

BRITISH PHYSICS OLYMPIAD 2005

COMPETITION

Paper 2 - 5th November 2004

3 hours plus 15 minutes reading time.

There are NINE questions in this paper.

The marks for each section are given on the right hand side of the page.

- FOUR questions are to be attempted. Formulae sheets may be used.
- QUESTION 1 IS COMPULSORY. It is expected that students will spend 75 minutes on this question. The total mark allocated to the question is 66. Students can attempt any, or all, of the sections. However the maximum total mark awarded will be 40.
- THREE of the remaining eight questions should be attempted. Students are recommended to spend 35 minutes on each of these questions. The maximum mark for each of these questions is 20.

Useful data:

Speed of light	c	3.00×10^8	m s^{-1}
Planck constant	h	6.63×10^{-34}	J s
Electronic charge	e	1.60×10^{-19}	C
Mass of electron	m_e	9.11×10^{-31}	kg
Acceleration of free fall	g	9.81	m s^{-2}
Gravitational constant	G	6.67×10^{-11}	$\text{N m}^2 \text{kg}^{-2}$
Gas constant	R	8.31	$\text{J mol}^{-1} \text{K}^{-1}$
Radius of the Earth	R_E	6.38×10^6	m
Mass of Earth	M_E	5.97×10^{24}	kg
Density of air at STP	ρ_{air}	1.29	kg m^{-3}
Density of helium at STP	ρ_{He}	1.78×10^{-1}	kg m^{-3}

Q1

- a) An accurate thermometer, of heat capacity 20.0 J K^{-1} , reads $18.0 \text{ }^\circ\text{C}$. It is then placed in 0.250 kg of water and both reach the same final temperature of $50.0 \text{ }^\circ\text{C}$. Calculate the temperature of the water before the thermometer was placed in it. The specific heat capacity of water is $4200 \text{ J kg}^{-1} \text{ K}^{-1}$.

[4]

- b) The potential in volts, V , across a new material varies with the current in amperes, I , according to the relation

$$V = (6I - 3I^2 + 2I^3)/6,$$

from $I = 0$ to $I = 5$.

Determine the resistance, for (i) $I = 2$, and (ii) $I = 0$.

For a light bulb and a LED:

- (iii) sketch graphs of voltage V against current I
(iv) explain the variation in each case.

[6]

- c) (i) Why is there a force of net attraction between a charged particle and an uncharged isolated copper sphere some distance away? Why does this force increase if the sphere is earthed?
(ii) Why can a ladder be leaned at an angle on a rough floor against a smooth wall, but not on a smooth floor against a rough wall?
(iii) Why must a hole in a pinhole camera be neither too small nor too large?
(iv) Why does a blue cloth look dark when viewed in sodium light?

[7]

- d) (i) Explain why light can be polarized but sound cannot. How would you distinguish, experimentally, between partially and fully polarized light?
(ii) The equation of a wave, with displacement y , moving along the x -axis at time t is given by

$$y = A \sin 2\pi(\beta t - x/\gamma),$$

where A, π, β and γ are constants.

What are the dimensions or units of A, π, β , and γ ?

Why does this equation represent a wave moving with constant velocity?

[7]

e) Wet clothing at 0°C is hung out to dry. The air temperature is 0°C and there is a dry wind blowing. After some time it is found that some of the water has evaporated and the water remaining on the clothes has frozen. The specific heat of fusion of ice is 333 kJ kg^{-1} and the specific latent heat of evaporation of water is 2500 kJ kg^{-1} .

