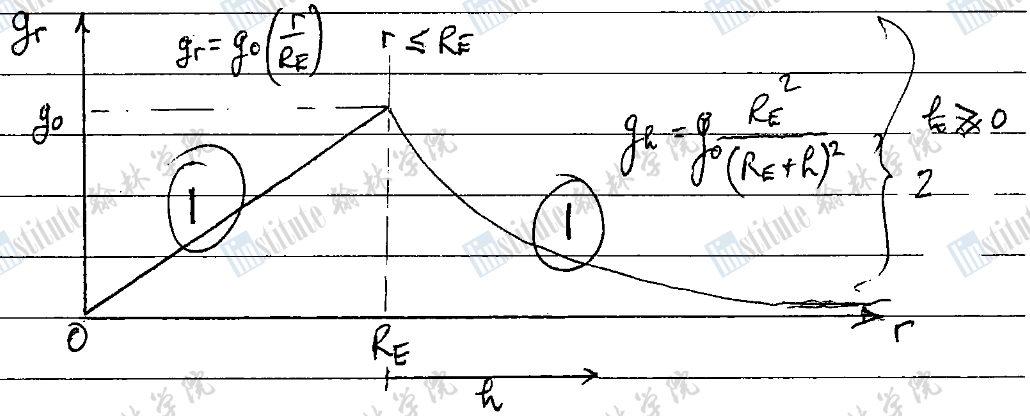
(ii) Below the Earth's surface force on mass m due to sphere radius r, M_r,
Thus

$$m g_r = \frac{G m M_r}{r^2} \quad 1$$

But $$M_r = \left(\frac{r}{R_E}\right)^3 M_E \quad 1$$

So $$g_r = \frac{G M_E r}{R_E^3} = g_0\left(\frac{r}{R_E}\right) \quad 1$$

(iii)



(iv) The Earth does not have uniform density, but it is spherically symmetric, with increasing density towards its centre, consequently the equations in (i) and (ii) do apply, providing in (ii) the M_r is the mass within the sphere of radius r and the subsequent equations are modified to take this into account 1 1

[9]

2(b) (i) For a satellite in circular motion, mass m, angular frequency ω,

$$m(R_E+h)\omega^2 = \frac{GM_Em}{(R_E+h)^2} \quad 1$$

where

$$\omega = 2\pi/(2.0\times 60\times 60)$$

$$(R_E+h)^3 = \frac{GM_E}{\omega^2}$$

$$= \frac{(6.67\times10^{-11})(5.97\times10^{24})}{(2\pi/2.0\times60\times60)^2} \quad 1$$

$$= (6.67)(5.97)10^{13}\left(\frac{7.20\times10^3}{2\pi}\right)^2$$

$$= 5.229\times10^{20}\ m^3 \quad 1$$

$$R_E+h = 8.056\times10^6\ m$$

$$h = (8.056-6.38)10^6\ m$$

$$h = 1.68\times10^6\ m = 1.68\times10^3\ km \quad 1$$

(ii) Angular velocity of the Earth $= 2\pi/(24\times60\times60)$ rads/s $\frac{1}{2}$

Angular velocity of satellite $= 2\pi/(2\times60\times60)$ rads/s $\frac{1}{2}$

Angular velocity of satellite relative to the Earth $= \frac{2\pi}{60\times60\times2}\left[1-\frac{1}{12}\right]$ $\frac{1}{2}$

$$= 2\pi(11)/(12\times60\times60)$$

$$= 8.000\times10^{-4}\ s^{-1} \quad \frac{1}{2}$$

(iii) In the diagram

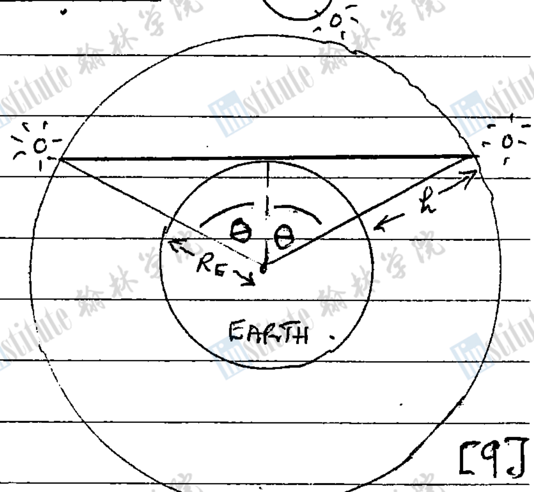
$$\cos\theta = \frac{R_E}{R_E+h} \quad 1$$

