Summary

In this paper, we propose a dynamic global network model to develop a comprehensive analysis on the health of planet Earth. We choose **ocean pollution** as a measure of the global health and eight coastal water areas as the nodes. The propagation of the pollutants is the link connecting the nodes. Three models have been designed, focusing on and providing full-scale solutions to the following issues:

- Develop a global network model concerning ocean pollution.
- Predict the variation of polluting patterns in different situations.
- Analyze the network structure of the model.

We set up a **Pollution Diffusion Model** to analyze the performance of one node and the interaction between two nodes. We first build up a Convection Diffusion Equation to evaluate how the pollutants spread in the ocean. Then, we estimate the variation of the pollutants for different development patterns and assume an unexpected environmental event to see the reaction of the nodes. Finally, we propose a method to justify the model.

We build an **MIMO Control System Model** based on control theory to predict future ocean health and analyze the network structure. A block diagram has been used to represent the relationship in the system. We apply MATLAB Simulink to simulate the system and try several combinations of inputs, which vary according to the development pattern and government policy, to evaluate the system response in the future. In addition, we conduct sensitivity analysis by eliminating one node each time respectively and test the system response to noise signal which appears uncertainly.

We introduce **Social Network Analysis Model** to find out the critical nodes. Based on the original indicator-Closeness Centrality in general analysis, we divide all the nodes into two parts and put up with a similar assessment standard called “Influencing/Influence Centrality”. In this way, we obtain the importance rank of the nodes. The result shows that the indicator agrees with real condition. Subsequently, we propose some suggestions on policy making on ground of our former analysis so as to better Earth’s health.

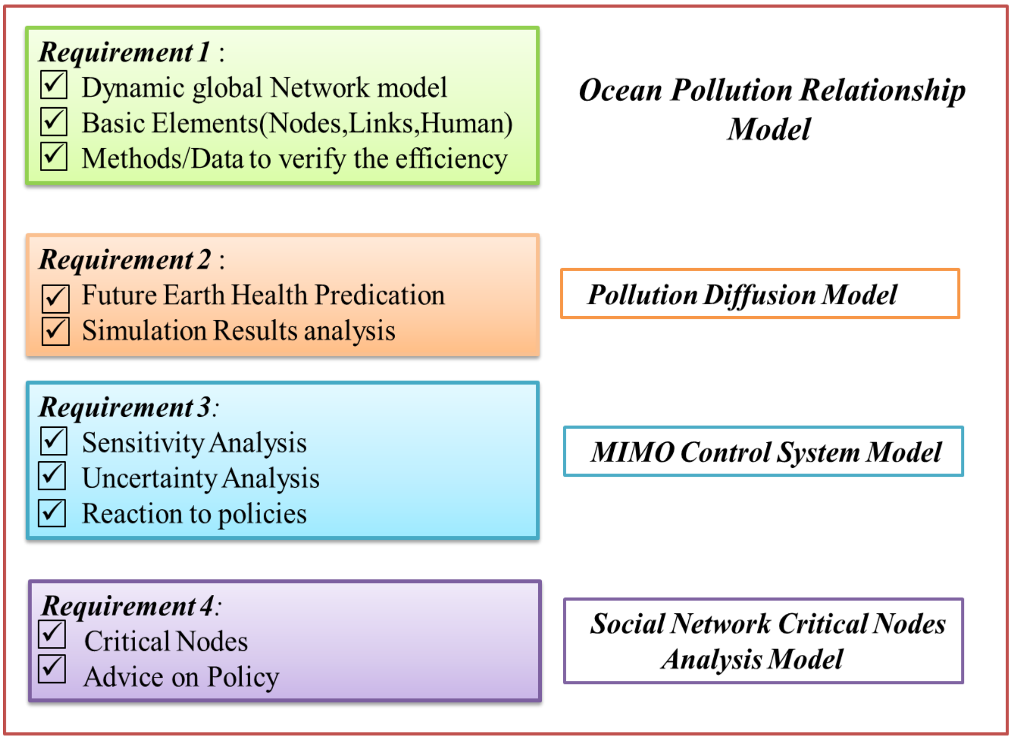
You Pollute, I Polluted !

Introduction

Nowadays we are troubled by various environment problems. Many works have been done to forecast the Earth's health, while most of the models are only established in isolated regions and tend to ignore local relationships[3]. Our aim is to build a dynamic global network model to analyze one aspect of Earth's heath and finish the requirements. From various health measures, we choose to study on global ocean pollution[2]. Almost everyone knows the catastrophe of massive oil leak in the Gulf of Mexico.[1]

Our Ocean Pollution Relationship Model contains three sub-parts: Pollution Diffusion Model, MIMO Control System Model and Social Network Critical Nodes Analysis Model.

The structure of our essay is as follows:



Assumptions

●We abstract all the global coastal areas into eight nodes and regard them as points.

●Minor ocean currents are neglected.

●We do not consider the diffusion in intersecting surface.

●The path of pollutant transmission from one node to another is one-dimensional.

●The pollutants transmitted from one node to another will not affect other nodes any more.

●The temperature of ocean water is a stable and thus the attenuation coefficient of pollutants is a constant.

●The velocities of different ocean currents are regarded as the same.

[1] http://planetgreen.discovery.com/tech-transport/gulfofmexico-oilspill-whatwhenwhere-whatyoucando.html

●We can calculate several nodes' mutual effect by summing up the effect of separate node.
●The condition of the ocean remains unchanged.
●In Social Network analysis model, the decay rate is proportional to the amount of pollutants and inverse proportional to the distance of two nodes.

Notations

$C(x,t)$	the amount of pollutants produced by one node at position *x* and time *t*
u	the velocity of the ocean current, a constant
D_{AB}	the molecular diffusion coefficient
W	the external input of pollutants to one node
S_k	the amount of pollutants degradation
P_i	the adjusted amount of pollutants after considering links between nodes
K	the attenuation coefficient of pollutants in the ocean
$C_c(i)$	Closeness Centrality of i^{th} node
d_{ij}	the shortest distance from node *i* to node *j*

Overview

There are numerous factors that would affect the Earth's health, such as the stratospheric ozone depletion, ocean acidification and global warming. More aspects concerning Earth's health could be reviewed in ref.2. Taking the interaction between different local regions into consideration, we choose ocean pollution as our measure to analyze to what extent the Earth stays in healthy condition and what should we do to slow down the course of environmental deterioration. To be more specific, since ocean pollution usually consists of acidification, eutrophication and plastic debris, we focus on eutrophication, which is characterized by the over-emission of nitrogen and phosphorus.[2]

Dynamic Global Ocean Pollution Network

Nodes Selection Method

We choose our nodes based on the criteria that the nodes should be the major polluted areas around the globe and should be in touch with the ocean current. The following picture is a map of human's overall impact on marine ecosystem[1].[3]

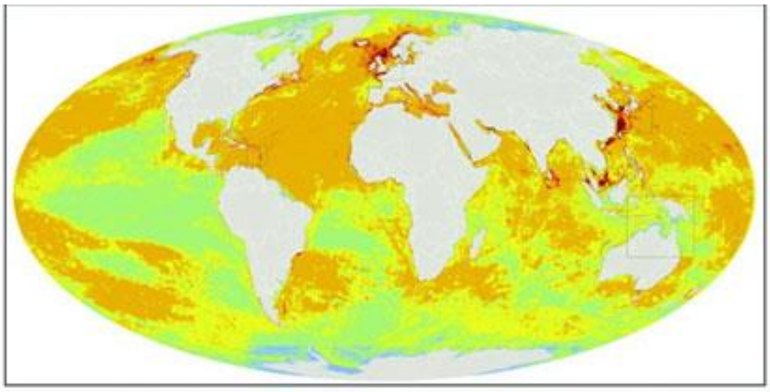


Figure 1 human's overall impact on marine ecosystem

[2] http://en.wikipedia.org/wiki/Marine_pollution#Types_of_pollution
[3] http://www.guardian.co.uk/science/2008/feb/14/ocean.ecosystems

At the same time, we refer to the general pattern of ocean currents, which is defined by the following picture.[4]

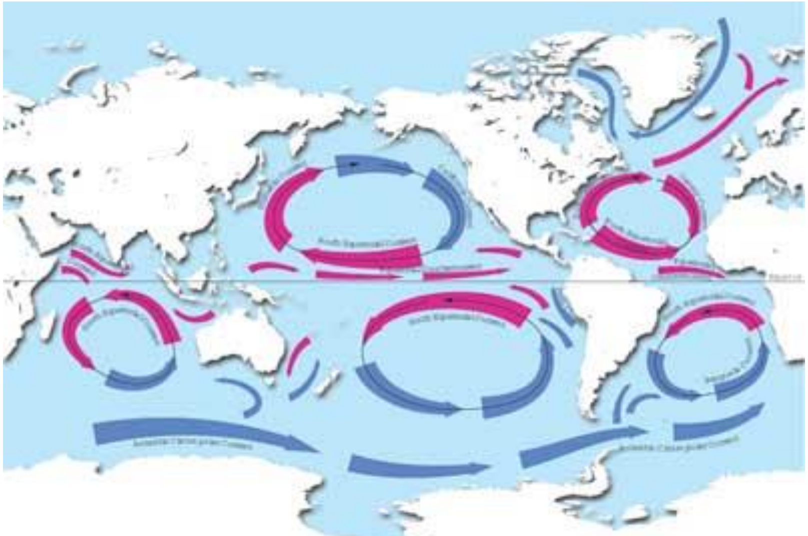


Figure 2 Concise Distribution of Ocean Currents

Considering that the coastal areas are the places to be influenced by ocean pollution as well as the generator of the pollutants, and referring to the two pictures shown above, we determine the 8 nodes for analysis in our model.