- (i) What is the source of energy required to evaporate the water? Explain the mechanism for evaporation.
- (ii) Estimate the fraction, by mass, of water originally in the clothing that freezes. [5]

f) A bullet leaves the 1.20 m barrel of a rifle at a horizontal speed of 310 m s^{-1} :

- (i) Calculate the acceleration of the bullet in the barrel assuming it to be constant.
- (ii) If the bullet emerges from the barrel after two complete rotations, estimate its final rotational speed in radians per second.
- (iii) What advantage has a barrel that causes the bullet to rotate?
- (iv) In practice the acceleration of the bullet is not constant. Sketch a graph of the expected variation, with distance, and explain the variation. [5]

- g) (i) A helium balloon has a volume of 512 m^3 . What is the largest mass that can be raised by the balloon at STP if the balloon has negligible mass?
- (ii) A child, restrained by a seat belt, is travelling at constant velocity in a bus and holds a vertical string with a helium balloon at the end. Explain what happens to the balloon when the bus decelerates and finally comes to rest. [5]

h) The acceleration, a , of a particle, velocity v , along a straight line can have the following forms:

- (i) g
- (ii) $g - kv$
- (iii) $-\omega\sqrt{b^2\omega^2 - v^2}$,

g is the constant acceleration of free fall, v is the velocity, k , ω and b are constants. Sketch graphs, for each case, assuming $v = 0$ at time $t = 0$, of:

- (1) a against v
- (2) v against time, t .

[8]

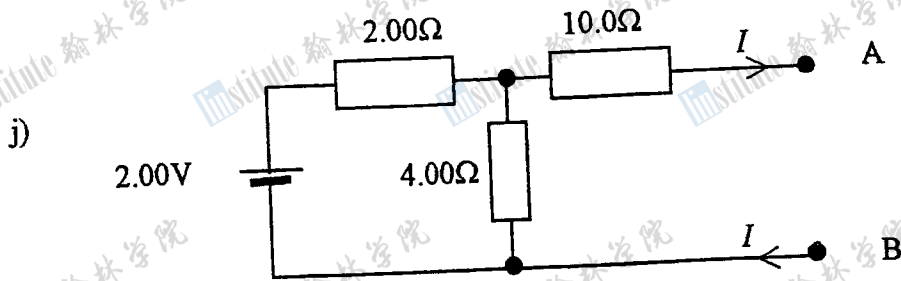


Figure 1.j

For the circuit in Figure 1.j determine:

- (i) the current $I = I_0$, between A and B, when A and B are short circuited
 (ii) the potential difference, V_{AB} , across AB when A and B are on open circuit.

If a resistor, resistance R , is connected across AB the current I is identical to that in a circuit with voltage source V_{AB} in series with R and a further resistance $r = V_{AB}/I_0$.

Hence solve:

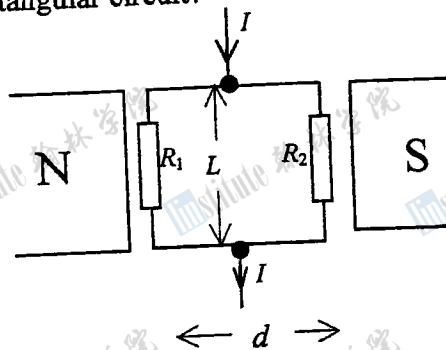
- (iii) A lamp, resistance 8.50Ω , is connected across AB. Determine the current through the lamp. [6]

- k) The mass and radius of Mars are $0.1074 M_E$ and $0.5326 R_E$, where M_E is the mass of the Earth and R_E is its radius. Determine the escape velocity from the surface of Mars in terms of its value on Earth, v_E . [3]

- l) Two identical elastic spheres, each of mass m , are at rest on a smooth table and touch each other. They are struck symmetrically by an identical sphere, but of mass M , having a velocity u , perpendicular to the line of centres of the two stationary spheres. The mass M comes to rest immediately after the collision. Determine the ratio M/m . [4]

- m) Figure 1.m shows a current-carrying circuit in which the current I splits to pass through two parallel resistors of resistance R_1 and R_2 . The resistors form the vertical arms of the rectangular part of the circuit of length L and width d . The rectangular arrangement is in the plane of a uniform horizontal magnetic field of flux density B produced by two pole pieces. What is the torque on the rectangular circuit?