$$= \frac{6.38\times10^6}{8.056\times10^6}$$

$$= 0.792 \quad 1$$

$$\theta = 37.63^\circ$$

$$\therefore 2\theta = 75.3^\circ \quad 1$$



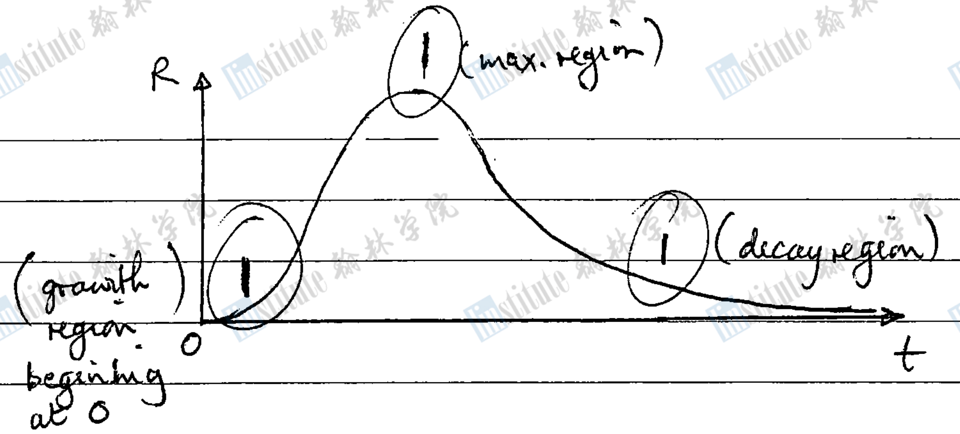
[9]

(c) The gravitational field of the Moon, and to a lesser extent the Sun, will perturb the circular motion and limit the largest periods. Viscous effects of the Earth's atmosphere will reduce the energy of the satellite and limit the smallest possible periods of a satellite

[2]

Q3

a)



[3]

b) (i) $R = \frac{\lambda_1}{(\lambda_2-\lambda_1)}\left[e^{-\lambda_1 t} - e^{-\lambda_2 t}\right]$

For small t

$$e^{-\lambda t} = 1 - \lambda t + \cdots$$

Giving

$$R = \frac{\lambda_1}{(\lambda_2-\lambda_1)}\left[(1-\lambda_1 t+\cdots) - (1-\lambda_2 t+\cdots)\right] \quad 1$$

$$= \frac{\lambda_1}{(\lambda_2-\lambda_1)}\left[(\lambda_2-\lambda_1)t+\cdots\right]$$

$$\underline{R = \lambda_1 t + \cdots} \quad 1$$ (accept proof if λ_1 neglected with respect to λ_2)

(ii) For large t and $\lambda_2 > \lambda_1$ the second exponential term can be neglected, then 1

$$\underline{R = \frac{\lambda_1}{(\lambda_2-\lambda_1)} e^{-\lambda_1 t}} \quad 1$$ (accept $\frac{\lambda_1}{\lambda_2}e^{-\lambda_1 t}$, where λ_1 neglected in denominator)

[4]

c) (i) $R = \lambda_1 t$

Plot R against t 1 for small t, in the region where $e^{-\lambda t} \simeq 1 - \lambda t$

This will be a straight line gradient λ_1. Hence λ_1 determined by measuring the gradient. 1

(ii) $R = \frac{\lambda_1}{(\lambda_2-\lambda_1)} e^{-\lambda_1 t}$ (I)

Thus

$$\ln R = \ln(\lambda_1/(\lambda_2-\lambda_1)) - \lambda_1 t \quad 1$$

Plot $\ln R$ against t for values of t where $e^{-\lambda_2 t} \ll e^{-\lambda_1 t}$

This is a straight line gradient $(-\lambda_1)$, which has previously been determined, and intercept $\ln[\lambda_1/(\lambda_2-\lambda_1)]$. Knowing λ_1, λ_2 can be determined from this intercept 1

ACCEPT SOLU. WHERE λ_1 NEGLECTED IN DENOMINATOR OF (I) eg if intercept $k = \ln[\lambda_1/(\lambda_2-\lambda_1)]$ 1

$$\frac{\lambda_1}{(\lambda_2-\lambda_1)} = e^{k}$$

$$\underline{\lambda_2 = \lambda_1 e^{-k} + \lambda_1 = \lambda_1(1+e^{-k})}$$ 1

[3]

d) $R = \frac{N_2}{N_0}$ so $\frac{N_2}{N_0} = \frac{\lambda_1}{(\lambda_2-\lambda_1)}\left[e^{-\lambda_1 t} - e^{-\lambda_2 t}\right]$ 1

Also

$$N_1 = N_0 e^{-\lambda_1 t}$$ 1

For $\lambda_2 \gg \lambda_1$ and $e^{-\lambda_1 t} \gg e^{-\lambda_2 t}$ (suitable long times)