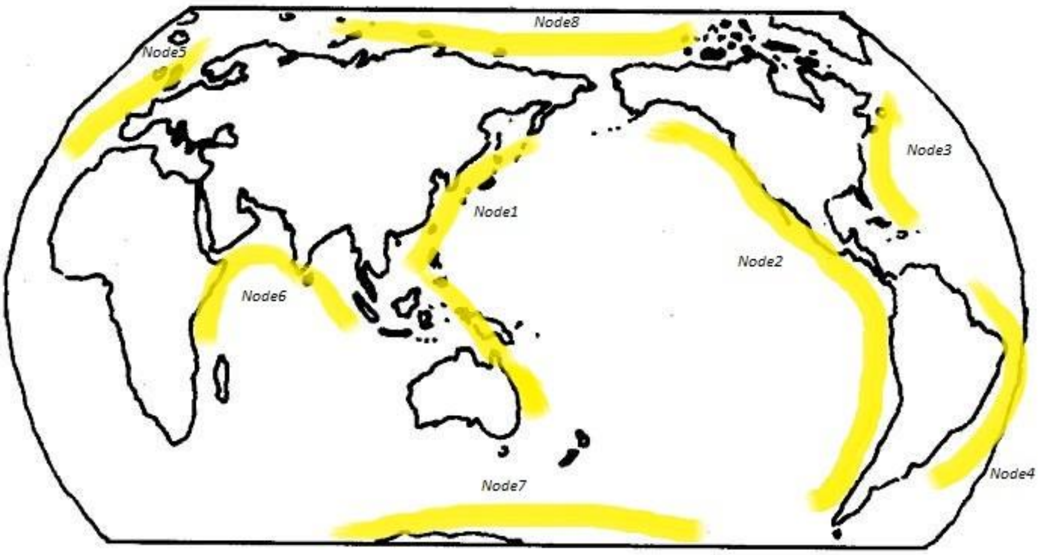


Figure 3 Nodes Selection

Node 1: Asian-Pacific coastal area, including Japan Sea, East China Sea, South China Sea, etc.

Node 2: West America coastal area, including the Bering Sea, etc.

Node 3: East North-America coastal area, including the Caribbean, the Gulf of Mexico, etc.

Node 4: East South-America coastal area

Node 5: West Europe coastal area, including the Mediterranean, the North Sea, etc.

Node 6: the Indian Ocean area, including the gulf, the Red Sea, etc.

Node 7: Antarctica coastal area

Node 8: the Arctic Ocean area

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4 http://www.kidsgeo.com/geography-for-kids/0145-ocean-currents.php

Since the ocean current is the driving force of pollutant propagation, we develop a graph according to the direction of ocean current. In the directed graph below, an edge represents the link between two nodes, which means the sink node will be affected by the emission from the source node. Besides, the weight for each edge represents the distance between the linked nodes.

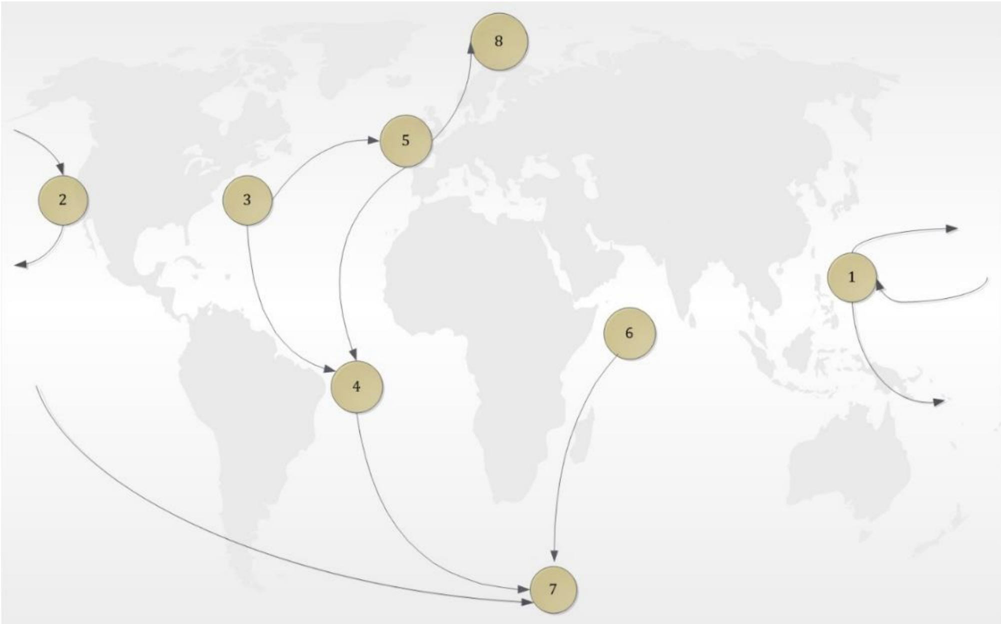


Figure 4 Links between Nodes

Model 1: Pollution Diffusion Model

For each node, we have already obtained the amount of pollutants to be released for each unit time. However, because of the interaction among nodes, the nodes will be affected by both the pollutants from themselves and those from other nodes as long as the ocean current enables the transmission. Since we assume that the pollutants received by one node will not be transmitted to another node any more, we define a binary function $C(x,t)$ to indicate the amount of pollutants produced by one node at position *x* and time *t*. In addition, the pollutants will be degraded by ocean self-purification. From the analysis above, we adapt the Princeton Ocean Model (POM) to solve the problem. [7]The original POM for water quality is a three-dimensional partial differential equation. However, the pollutants are always gathered on the surface of the ocean and so the depth could be neglected. Besides, ocean currents have their particular flow direction; thus, a one-dimensional model is precise enough to simulate the propagation of pollutants.[6]

$$\frac{\partial C}{\partial t} + u\frac{\partial C}{\partial x} = D_{AB}\frac{\partial^2 C}{\partial x^2} + W + S_k$$

Where *u* is the velocity of the ocean current[5], D_{AB} [11]is the molecular diffusion coefficient[6], *W* is the external input, and S_k is the amount of degradation.

After solving the partial differential equation for each node, we will obtain the amount of

[5] http://baike.baidu.com/view/46767.htm
[6] http://www.ymhb.gov.cn/info/content.asp?infoId=2431

pollutants at any position and time if there is only one node existing separately. However, we have not considered the inter-relationship between nodes yet. In this problem, we focus on the effect one node poses to the others, so we calculate the value of $C_i(x_{ij}, t)$ for each node *i*, where x_{ij} is the distance between node *i* and node *j*, and for each node *j*, sum up the amount of pollutants from *i* to *j*. Take node 1 as an example. The amount of total pollutants affecting node 1, which is the amount of pollutants gathered in coastal water, can be calculated as follows.

$$P_1 = C_1(0, t) + \sum_{i=2}^{5} C_i(x_{i1}, t)$$

The variable P_i is the indicator that we will use to judge the health condition of the Earth. It is the ultimate amount of pollution after taking links between nodes into account.

Model Justification Method

This pollution diffusion model provides us with the amount of pollutants that spreads across the globe by using the data about how many pollutants have been released from a given set of nodes. Thus, in order to verify the model to see how precisely it can predict the health condition, we should search for some data mainly from two aspects, namely the amount of pollutants released by the nodes and the concentration of the pollution in the coastal water around the nodes. The former is used to provide suggestions for policy making and the latter can help us assess the pollution level and damage extent to ecosystem.