Figure 1.m



[6]

Q2

- a) (i) Derive an expression for the acceleration due to gravity at a height h above the Earth's surface, g_h , in terms of g_0 , the acceleration due to gravity at the surface, and R_E the radius of the Earth. Assume the Earth is a sphere of uniform density.
- (ii) Repeat the above calculation for a point below the Earth's surface at a distance r from the centre of the Earth.
- (iii) Sketch a graph of the variation of the acceleration due to gravity with distance from the centre of the Earth.
- (iv) Comment on the assumption that the Earth has a uniform density. [9]
- b) An Earth satellite, in a circular orbit, has a rotational period of 2.00 hours. It is in the plane of the Earth's equator, moving in the same rotational direction as the Earth.
- (i) How far above the Earth's surface is the satellite ?
- (ii) Calculate the angular velocity of the satellite relative to that of the Earth.
- (iii) Determine the angle, measured about the centre of the Earth, through which the satellite will be visible to an observer at the equator. [9]
- c) What limits the largest and smallest possible periods of an Earth satellite ? [2]

Q3

An element A is radioactive, with decay constant λ_1 , and decays to element B. B decays, with decay constant λ_2 , where $\lambda_2 \gg \lambda_1$, to the stable element C. At time t the number of atoms of A is $N_1(t)$ and the number of atoms of B is $N_2(t)$. N_0 is the total number of atoms. The ratio $R = (N_2/N_0)$ can be shown to be given by

$$R = \frac{\lambda_1}{(\lambda_2 - \lambda_1)} [e^{-\lambda_1 t} - e^{-\lambda_2 t}]$$

a) Sketch a graph of R as a function of t . [3]

b) Obtain the approximate variation of R with t for:

(i) t is small

(ii) t large [4]

c) Using extensive experimental data, from measured values of R and t , how would you graphically determine:

(i) λ_1 ?

(ii) λ_2 , knowing λ_1 ? [6]

d) From the equation for R above deduce the approximate relationship

$$N_2 \lambda_2 = N_1 \lambda_1$$

State the conditions under which it is valid. [5]

e) Sketch the behaviour of $N_3(t)$, the number of C atoms, against t . [2]

Useful relation for small x

$$e^x = 1 + x + \dots$$

Q4

a) A particle, mass m , rotates in a circle of radius r with constant speed s .

(i) Determine the work done by the particle in one revolution.

(ii) State the force acting on the particle.
How does it vary during the motion?

[4]

b) A particle, mass m and charge Q , enters a constant magnetic field, B . The field is directed in the z - direction. It initially has velocity components along the Cartesian x -, y -, and z - axes of u , v , and w respectively, and is located at the origin.

(i) What is the magnitude of its initial velocity component, v_{xy} , in the $x - y$ plane?

Write down:

(ii) its position along the z - axis at time t

(iii) the radius of curvature, r , in any $x - y$ plane

[5]

c) (i) What is the period, T , of the motion ?

(ii) Explain how the motion alters if B is increased in magnitude.

[4]

d) Sketch graphs giving the variation with time, t , of the following:

(i) x and z displacements

(ii) v_x and v_z , the Cartesian velocity components

(iii) a_x and a_z , the Cartesian acceleration components

[5]

e) How will reversing the direction of B affect the motion?

[2]

Q5

a) Two identical insulating spheres, A and B radius a , have uniform charge density, each with a total negative charge $-Q$. Their centres are $6a$ apart. The origin of coordinates, O, is midway between the centres. The x - axis is along the line of centres.

- (i) What is the electrostatic potential, V , at O?
- (ii) At what points along the x - axis is the potential equal to that at O?
- (iii) Sketch the electrostatic equipotential curve of magnitude V in the $x - y$ plane and show the spheres on your diagram.

[7]

- b) (i) Indicate, in a diagram, the point/s on the surface of B with the highest potential, V_H , and calculate its value.
- (ii) What is the lowest potential "barrier", V_L , that has to be surmounted by a positively charged particle launched from B in order to reach A?
- (iii) What is the minimum speed with which a positive gas ion, mass m , and positive charge q , emitted from the surface of B, can be captured by A?

[8]

c) Calculate the force acting on a particle with charge q , mass m , on the y - axis.