$$\frac{N_2}{N_1} = \frac{\lambda_1}{(\lambda_2-\lambda_1)}$$

$$\simeq \frac{\lambda_1}{\lambda_2}$$ 1 as $\lambda_2 \gg \lambda_1$

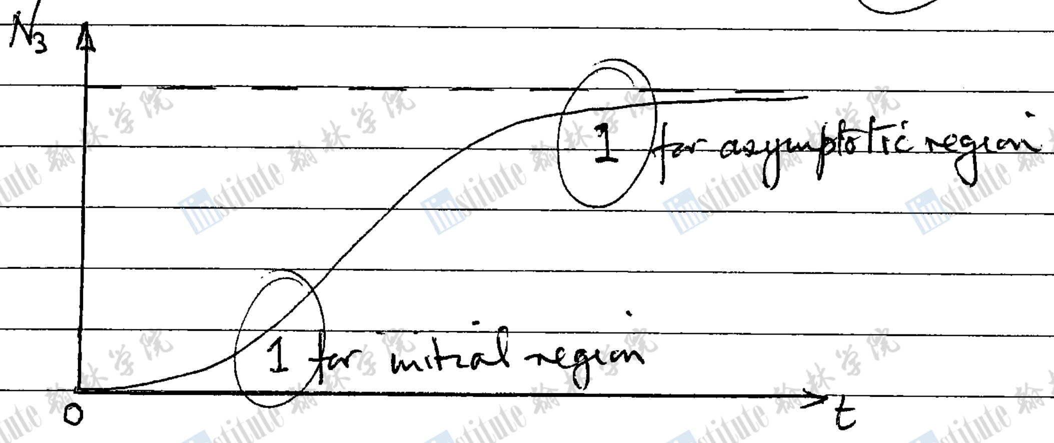
$$\therefore \underline{N_2\lambda_2 = N_1\lambda_1}$$

Conditions for validity: $\lambda_2 \gg \lambda_1$ 1

$e^{-\lambda_1 t} \gg e^{-\lambda_2 t}$ 1

[5]

(e)



[2]

$N_3 = N_0 - N_1 - N_2$

Q4

a) (i) Force on particle is perpendicular to its displacement hence work done is zero. Work done in one revolution is zero. 1 1

(ii) Force acting on particle $\frac{mS^2}{r}$ 1 towards centre of circle radius r.
The direction only varies uniformly with time 1

[4]

b) (i) $V_{xy}^2 = u^2 + v^2$

$$\underline{V_{xy} = \sqrt{u^2+v^2}} \quad 1$$

(ii) $\underline{z = wt}$ 1

(iii) $\frac{mV_{xy}^2}{r}$ 1 $= BQ V_{xy}$ 1

$$r = mV_{xy} / BQ \quad \text{I}$$

$$\underline{r = \frac{m\sqrt{u^2+v^2}}{BQ}} \quad \text{II} \quad 1$$

[5]

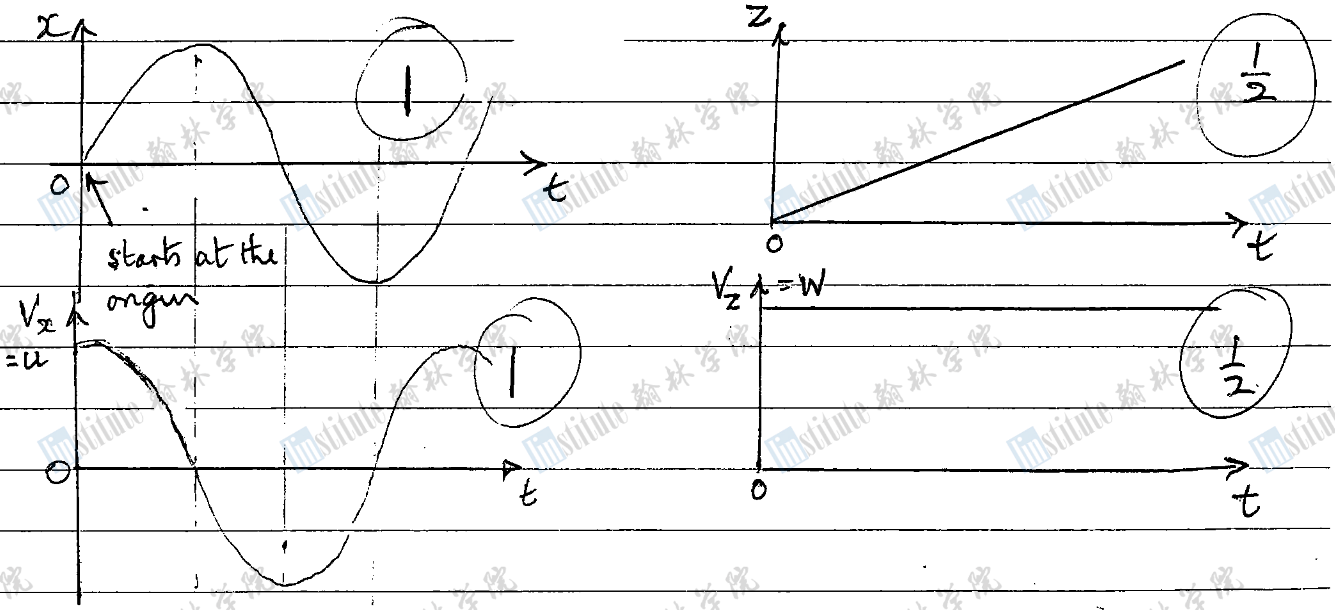
c) (i) $T = \frac{2\pi}{\omega} = \frac{2\pi r}{V_{xy}}$ 1