For most of the countries, the total amount of pollutants released every year can be found easily since such data are generally required to be publicized. However, in order to collect statistics on the concentration of pollution in coastal water, samples should be obtained at every node area and corresponding detection and experiment should be made specially. However, no such data are available now since there do not exist global detection of the concentration systematically.

Facing this difficulty in collecting enough data, we design a scheme to accomplish the goal, and it can be conducted if permitted. We need to set up some observation stations on the coastline, testing the concentration of pollutants regularly. The observation stations should be uniformly separated around the coastal water in order to eliminate the possibility of randomness. We need to perform such observation for all the nodes in our model, and the collection process might last for about several years. Besides, we record the amount of released pollutants annually in the meantime. By taking the data of released pollutants into the model, we acquire the expected values of the pollution concentration. Through comparing the prediction with the actual values, we will be able to verify our model. If the output of the model could match the actual value quite well, then the model can be used to predict the concentration of pollution in coastal waters in the future.

In conclusion, we have to obtain the concentration data for several years in global nodes we set, and thus compare the prediction with actual tendency of pollution level. We could finally check whether our model could be put into practice in global ocean pollution networks and policy making.

Solution to the partial differential equation

We use Finite Differential Method[7] and choose Four Point Implicit Difference Scheme. In this way, we can obtain a numerical solution to the equation described in Model Design. We assume a discrete release of the pollutants so that an easier way to analyze the influence of each node could be achieved. Later on, we will take the interaction among those nodes into consideration to form a refined model solution. In the equation, W is the original release of the pollutants while $S_k = -KC$ where K is the attenuation coefficient[9]. By applying the difference method, the initial equation can be transformed into the following form[10]:

$$\frac{C_i^{j+1} - C_i^j}{\Delta t} + u\frac{C_i^j - C_{i-1}^j}{\Delta x} = D_{AB}\frac{C_{i-1}^{j+1} - 2C_i^{j+1} + C_{i-1}^{j+1}}{\Delta x^2} - \frac{1}{2}K(C_i^{j+1} + C_{i-1}^j)$$

Substitute the boundary condition so that the equation becomes a tridiagonal equation. We use Thomas algorithm[8] to obtain the numerical solution.

It equals to

$$\alpha_i C_{i-1}^{j+1} + \beta_i C_i^{j+1} + \gamma_i C_{i+1}^{j+1} = \delta_i \quad (i = 1,2,\cdots,n)$$

Here $\alpha_i = -\frac{E}{\Delta x^2}$; $\beta_i = \frac{1}{\Delta t} + \frac{2D_{AB}}{\Delta x^2} + \frac{K}{2}$; $\gamma_i = -\frac{D_{AB}}{\Delta x^2}$; $\delta_i = C_i^j\left(\frac{1}{\Delta t} - \frac{U}{\Delta x}\right) + C_{i-1}^j(\frac{u}{\Delta x} - \frac{K}{2})$

The Matlab codes[8] are attached in the Appendices.

Contamination Condition variation under different development pattern

Currently, different countries have different development patterns, some with a high pollution emission amount while some with the opposite. In our model, even though we distinguish the nodes according to the oceans but not the countries, the development patterns quite resemble the major influential countries' pollution emission mode in a given region. For example, in the region of Western Europe, the emission of pollutants hasn't been increasing too quickly, however, in the region of Asia, which mostly includes developing countries, the amount of pollutants release to ocean grows very fast. Thus, it is reasonable to list out some particular emission patterns to see how our model will react to such patterns. Since it is almost impossible to collect all the data needed to run our model, we simply design some growth equations to analyze how the health condition changes with time. We should note that the data used in the model are not real data, but we could still find something by simulation. A more precise and realistic solution could have been achieved if we had access to real data.

Basically, the development patterns can be described as the following forms.

1) Constant function

$$x(t) = C$$

2) Linear function:

$$x(t) = kt + b$$

3) Quadratic function

$$x(t) = ax^2 + bx + c$$

4) Exponential function

$$\mathrm{x(t) = C_0 e^{\sigma t} + B_0}$$

5) Logarithmic function

7 http://wenku.baidu.com/view/44c3baf8770bf78a652954ee.html
8 http://www.pudn.com/downloads355/sourcecode/math/detail1544589.html

$$x(t) = C_0 log_\alpha t + B_0$$

We apply these five types of function into our model. The output of the model is a binary function $C(x,t)$ so after solving the model, we would acquire the amount of pollutants in any place and at any time. In order to get to know how badly the pollutants affect the countries in the networks, we regard the pollutants near the land as our indicator. The picture below shows how the health condition will change under different pollution emit patterns.

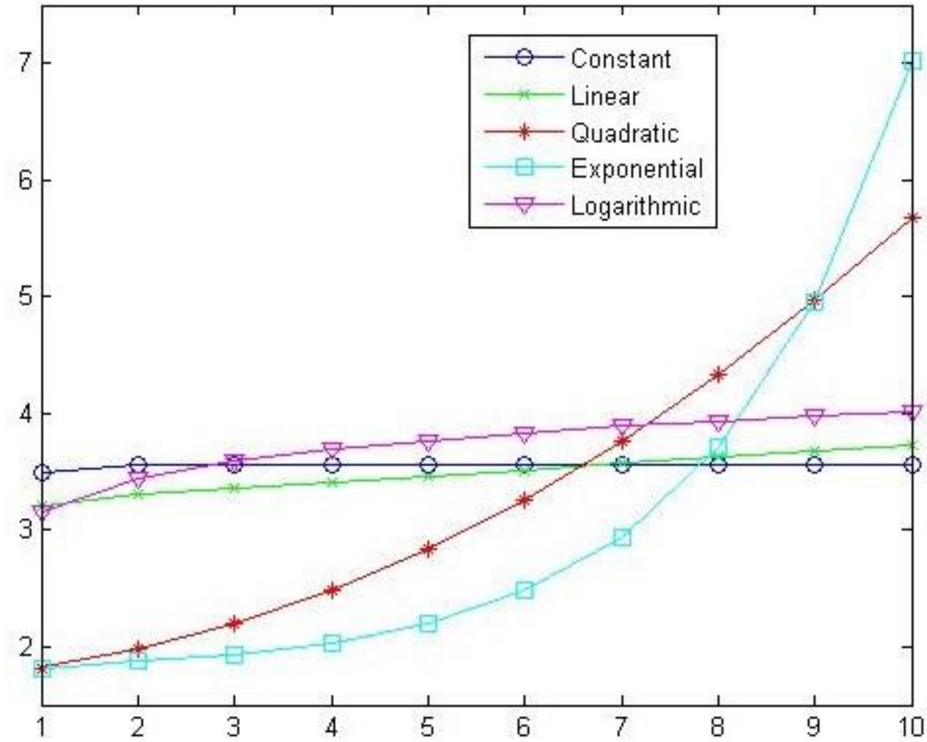


*Figure 5 Pollutants with Different Development Patter*ns

From the picture we know that the increase in the aggregation of pollutants in the coastal water quite resembles the increase in the emission of pollutants. Even though the ocean has the ability of self-purification, these results reflect how it works considering self-purification in the ocean, which can be characterized by attenuation coefficient. If the emission is stable the pollution level remains constant and the more pollutants discharged into the ocean, the more threat to the Earth. The pollution growing type is in accord with emission development.

Unexpected Environmental Event

There are usually some environmental events that are out of our expectation. Recalling the massive leakage of oil in the Gulf of Mexico in 2010, we believe that the ability to evaluate the influence of unexpected environmental events is quite important, both for the technicians and for policy makers. Our model is able to analyze the propagation of pollutants so it is reasonable to predict how many and how quick the pollutants travel from one node to the others if one of the nodes encounters a massive release of pollutants. The following picture shows how the pollutants might transmit with time.