[5]

Q 6. There are three forms of crystal that have a cubic lattice structure:

- (i) simple cubic (SC)
- (ii) body centred cubic (BCC)
- (iii) face centred cubic (FCC)

The SC has atoms at the cube corners of each cell. The BCC, in addition, has an atom at the centre of each cubic cell. The FCC is a SC with additional atoms at the centre of each face of every cubic cell. An atom that is shared between a number of cubic cells only makes a fractional contribution to any one cell.

a) Determine the number of atoms per cubic cell in cases (i), (ii) and (iii). [4]

b) The length of the sides of the cubic cell is a . Determine for each of the three crystals the distance D between the centres of any two atoms that are nearest neighbours and the number, N , of nearest neighbours located around any atom. [8]

c) X-ray diffraction spots are produced by crystal lattices. They are due to interference produced by parallel layers of atoms that reflect the rays. The diagram, Figure 6.1, shows two such layers separated by a distance d . The two parallel rays at an angle θ to the plane of the atoms are reflected and interfere.

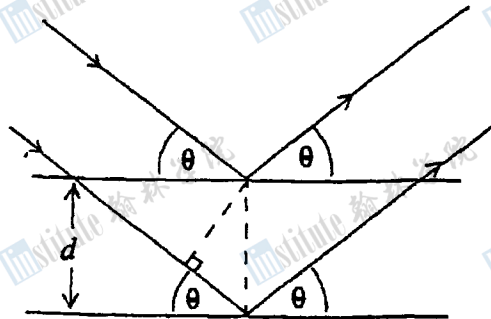


Figure 6.1

(i) Determine the path difference between the two rays.

(ii) Deduce the condition for constructive interference for X-rays of wavelength λ

[3]

d) X-rays of wavelength $\lambda = 1.24 \times 10^{-10}$ m are incident on the planes of atoms parallel to a face of the cubic cells of a SC lattice with $a = 5.00 \times 10^{-10}$ m. As θ increases from 0, the first diffraction spot appears at θ_0 and the highest order of spot occurs at θ_H . Determine θ_0 and θ_H .

[5]

Q7.

- a) The relativistic energy E of an electron with momentum p is given by

$$E^2 = p^2 c^2 + m_e^2 c^4.$$

- (i) What is understood by the rest energy of the electron?
(ii) Deduce an expression for the kinetic energy, T , of the electron.

- b) A photon of frequency f is scattered through an angle of 90° by an initially stationary electron. After the collision the photon has a frequency f' . The electron acquires relativistic momentum p directed at an angle θ to the incident photon. [4]

- (i) Write down the conservation equations for this collision.
(ii) Prove that

$$c^2 p^2 = h^2 (f^2 + f'^2)$$

and

$$f' = \left(1 + \frac{hf}{m_e c^2}\right)^{-1} f.$$

Momentum of a photon, frequency f , is hf/c . [13]

- c) What can be deduced about f' , θ and p when $hf \ll m_e c^2$? [3]

Useful relationship: $\sin^2 \theta + \cos^2 \theta = 1.$

Q8

a) Figure 8.1

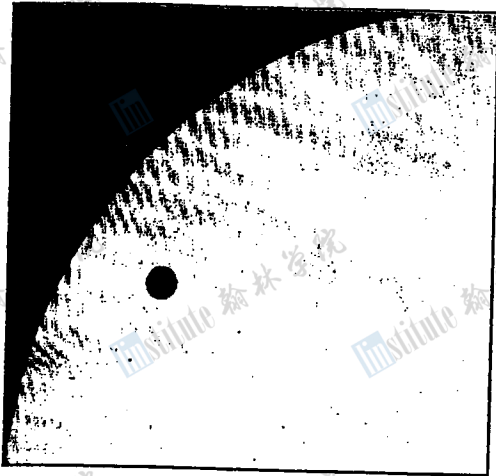
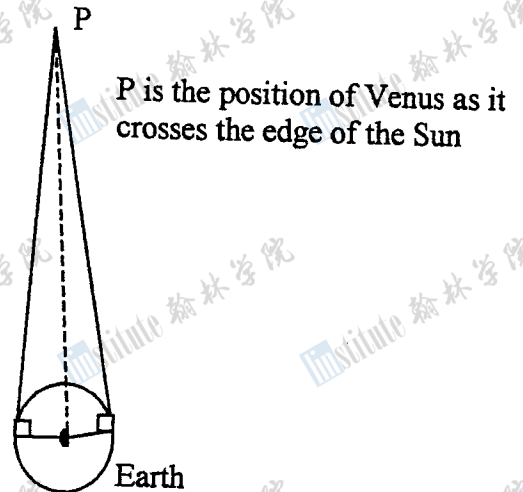