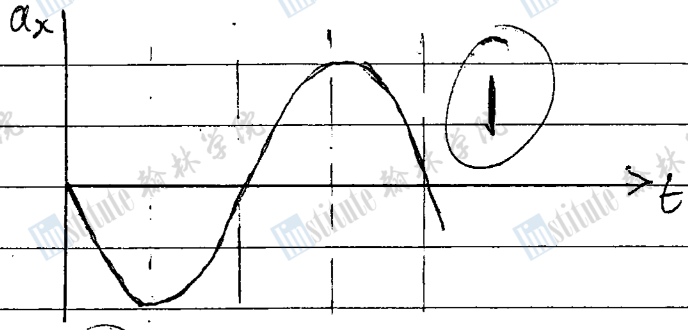
$$\therefore \underline{T = \frac{2\pi m}{BQ}} \quad 1 \text{ from I} \qquad \text{III}$$

(ii) If B increases, from II, r decreases 1 and, from III, T decreases 1

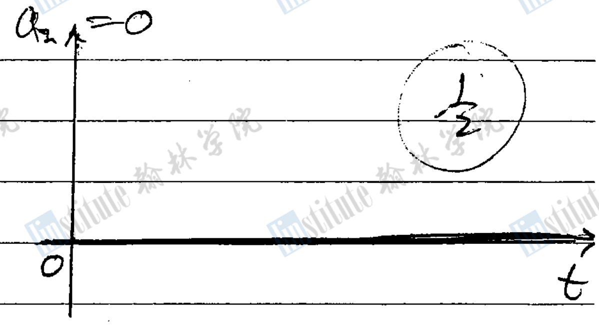
[4]

d) (i)





1



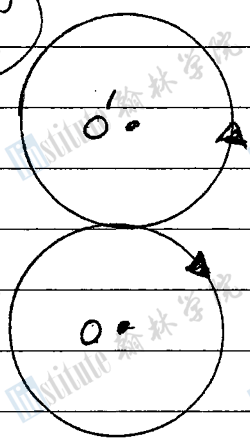
$\frac{1}{2}$

$\frac{1}{2}$ mark for initial correct displacements $x=0$ & $z=0$.

[5]

e) From II reversing B will change the centre of rotation in the x-y plane from O to O' in the diagram. 1

The motion along the z-direction will be unchanged. 1



x-y plane

[2]

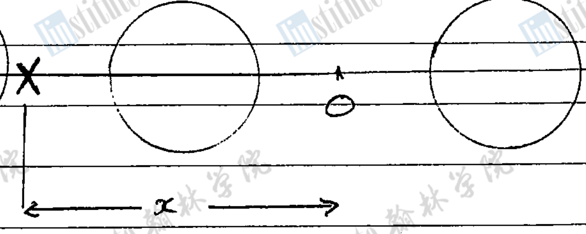
a) (i) Electrostatic potential at O $= -\frac{Q}{(4\pi\varepsilon_0)3a} - \frac{Q}{(4\pi\varepsilon_0)3a} = \frac{-2Q}{(4\pi\varepsilon_0)3a}$ 1

(ii) Let the point/s be at distance x from O, at X

Then

$$-\frac{2Q}{(4\pi\varepsilon_0)3a} = \frac{-Q}{4\pi\varepsilon_0(x-3a)} + \frac{-Q}{4\pi\varepsilon_0(x+3a)} \quad 1$$

$$= \frac{-2xQ}{4\pi\varepsilon_0(x^2-9a^2)}$$



Hence

$$x^2 - 9a^2 = 3xa$$

$$x^2 - 3xa - 9a^2 = 0$$

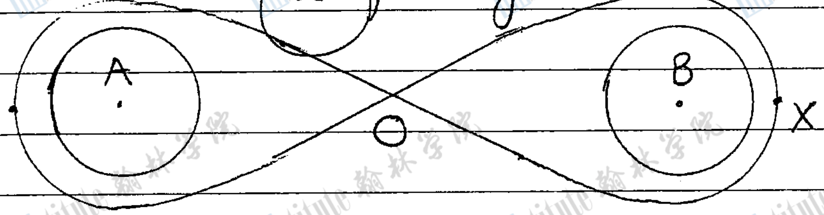
$$x = \tfrac{1}{2}\left[3a \pm \sqrt{9a^2 + 4.9a^2}\right] = \frac{3a}{2}\left[1 \pm \sqrt{5}\right] \quad 1$$

Negative value corresponds to point inside sphere where equation not valid. There are thus two points at distance from O of 1

$x = \frac{3a}{2}\left[1+\sqrt{5}\right]$ to left and right of O! 1

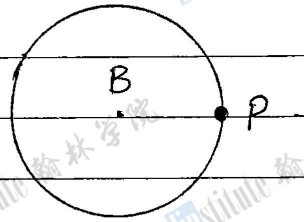
(iii) 2 for diagram

$V = -\frac{2Q}{(4\pi\varepsilon_0)3a}$



[7]

b) (i) Point P has highest potential 1



$$V_H = -\frac{Q}{(4\pi\varepsilon_0)7a} - \frac{Q}{(4\pi\varepsilon_0)a} \quad 1$$

$$V_H = -\frac{8Q}{(4\pi\varepsilon_0)7a} \quad 1$$

(ii) To reach A the positive charge must pass through the potential barrier given in (iii). This is the lowest potential barrier; $V_L = -\frac{2Q}{(4\pi\varepsilon_0)3a}$ 1