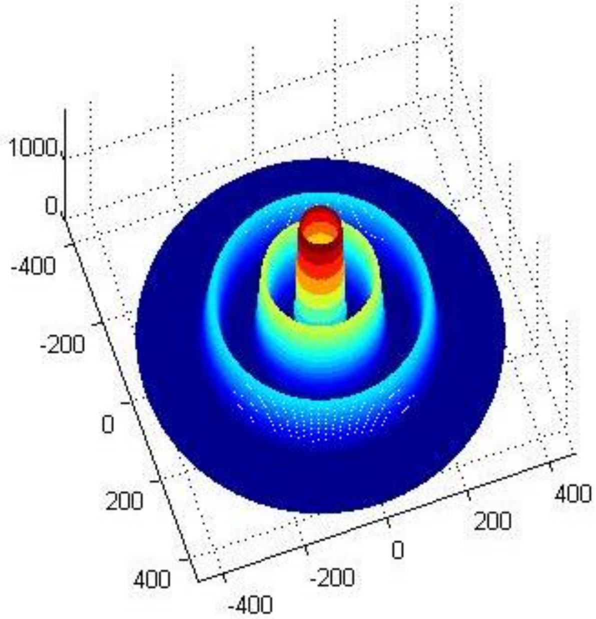


Figure 6 3D Simulation on the Transmission of Pollutants

The center of the concentric circles is the source node, which encounters an unexpected environmental event recently with large amount of release of pollutants. We raise the amount to 500 times to that of in normal conditions. We have three samples with respect to time so there are three humps in the figure. The pollutants are first emitted from the center and then spread out. The hump closest to the center is the data sampled first, with the highest intensity. After some time, the hump transmits to a more distant point, with lower intensity. This is because of the diffusion and the self-purification of the ocean. Even though the attenuation rate is quite high, when the source node provides excessive amount of pollutants, other nodes can be influenced greatly. Simulation above does not take real data so it could just be used as a relative condition. If our model verification plan could be conducted, we would be able to provide prediction more practically.

Model 2: MIMO Control System Model[14]

Overview

In the first model, we have already defined our nodes and the links between each two nodes. We assign node 1 to node 6 to be the coastal areas near the continents which have been influenced by human impact. Because human activities mainly occur in these areas, these 6 nodes appear to be the source of global pollution. Besides, we define two other nodes, node 7 and node 8, to represent the Arctic and the Antarctic respectively. These two regions, with almost no human activities occurring, cause little damage to the global health.

If the eight nodes were not linked together by some means, which are the ocean currents in our model, the Arctic and the Antarctic would not have been contaminated like what they appear to be nowadays. The health condition of the North Pole and the South Pole have been worsening since the Industrial Revolution. The main causes of the deteriorating environment can be categorized as oil pollution, persistent organic pollution, and flotsam.[9]

[9] http://www.coolantarctica.com/Antarctica%20fact%20file/science/threats_pollution.htm
http://www.scholastic.com/teachers/article/arctic-pollution

The first model focuses on the influence on coastal water by one node and the relationship between two nodes. This forms the basis of the second model. From the results about how one node can affect others, we could conclude that the first six nodes are not transmitting large amount of pollutants to the two poles because of the limitation of distance, the velocity of ocean currents, and the ocean self-purification. However, small amount of pollutants, with the cumulative effect of time, could cause great damage to the Polar Regions. Thus, we use the health condition of Polar Regions to measure the health condition of the Earth. This method could be sound because the health condition of the Earth must have been collapsed if the Polar Regions were badly damaged.

Model Design

According to the introduction in the Overview and based on the results in the first model, we decide to link the eight nodes and try to draw a block diagram to illustrate the relationship. Because the relationship, which is characterized by how the pollutants can be transferred to other nodes, can be properly described as a transfer function. In this model, we aim to use the Control Theory, transfer function and block diagram being its subjects, to solve the problem. By consulting to the first model, we find that the propagation of the pollutants is almost linear. Besides, the oceanic condition is not going to change sharply in 50 to 100 years. Thus, we assume the system to be linear and time-invariable, that is, a LTI system. [14]

In this LTI system, we attach six inputs, *u1~u6*, to node 1 to node 6, and two outputs, *y1~y2*, to node 7 and node 8. Therefore, by judging the condition of the two outputs, we could analyze the health condition of the Earth. We have already drawn some links between these eight nodes in the foregoing paragraphs, and the block diagram of the system is presented in the following.

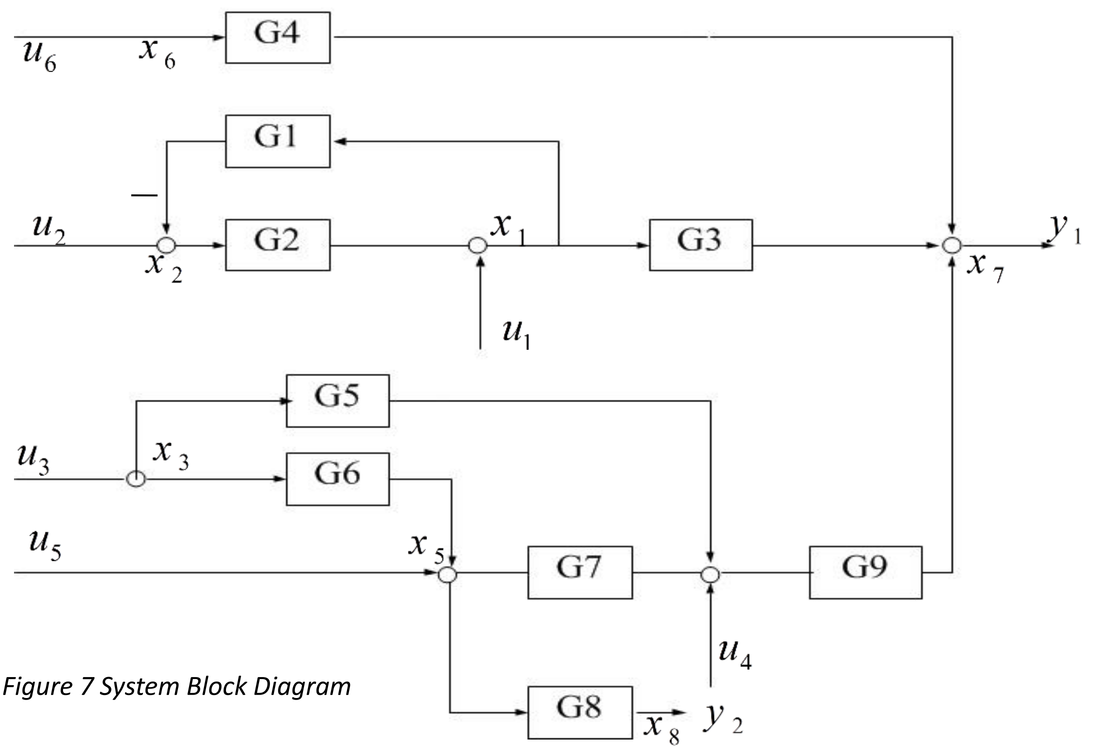


Figure 7 System Block Diagram

In the block diagram, variables with the letter u represents the inputs, which is the flow of the pollutants. Variables with the letter x is the state variable for each node, which represents the health condition of the node. There are several blocks in the diagram, with a notation G_i in it. This is the transfer function of the pollutant propagation for every two nodes. According to the control theory, if the input function is *x(t)* and the transfer function is *g(t)*, then the output of the system is the convolution result of *x(t)* and *g(t)*, that is, *x(t)*g(t)*. If we apply Laplace Transform to the system, the result function is *X(s)G(s)*.

By solving the block diagram, we obtain the relationship between the input and the output. The following is the transfer function in Laplace form.

$$Y_1(s) = \frac{G_3(s)U_1(s)}{1+G_1(s)G_2(s)} + \frac{G_3(s)G_2(s)U_2(s)}{1+G_2(s)G_3(s)} + [G_5(s)+G_6(s)G_7(s)]G_9(s)U_3(s) + G_9(s)U_4(s) + G_7(s)G_9(s)U_5(s) + G_4(s)U_6(s)$$

$$Y_2(s) = G_8(s)U_5(s) + G_6(s)G_8(s)U_3(s)$$

Solution to the Model

We use Matlab Simulink to conduct some simulations to analyze the network structure. The first step is to assign some functions to the inputs. We simply divide them into two categories, namely the step function and the ramp function. The step function means that the pollutants are released constantly at a certain level while the ramp function means that the pollutants rise linearly. The functions we use in the simulation have been listed in the following table. The parameters of the functions can be obtained from the results of model one.

Input 1	$x(t) = 2t$	Input 4	$x(t) = 1.5t$
Input 2	$x(t) = 6$	Input 5	$x(t) = 8$
Input 3	$x(t) = 8$	Input 6	$x(t) = 2t$

Table 1 Input Functions

Noticing that the propagation of the pollutants has two characteristics, namely attenuation and hysteresis, the transfer functions generally have the form of $G(s) = Ke^{-\tau s}$, where K is the attenuation ratio and τ is the time constant. Noting that the exponential function is not easy to solve, we apply Pade Approximate Formula[15]

$$e^{-\tau s} \approx \frac{1-0.5\tau s}{1+0.5\tau s}$$

The simulation diagram is as follows.