Figure 8.2



- (i) Figure 8.1 is a photograph of the recent transit of Venus. The planet Venus is shown crossing the face of the Sun. Determine, by measurement, the ratio, λ , of the photographic area of the full face of the Sun to that produced by Venus. State the accuracy of the result.
- (ii) Calculate λ , with the associated accuracy, using the data in the Table 8.1.

Body	Radius/m	Orbital period (round the Sun) /s	Distance from the Sun/m
Sun	6.96×10^8		
Venus	6.05×10^6	1.94×10^7	1.08×10^{11}
Earth	6.38×10^6	3.16×10^7	1.50×10^{11}

Table 8.1

[6]

b) Assume here that the orbits of the Earth and Venus are coplanar.

(i) Determine the interval, in years, between successive transits using data in Table 8.1. Comment on the result in the light of astronomical observations.

(ii) Calculate the transit time for Venus to complete a diameter of the Sun.

[7]

c) In the 18th century the transit of Venus was observed using two telescopes separated roughly by the diameter of the Earth. This enabled the distance between Venus and the Earth to be determined. Figure 8.2 illustrates the simplest placing of the telescopes.

(i) What is the angle between the telescopes?

(ii) Why was it essential that both telescopes observed the point when the image of Venus touched that of the edge of the Sun?

(iii) What were the difficulties associated with the timing of the measurements? [4]

d) Calculate the smallest velocity component, v_s , perpendicular to its orbit, required by Venus so that the next expected transit does not occur. [3]

Q 9
a)

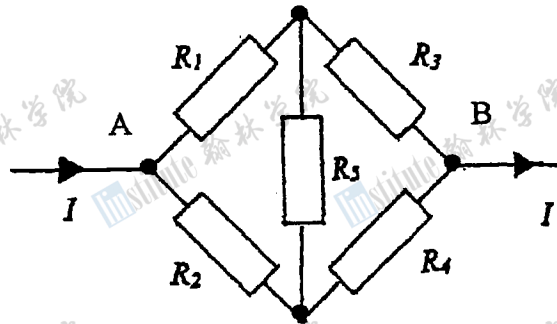


Figure 9.1

The circuit in Figure 9.1 consists of five resistors, with resistances R_1 , R_2 , R_3 , R_4 and R_5 . The current I enters at A. By choosing appropriate values of R_2 and R_3 , in the range zero to infinity, reduce the circuit to the following, indicating the values of R_2 and R_3 and giving a circuit diagram:

- (i) three resistors in series
- (ii) three resistors in parallel with each other
- (iii) two equal resistors in parallel

[8]

b)

For each network in (a) calculate:

- (1) The total resistance across AB
- (2) The power dissipated in R_5

[6]

c)

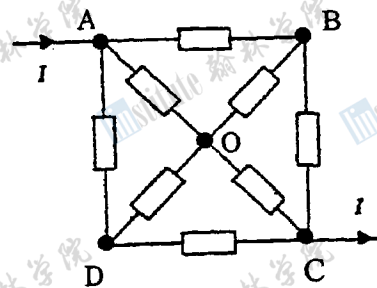


Figure 9.2

The network in Figure 9.2 consists of eight resistors each with resistance r .

- (i) Explain why B and D are at the same potential.
- (ii) Determine the total resistance between A and C.

[6]