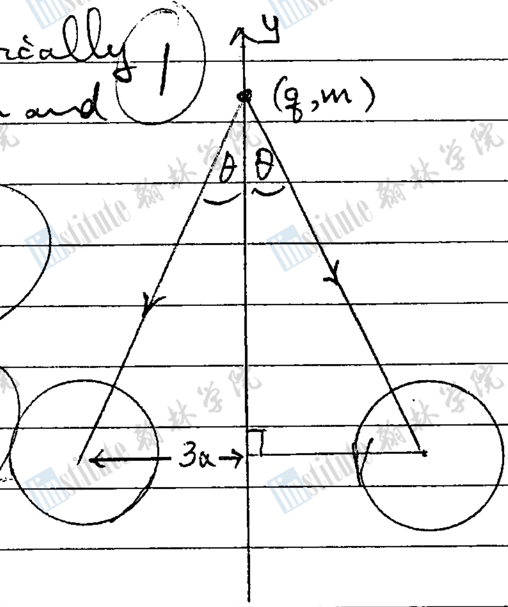
(iii) Minimum speed v required by a charge q at P to overcome the barrier V_L is given by the conservation of energy equation 1

$$1 \quad \tfrac{1}{2}mv^2 - \frac{8Qq}{(4\pi\varepsilon_0)7a} = -\frac{2Qq}{(4\pi\varepsilon_0)a3} \quad 1$$

$$v = \sqrt{\frac{20}{21a}\frac{Qq}{(4\pi\varepsilon_0)m}} \quad 1$$

[8]

c) Resultant force, F, is by symmetry vertically downwards. Using notation in diagram and resolving vertically 1



$$F = \frac{2Qq}{(4\pi\varepsilon_0)(y^2+9a^2)} \cos\theta \quad 2$$

$$= \frac{2Qq}{(4\pi\varepsilon_0)(y^2+9a^2)} \frac{y}{\sqrt{(y^2+9a^2)}} \quad 1$$

$$F = \frac{2Qqy}{(4\pi\varepsilon_0)(y^2+9a^2)^{3/2}} \quad 1$$

[5]

a) (i) 1 — 1

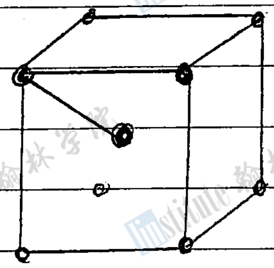
(ii) 2 — 1

(iii) 4 — 2 [4]

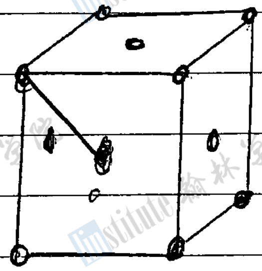
b) (i) $D = a$ — $\frac{1}{2}$ — no proof — 1 — $N = 6$ — $\frac{1}{2}$

(ii) $D = \sqrt{\left(\tfrac{1}{2}a\right)^2 + \left(\tfrac{1}{2}a\right)^2 + \left(\tfrac{1}{2}a\right)^2} = a\frac{\sqrt{3}}{2}$ — 2 — 1 — $N = 8$ — 1

(iii) $D = \sqrt{\left(\tfrac{1}{2}a\right)^2 + \left(\tfrac{1}{2}a\right)^2} = a\frac{\sqrt{2}}{2}$ — 2 — 1 — $N = 12$ — 2



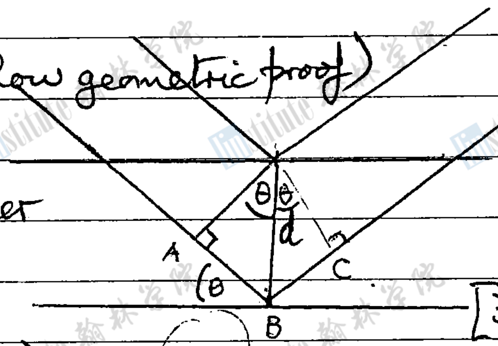
BCC



FCC

[8]

c) (i) Path difference (ABC) $= 2d\sin\theta$ 2 (must show geometric proof)



(ii) $2d\sin\theta = n\lambda$ 1 where n integer [3]

d) (i) $2d\sin\theta_0 = \lambda$ (case $n=1$) 1

Now $d = a = 5.0 \times 10^{-10}$ m

Substituting $\lambda = 1.24 \times 10^{-10}$ m into formula for θ_0

$$\sin\theta_0 = \frac{1.24 \times 10^{-10}}{10^{-9}} = 0.124$$

$$\underline{\theta_0 = 7.1^\circ}$$ 1

(ii) Highest order of diffraction occurs when $\frac{n\lambda}{2d}$ just less than max. value of $\sin\theta$, which is 1 1.