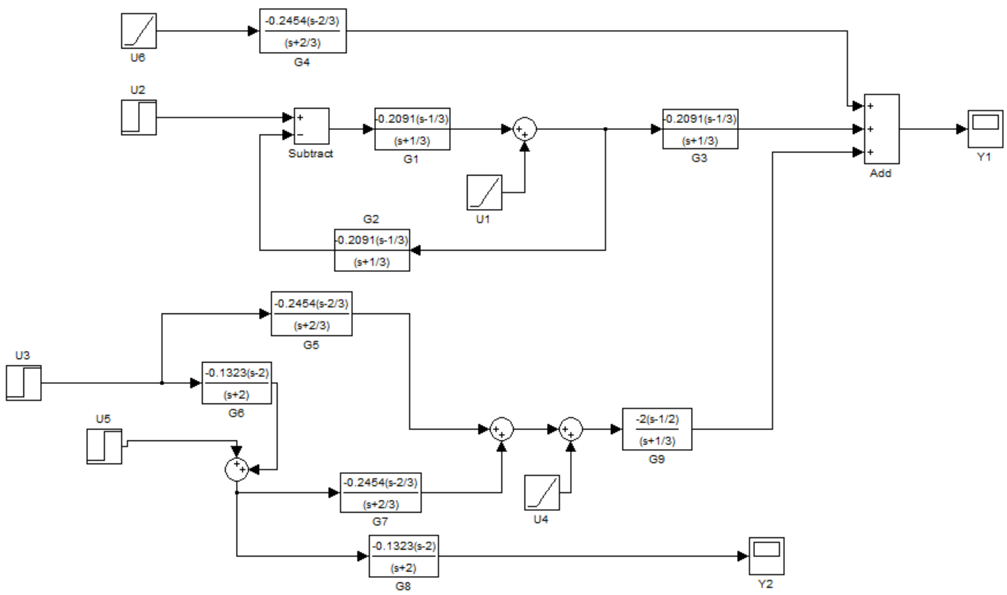
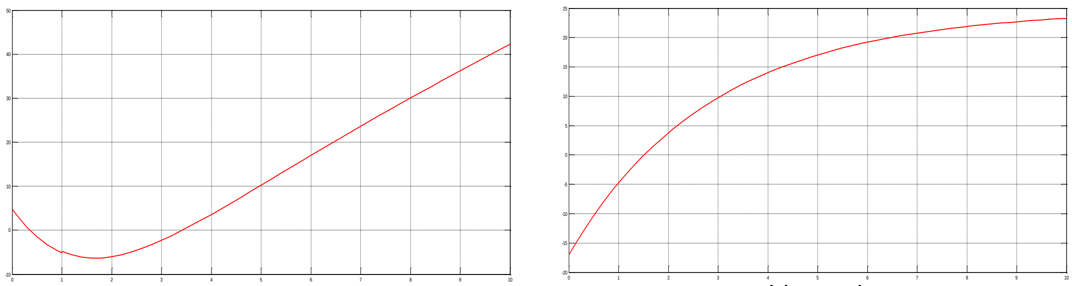


Figure 8 Simulink Block Diagram

Health Prediction Results

Simulation for the Antarctic

We run the simulation using the data provided in the previous section and obtain the development pattern for the Antarctic, indicated by the following figure (Growth Mode). The figure shows that the response begins to rise after about 3 unit time interval and keeps rising with time. The figure informs us that the health condition is getting worse if the emission of pollutants keeps rising, even though it rises linearly. As time goes by, the degree of pollution will reach a tipping point sooner or later.



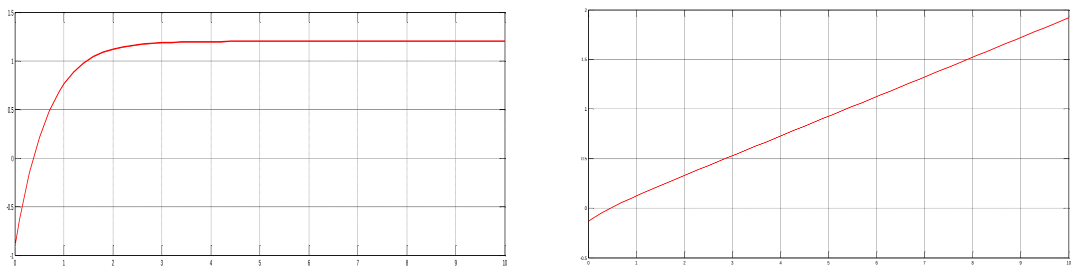
A. Growth Mode B. Stable Mode

Figure 9 Simulation Results for the Antarctic

However, if human change the development pattern so that the emission of pollutants can be constrained to a certain level, the health condition will turn out to be stable, as shown in figure B. Thus, as long as we can cut down the emission to some extent, the health condition will remain below the tipping point.

Simulation for the Arctic

Only two nodes, node 3 and node 5, affect the condition of the Arctic. According to the initial input, the variation of the health condition can be described by figure A presented in the following. However, if the development pattern changes, for example, to ramp function, the health condition will be worse.



A. Stable Mode B. Growth Mode

Figure 10 Simulation Results for the Arctic

Sensitivity Analysis

In order to test how sensitive is the model to any changes in the network, we deliberately set the input functions to be 0 for all the six nodes respectively, and the results are as follows.

Antarctic:

We run the simulation for the Antarctic and the results are presented in the following figure.

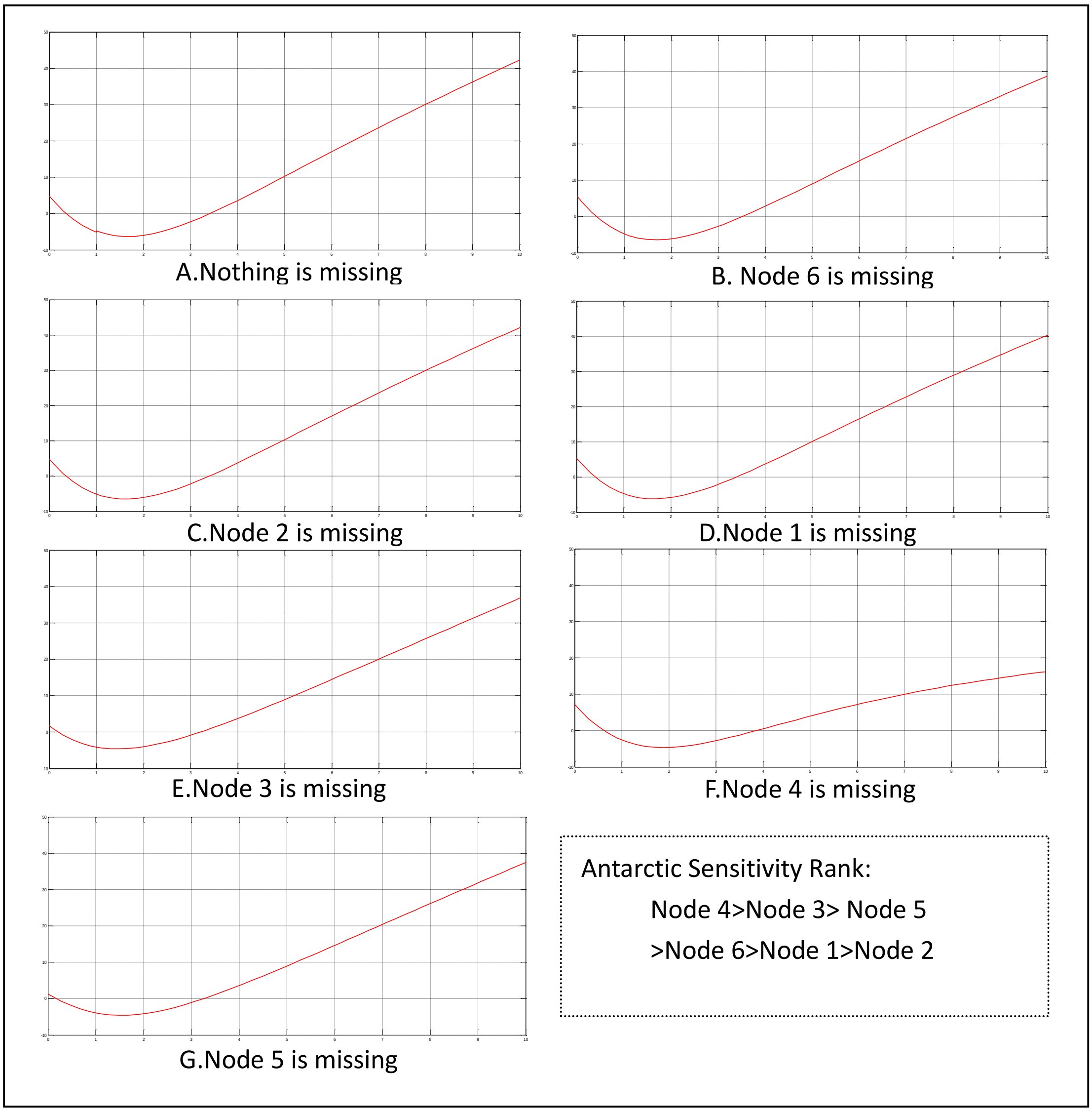


Figure11 Sensitivity Analysis for the Antarctic

Evidently, the health condition of the Antarctic is most sensitive to the variation of node 4 (East Latin America and coastal area). The Antarctic and node 4 are quite close geographically and the ocean currents help with the propagation. Thus, in order to protect the fragile environmental condition in the Antarctic, we should pay much attention on the ocean pollution in the South America coastal area.

Even though other nodes seem to be less important in this simulation, we feel reluctant to say they are unimportant in the global health. The data used in the simulation is not real data since such statistics cannot be found. We simply estimate the data in order to run the simulation. Also, the pollutants in the Antarctic are one of the measures to analyze the Earth’s health condition, but not the only one.

Arctic:

We run the simulation for the Arctic and the results are presented in the following figure.

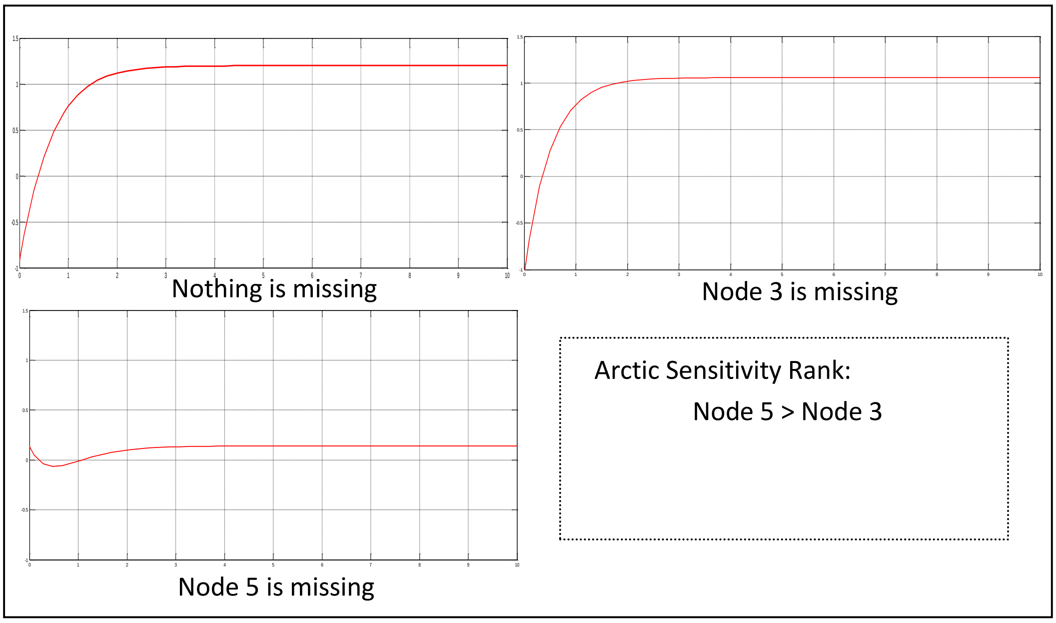


Figure 12 Sensitivity Analysis for the Arctic

The health condition of the Arctic is more sensitive to the variation of node 5 (West Europe coastal area), because geographically, West Europe is closer to the Arctic. In order to protect the environment in the Arctic, West Europe is the critical node.

Uncertainty Analysis

In our model, we simplify the input to some particular functions. However, there are lots of uncertainty in the real world. We randomly generate a set of noise signal and apply the signal to the most sensitive node and the least sensitive node respectively. The results for the Antarctic and the Arctic are as follows.

(1) **Antarctic:**

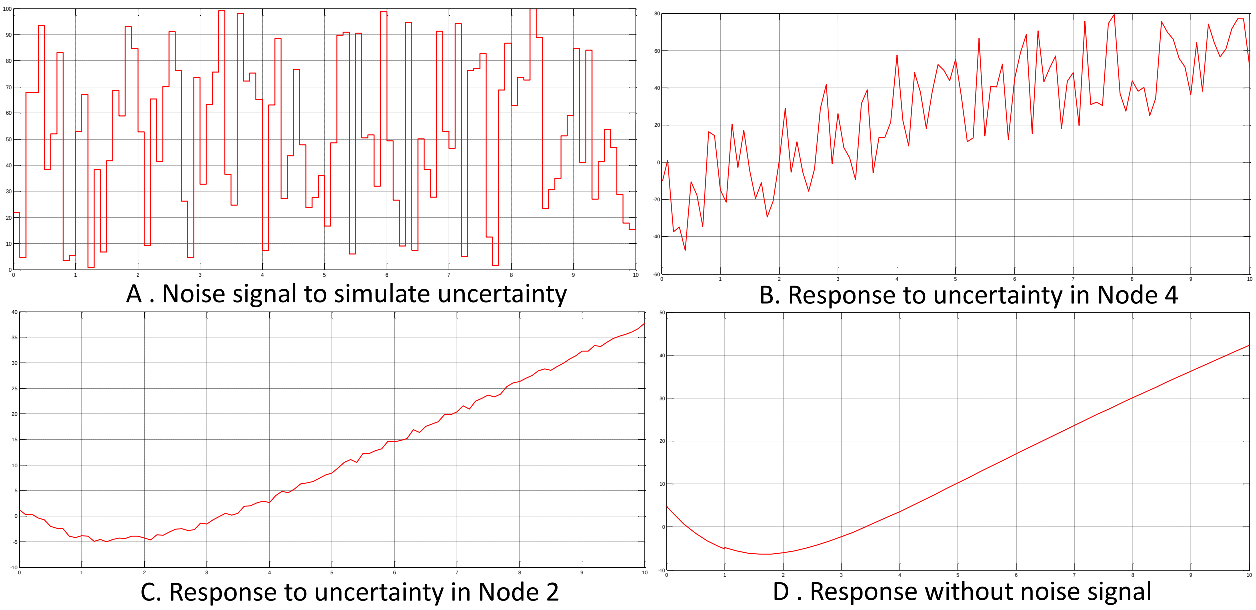
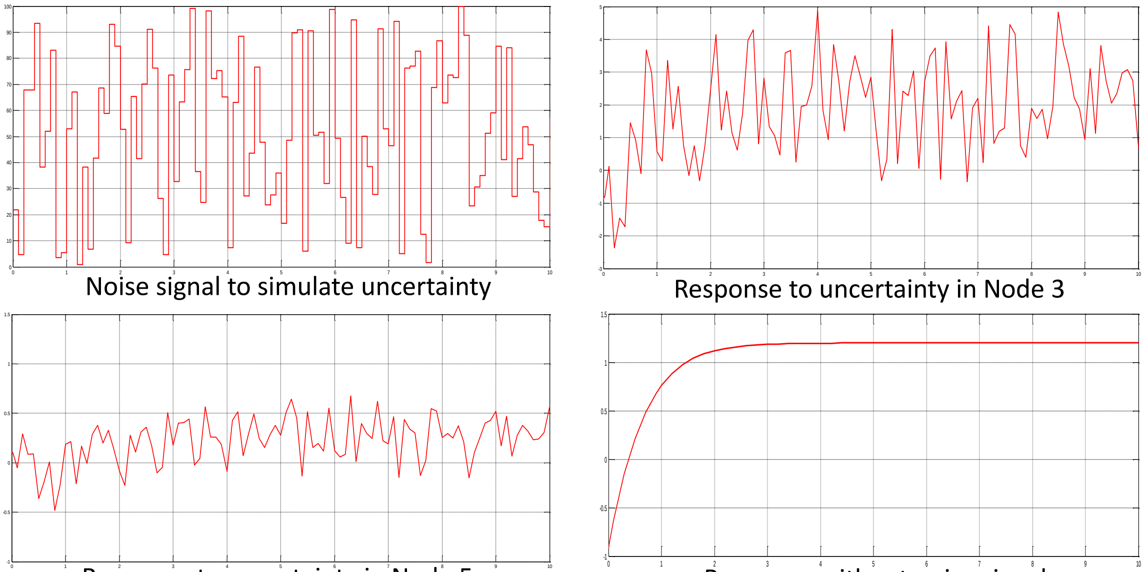


Figure 13 Uncertainty Analysis or the Antarctic

(2) **Arctic:**



Noise signal to simulate uncertainty

Response to uncertainty in Node 3

Response to uncertainty in Node 5

Response without noise signal

Figure 14 Uncertainty Analysis for the Arctic

By comparing the responses to the noise signal from the sensitive node and the non-sensitive node, we find that, when the noise is put to the sensitive node, the response in the Antarctic or the Arctic will act like noise; however, when the noise is put to the non-sensitive node, the response changes little.