As $\left(\frac{\lambda}{2d}\right) = 0.124$ this occurs when $n = 8$, 1. giving $\frac{8\lambda}{2d} = 0.99$

This occurs for

$$\sin\theta_H = 0.992$$

$$\underline{\theta_H = 82.7^\circ}$$ 1

[5]

Q7

a) (i) Rest energy is the energy when $p=0$ 1

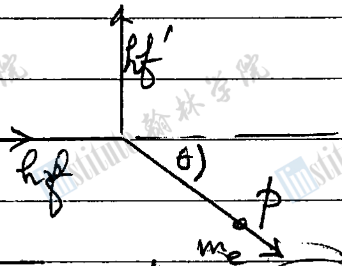
$$\underline{E_{rest} = m_e c^2}$$ 1

(ii) The kinetic energy T is $E - E_{rest}$ thus 1

$$\underline{T = \sqrt{p^2c^2 + m_e^2c^4} - m_e c^2}$$ 1

[4]

b) (i) Conservation equations



Horizontal momentum: $p\cos\theta = \frac{hf}{c}$ 2 I

Vertical momentum: $p\sin\theta = \frac{hf'}{c}$ 2 II

Energy equation: $hf + m_ec^2 = \sqrt{p^2c^2 + m_e^2c^4} + hf'$ 3 III

(ii) Squaring and adding I and II, using $\sin^2\theta + \cos^2\theta = 1$, 1

$$p^2 = \left(\frac{h}{c}\right)^2 (f^2 + f'^2)$$

$$\underline{c^2p^2 = h^2(f^2 + f'^2)}$$ 1 IV

From III $\quad h(f - f') + m_ec^2 = \sqrt{p^2c^2 + m_e^2c^4}$

Squaring, $\quad [h(f-f') + m_ec^2]^2 = p^2c^2 + m_e^2c^4$

From IV $\quad = h^2(f^2 + f'^2) + m_e^2c^4$ 1

Expanding LHS, $\quad h^2(f^2 + f'^2 - 2ff') + m_e^2c^4 + 2m_ec^2h(f - f') = h^2(f^2 + f'^2) + m_e^2c^4$

Cancelling, $\quad -2ff'h^2 + 2m_ec^2h(f - f') = 0$

$$-f'(fh + m_ec^2) + m_ec^2f = 0$$

$$f' = \left(\frac{hf + m_ec^2}{m_ec^2}\right)^{-1} f$$ 2

$$\underline{f' = \left(1 + \frac{hf}{m_ec^2}\right)^{-1} f}$$ 1 V

[13]

When $hf \ll m_e c^2$ equation (V) gives

$$f = f'$$ (1)

From equations (I) and (II)

$$\tan\theta = \frac{\sin\theta}{\cos\theta} = \frac{f'}{f} = 1$$ (as $f = f'$ above)

$$\therefore \theta = 45^0$$ (1)

Equation (IV) gives

$$p = \sqrt{2}\ \frac{hf}{c}$$ (1) as $f = f'$

[3]

Q8

a)

(i) Diameter of Venus shadow 4.0 mm ± 0.5 mm

Diameter of Sun 140 mm ± 15 mm (NOTE PICTURE DOES NOT SHOW A QUARTER OF SUN AREA)

$$\lambda = \left(\frac{140}{4}\right)^2 = 1225 \quad 1 \quad \text{(ACCEPT } 700 \rightarrow 1600\text{)}$$

[ERROR CALCULATION, NOT REQUIRED BY STUDENT GIVES

$$\frac{\Delta\lambda}{\lambda} = 2\left(\frac{0.5}{4.0} + \frac{15}{140}\right) \sim 45\%$$]

$$\lambda = 1225 \pm 500 \quad 1$$

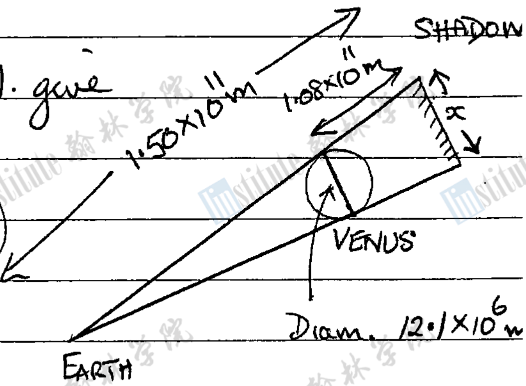
Accept error in range ±500 to ±200
No marks for smaller or larger error

(ii) Similar triangles (See diagram) and data from Table 8.1. give

$$\frac{x}{12.1 \times 10^6} = \frac{1.50 \times 10^{11}}{(1.50 - 1.08) \times 10^{11}} = \frac{1.050}{0.42}$$

$$x = 12.1 \times 10^6 \left(\frac{1.50}{0.42}\right) \quad 1$$

∴ Radius of shadow $= 6.05 \times 10^6 \left(\frac{1.50}{0.42}\right)$



Giving

$$\lambda = \left(\frac{6.98 \times 10^8}{6.05 \times 10^6}\right)^2 \left(\frac{0.42}{1.50}\right)^2 \quad 1$$

$$\lambda = 1042 \quad 1$$

[Error Calculation, not required by student, gives

$$\frac{\Delta\lambda}{\lambda} = 2\left[\frac{10}{6980} + \frac{10}{605} + \frac{1}{42} + \frac{1}{150}\right] \sim 0.05$$]

$$\lambda = 1042 \pm 52$$

(ACCEPT ANY ERROR IN RANGE 1 FOR ERROR 25 → 100, NO MARKS OTHERWISE)