Model 3: Social Network Critical Nodes Analysis Model

In our model, we abstract eight nodes and several links to represent important parts and the interactions between them. After we have drawn the nodes and the links (we use line segments with edge weight to show the links), it occurs to us that the relationship resembles a social network[10]! Just imagine there are eight people in the world, some of them have relationship with each other. Since we consider pollutants in the model, the relations are something not that good. We can just compare the links to the situation where A lends money from B or A gives B a big fist. In a word, we need to find out who is the most unpopular person and give them a relative rank, which helps to determine the critical node and list the nodes according to their pollution level.
Here we use NodeXL[5] as our analysis platform. The social network can be drawn as follows.

[10] http://en.wikipedia.org/wiki/Social_network_analysis

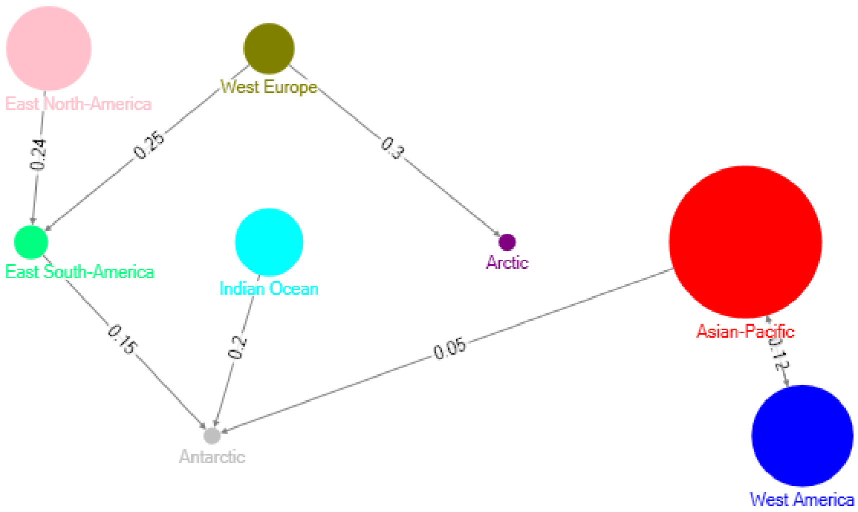


Figure 15 Social Network Directed Graph in NodeXL

In this picture, we use circles to represent nodes and the area is proportional to the amount of pollutants every node releases. The graph is directed and it means it may be one-way influence just like Amy is a fan of a singer in the twitter while the singer may not follow Amy's twitter. And every link has an edge weight which represents the decay rates of pollutants related to the distance and the material attribute.

As we know, there are several evaluation indicators in social network analysis such as in-degree, out-degree, betweenness centrality, closeness centrality, eigenvector centrality and so on. In order to assess the degree to which one node's pollutant influences others, we have to choose one evaluation indicator.

Problems to be solved

Nowadays, there are many kinds of networks such as communication networks, biological networks and social networks. Among these networks there already exist some methods to pick up the critical node. Their core idea is to analyze how the network would work if one node were deleted. For instance, if one node is removed from the network, other nodes can still connect with each other normally. In this circumstance, we say that the node is not critical. On the contrary, if most part of the connections are cut off because of the deletion of a node, it is obviously quite crucial in the whole network.

However, what we value most is not whether the network can still be connected if one node is removed. We set models to find out which node does more harm to others. Our aim is different from connectivity.

Besides, general method to pick up the critical nodes can be applied when all the edges and vertices do not have weights. As for our directed graph, we both have the edge and vertex weight. In this way, common methods cannot solve our problem. So we put up with a new concept to evaluate the importance of nodes in the directed graph based on the core idea of "Closeness Centrality".

Closeness Centrality

Closeness Centrality[11] is defined as the sum of the shortest distance between a specific node and the rest nodes in the whole graph. Thus we can use the indicator to analyze to what degree the node has influence on other nodes through social network. Here is the calculation formula[12]:

$$C_c(i) = \sum_{j \neq i} d_{ij}$$

Where i stands for the i^{th} node and d_{ij} stands for the shortest distance from i to j.
Node with smaller closeness centrality has better scope to receive information. What we want to emphasize is that under this definition if one node has the smallest closeness centrality it does not necessarily mean it is the core node. It just shows from the node we can easily receive information from other nodes.

Adjustment to Closeness Centrality

Now it comes to our improvement! We transform the meaning to some extent but still base our thought on the idea of Closeness Centrality. We give a calculation method in directed networks and we call the new indicator "Influencing Centrality and Influenced Centrality". In undirected networks the calculation is easier.

We divide all the nodes into two categories. The first are nodes which can influence others and the rest are those who can only be influenced. In the global network, the Arctic and the Antarctic belong to the second category. For them, we use "Influenced Centrality". Take the Arctic as an example, West Europe transmits pollutants there directly and East North-America influences it indirectly though West Europe. Notably, every node has pollutant emission and each edge has decay rates as their weights. So the first step is to combine the two weights. We multiply the amount of pollutant by decay rate as one node's influence to the other. If one path has two or more edges, the decay rates should be multiplied. Besides, we should take all the path from i to j into consideration instead of choosing the shortest one. In this way, we both consider the in-degree of influenced node and also all the possible approaches to affect it.
As for nodes who affect others, we will consider out-degree instead of in-degree and then we could obtain "Influencing Centrality". That is to say, consider all the viable paths and add all the influences it poses to others.

In order to emphasize our adjustment to Closeness Centrality, we list the similarities and differences between IC and CC (IC is the abbreviation of Influencing/Influenced Centrality and CC is that of Closeness Centrality).

Similarities:

- They could both be used in weighted networks. The evaluation is based on the distance of path.
- Two indicators start from a certain node and assess the node's impact on the others.

[11] http://en.wikipedia.org/wiki/Centrality#Closeness_centrality
[12] http://wenku.baidu.com/view/e71b50f804a1b0717fd5ddf8.html

Differences:

● **Abstract Meaning**

CC stands for message proximity.[13] A node has the shortest total distance to all the rest. However, the node with the smallest CC is not necessarily the critical node. It just shows that the node has the best scope to detect information from other nodes.

One node with the biggest IC is the node who poses the most impact on others. It is to say the node is the critical node. It takes all the possible links into consideration.

● **Shortest path or not**

CC just adds all the shortest paths while IC combines all the viable paths.

● **If one path has two or more edges**

CC just adds the edge weights up while IC has to multiply the weight of the first edge by the one of the second edge and thus we could get the adjusted weight.

Apart from the Antarctic and the Arctic (both of them will not release pollutants and they do not need to control pollution emission), we calculate the other nodes' Influencing Centrality and give them a rank. What is different from Closeness Centrality is that the bigger our

Influencing/Influenced Centrality is, the more influential the node will be.
The result is listed in the table below.

Node	**Influencing Centrality**	**Rank**
Asian-Pacific	15	3
West America	8.82	5
East North-America	40.635	1
East South-America	4.5	6
West Europe	23.5	2
Indian Ocean	10	4

Table 2 Rank of influencing nodes

Node	**Influenced Centrality**	**Rank**
Antarctic	24.255	1
Arctic Ocean	17.4	2

Table 3 Rank of influenced nodes

(Note: Due to the lack of realistic data, we just give a relative assessment.)

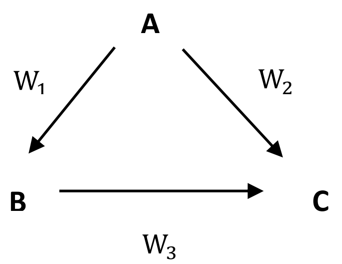
Analysis of the IC table

From the result above, we conclude that the IC criterion is quite realistic and reasonable. The rank shows that the most influential node is East North-America. It is not only because East North-America releases a large amount of pollutants but also it has many connections with other

[13] http://blog.sina.com.cn/s/blog_4c98b96001009a4p.html

nodes and the distances between them are relatively short. Reasons from three aspects all lead to the first rank. Notably, Asian-Pacific has the largest amount of pollution emission and it ranks third in the influencing nodes. It is because the Pacific Ocean is quite large and it has relatively few connections with other nodes.
As for the influenced areas, we notice that the Antarctic is in great danger of pollution because almost every influencing node could pose threat to it. Furthermore, the ecosystem there is quite vulnerable due to the low environmental bearing capacity and the simple food chain.
In conclusion, the critical node in our model is East North-America. Asian-Pacific is also important and the ecosystem safety of the Antarctic should be highlighted.