[6]

b) (i) Relative angular velocity $\omega = \omega_V - \omega_E$ 1 (E = EARTH, V = VENUS)

$$= 2\pi\left(\frac{1}{T_V} - \frac{1}{T_E}\right) \quad \text{Periods } T$$

$$= 2\pi\left(\frac{T_E - T_V}{T_E T_V}\right) \quad \tfrac{1}{2}$$

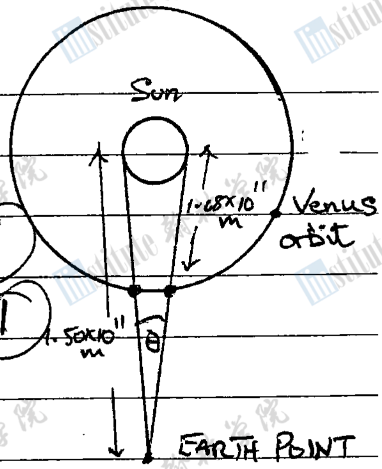
Sub^n. $T_V = 1.94 \times 10^7$ s, $T_E = 3.16 \times 10^7$ s

Relative period $T = \frac{2\pi}{\omega} = \frac{T_E T_V}{T_E - T_V} = 5.02 \times 10^7 \text{ s} = 1.59 \text{ years}$ 1

Comment: It is in fact much longer (~100 years) due to perturbations produced by other bodies in solar system $\tfrac{1}{2}$

(ii) Angular diameter of Sun at Earth

$$\theta = \frac{2 \times 6.96 \times 10^{8}}{1.50 \times 10^{11}} = 9.28 \times 10^{-3} \text{ rads.} \quad 1$$



Time of transit (see diagram) $= \dfrac{\text{Dist. travelled by Venus}}{\text{Orbital Circumference}} \times T$ 1

$= \dfrac{(1.50 - 1.08)10^{11}\,\theta}{2\pi\,(1.08)10^{11}}\,(5.02 \times 10^{7})\,s$ 1

$= \dfrac{\theta}{2\pi}\left(\dfrac{0.42}{1.08}\right) 5.02 \times 10^{7}\,s$

As $\theta = 9.28 \times 10^{-3}$ ↳ $= \dfrac{9.28 \times 10^{-3}}{2\pi}\left(\dfrac{0.42}{1.08}\right) 5.02 \times 10^{7}\,s$

$= \underline{8.0 \text{ hours}}$ 1

[7]

c) (i) Angle between telescopes to observe shadow of Venus crossing Sun

$$\phi \simeq \frac{\text{Diameter of Earth}}{\text{Dist. of Earth from Sun}}$$

$= \dfrac{2 \times 6.38 \times 10^{6}}{1.50 \times 10^{11}}$ rads. $= \underline{8.51 \times 10^{-5} \text{ rads}}$ 1 EITHER RESULT OK

$= \underline{4.87 \times 10^{-3} \text{ degrees}}$

(ii) So that clocks could be synchronized 1

(iii) In 18th century there was no universal time; different places had differing times. Thus (c)(ii) was a way of synchronizing clocks at two observatories. 1

[3] ~~[4]~~

d) If during the interval between transits Venus has a velocity V_s perpendicular to its orbit, and as a result is displaced by $\frac{1}{2}(9.28 \times 10^{-3})$ rads. as observed from Earth (half ang. diam. of Sun at Earth), Venus will not pass in front of the Sun.

4.64×10^{-3} rads. in 5.02×10^{7} s 1 (1.59 years)

ie $(4.64 \times 10^{-3})(0.42 \times 10^{11})$ m in 5.02×10^{7} s. 1

The speed

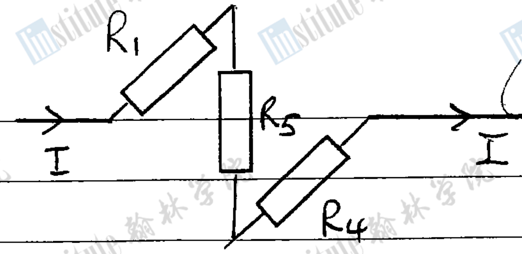
$$V_s = \left(\frac{4.64 \times 10^{-3} \times 0.42 \times 10^{11}}{5.02 \times 10^{7}}\right) ms^{-1} \quad 1$$

$= \underline{3.9 \text{ ms}^{-1}}$ 1

[4] ~~[3]~~

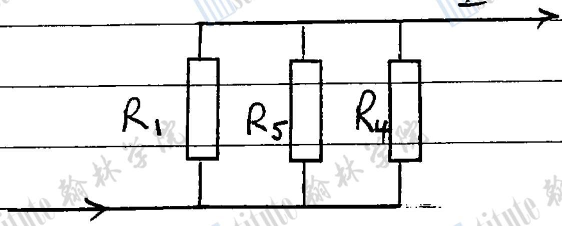
Q.9

a) (i) $R_2 = R_3 = \infty$ 1



1

(ii) $R_2 = R_3 = 0$ 1

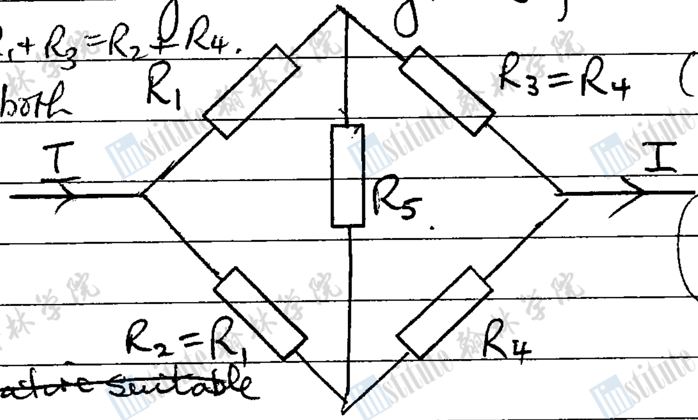


1

(iii) A balanced Wheatstone bridge, $R_1R_4 = R_2R_3$, 1 will produce a circuit with (R_2+R_3) in parallel with (R_2+R_4). A simple pair of values for R_2 and R_3 is $R_2 = R_1$ and $R_3 = R_4$. Any 1 other suitable set satisfying $R_1R_4 = R_2R_3$ is acceptable.*

Note: no current flows through R_5 for balanced bridge 1

*In addition we require $R_1 + R_3 = R_2 + R_4$. The only soln. satisfying both conditions is $R_2 = R_1$ & $R_3 = R_4$.



1

[8]

b 1 (i) $R = R_1 + R_5 + R_4$ 1

(ii) $R = \left(\frac{1}{R_1} + \frac{1}{R_5} + \frac{1}{R_4}\right)^{-1}$ 1

(iii) $R = \left(\frac{1}{R_1+R_4} + \frac{1}{R_1+R_4}\right)^{-1}$ 1

$= \frac{1}{2}(R_1+R_4)$ } or any acceptable alternative for R_2 & R_3

2 (i) $P_5 = I^2R_5$ 1

(ii) $P_5 = I^2\left(\frac{1}{R_1} + \frac{1}{R_5} + \frac{1}{R_4}\right)^{-2}\frac{1}{R_5}$ 1 ie "$\frac{V^2}{R_5}$"

(iii) $P_5 = 0$ 1 no current through R_5.

[6]

29

7 (i) By symmetry, the circuit is identical to that obtained by reflecting about the diagonal AC. Thus $V_B = V_D$; potentials at B and D identical

2

(ii) There are several different methods of solution.

Solution 1

Reversing the currents produces an identical circuit. However currents in OD and OB have changed direction. Since both circuits are identical, these currents must be zero; thus O at same potential as B and D. Thus currents, by symmetry, in ABC, AOC and ADC must be equal, each being $\frac{1}{3}I$.

Total resistance of 3 parallel resistors, resistance $r+r=2r$,

$$R = \left(\frac{1}{2r} + \frac{1}{2r} + \frac{1}{2r}\right)^{-1}$$

$$\underline{R = \frac{2r}{3}}$$

4

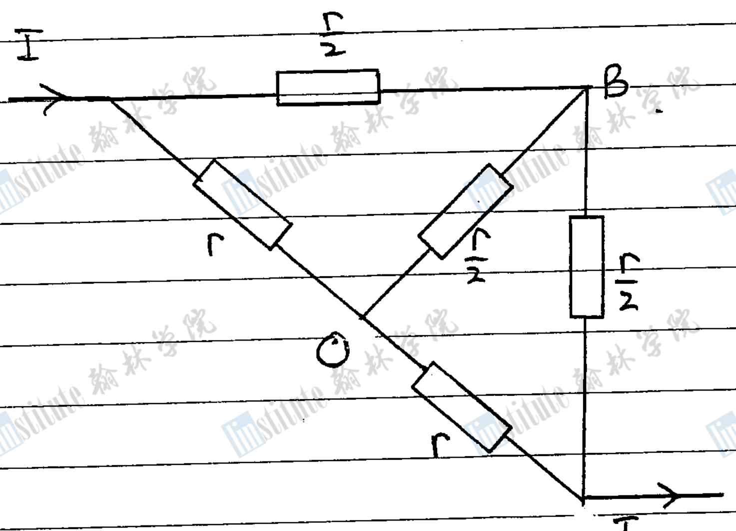
Solution 2

If only B and D are assumed to be at the same potential. Then

AD in parallel with AB; total resistance $\frac{r}{2}$

DO in parallel with OB; total resistance $\frac{r}{2}$

DC in parallel with BC; total resistance $\frac{r}{2}$



This is a balanced Wheatstone bridge with no current tho' O

Total resistance $(r+r)$ in parallel with $\left(\frac{r}{2}+\frac{r}{2}\right)$

4

$$\underline{R = \left(\frac{1}{2r} + \frac{1}{r}\right)^{-1} = \frac{2r}{3}}$$

[6]