An illustration of the calculation of IC:



●Direct influencing significance of A: $\mathrm{AW_1 + AW_2}$
●Indirect influencing significance of A: $\mathrm{AW_1W_2}$
●$\mathrm{W_1 \cdot W_2}$ shows the impact will decay after passing a bridge node.

Figure 16 An Example of the Calculation of IC

Suggestions based on analysis result

●As far as ocean pollutants are concerned, East North America coastal lies in a crucial position which has many links with other oceans. So the emission should be controlled there even though the amount of pollutants it released is not that large.

● Asian-Pacific area should also set a restriction to pollution discharge for its emission load is too high. To our relief, it does not have as many connections as East North America so the influence relatively decreases.

●Although the Antarctic does not emit pollution into water body, it is affected just as much as West Europe, which is really to our surprise. Besides, when taking the environment load into account, the Antarctic is more vulnerable than West Europe since it has a simple food chain.

●What we want to emphasize is the global influence of every node. If one node discharges pollution into the ocean, it not only does harm to the node itself but also poses threat to the global areas because of the existence of links. So it is the global areas that are supposed to be responsible for controlling ocean pollution instead of one certain node.

Strengths and Weaknesses

Strengths	*Weaknesses*
✓ Our model takes into account the nodes' connections and interactions and thus creates links to track relationships. The links show interregional influences. The network is concise and incorporates feedback loops. ✓ Our model relates the dynamic development of ocean contamination with pollution discharge pattern, which represents human effects on the ocean. ✓ Use control theory to analyze the network structure, sensitivity and uncertainty. ✓ We separate the Polar Regions from other nodes and use the health condition of them to measure that of the Earth. ✓ We introduce social network analysis into the evaluation of nodes' links. By making some adjustments to the existing indicator-Closeness Centrality, we make it possible to analyze directed graph and rank the nodes according to their impact on others, which helps us to pick out critical nodes. (Model III)	❖ Some of the minor ocean currents have been neglected and the selection of the nodes is not elaborate enough. ❖ We lack real data to testify our model so we can only give a plan to check the validity. ❖ We just give numeric solutions to the partial differential equations instead of analytic solution. When it comes to long distance and long period, the concentration of pollutants might be extremely small and the predicted result may probably be zero due to the rounding errors. (Model I) ❖ The transfer functions cannot describe the real situation precisely. ❖ The exact decay rates in social network analysis are not accessible.

In the future , we can do more jobs to make our model better :

- Our model is simplified to one-dimension. We will try to consider two or three dimensions to see whether the prediction will be better.
- We have already demonstrated the ability of our model to analyze the health condition of the Earth. Thus, the next step is to collect enough data to make our model more practical and present more useful information for policy makers.

Reference

[1] Benjamin S. Halpern *et al*. A Global Map of Human Impact on Marine Ecosystems. Science 319, 948 (2008).

[2] Johan Rockström *et al*. A safe operating space for humanity. Nature, Vol 461, 24 September 2009.

[3] Anthony D. Barnosky *et al*. Approaching a state shift in Earth's biosphere. Nature, Vol 486, 7 June 2012: 52-58.

[4] Evidence of Impending Tipping Point for Earth. ScienceDaily.
http://www.sciencedaily.com/releases/2012/06/120606132308.htm

[5] Derek Hansen and Ben Shneiderman, Marc Smith. Analyzing Social Media Networks: Learning by Doing with NodeXL. Draft (7/07/2009)

[6] Willem Hundsdorfer. Numerical Solution of Advection-Diffusion-Reaction Equations. Lecture notes, 2000, Thomas Stieltjes Institute.
[Online] Available: http://homepages.cwi.nl/~willem/Coll_AdvDiffReac/notes.pdf

[7] George L. Mellor. Users Guide for A Three-Dimensional, Primitive Equation, Numerical Ocean Model. This revision: June 2004.
[Online] Available: http://jes.apl.washington.edu/modsims_two/usersguide0604.pdf

[8] J. Scott Armstrong. Forecasting for environmental decision making. V.H. Dale and M.E. English, eds., Tools to Aid Environmental Decision Making, 1999, New York: Springer-Verlag, pp. 192-225.[Online] Available:
http://repository.upenn.edu/cgi/viewcontent.cgi?article=1006&context=marketing_papers

[9] Guo Ru, Li Yu-bin, Fu Guo. Controlling factors of degradation coefficient on organic pollutant in river. Journal of Meteorology and Enviroment. Vol 24, No.1 February 2008.

[10] Zheng Ning, Li Jian-wei, Chu Wei-pan. Solving one dimension diffusion?convection equation by Excel. Journal of Xi'an Univerisiy of Science and Technology. Vol.26, No.2, Jun. 2006.

[11] Wang Fajun. Determination of Marine Turbulence Diffusion Coefficents. Marine Science Bulletin. Vol 11, No.2, Apr. 1992.

[12] Li Qiufang. The Characteristic Difference Method for Solving Convection-Diffusion Equations. Xi'an: Xi'an Univerisity of Technology. 2003.

[13] Xia Shen. Study and Application of 3D Offshore Water Quality Model Based on POM. Nanjing: HoHai Univerisity. 2006.

[14] John J.D'Azzo, Constantine H.Houpis. Linear Control System Analysis and Design Convention and Modern (Fourth Edition). Peking: Tsinghua University Press. 2000, 91-138.

[15] Guo Qingwei. Recurring formula of quadratic Pade approximation's polynomials of e-x. Journal of Hefei University of Technology (Natural Science). Vol.24, No.3, Jun. 2001.

Appendix:Matlab code

The Performance of One Node

```
clear;clf;clc;
D=2;u=5;k=0.015;L0=10;Ct0=0;
t=0.2;h=1;
m=10;n=10;
tn=10;
C=zeros(m,n);
sum=zeros(tn,n);
for k=1:tn
    for i=1:m
        C0(i)=C(i,n);
    end
    C=calculate(C0,k);
    for i=1:n
        sum(k,i)=C(2,i);
    end
end
for i=1:(tn-1)
    diff(i)=sum(i+1,n)-sum(i,n);
end
value=sum(:,n);
plot(value,'mv-');
```

Sub-function to Calculate the Spread of Pollutants

```
function C=calculate(C0,p)
D=2;u=5;k=0.015;L0=9;Ct0=0;
t=0.2;h=1;
m=10;n=10;
for i=1:5
    C(1,i)=L0+log(p);
end
for i=6:10
    C(1,i)=Ct0;
end
for j=2:n
    C(j,1)=C0(j);
end
alpha=-D/h^2;beta=1/t+2*D/h^2+k/2;gama=-D/h^2;
a=alpha*ones(1,n);b=beta*ones(1,n);c=gama*ones(1,n-1);
a(n)=alpha-gama;b(n)=beta+2*gama;
```

```
d=ones(1,n);
for j=1:m-1
  d(1)=C(2,j)*(1/t-u/h)+C(1,j)*(u/h-k/2)-alpha*C(1,j+1);
for i=2:n
  d(i)=C(i,j)*(1/t-u/h)+C(i-1,j)*(u/h-k/2);
end
x=thomas(a,b,c,d);
C(:,j+1)=x;
end
for i=1:5
C(1,i)=L0;
end
for i=6:10
        C(1,i)=Ct0;
end
```

Thomas Algorithm

```
function x=thomas(a,b,c,d)
n1=length(b);
u0=0;y0=0;a(1)=0;
L(1)=b(1)-a(1)*u0;
y(1)=(d(1)-y0*a(1))/L(1);
u(1)=c(1)/L(1);
for i=2:(n1-1)
        L(i)=b(i)-a(i)*u(i-1);
        y(i)=(d(i)-y(i-1)*a(i))/L(i);
        u(i)=c(i)/L(i);
end
L(n1)=b(n1)-a(n1)*u(n1-1);
y(n1)=(d(n1)-y(n1-1)*a(n1))/L(n1);
x(n1)=y(n1);
for i=(n1-1):-1:1
        x(i)=y(i)-u(i)*x(i+1);
end
```

The Relationship between Two Nodes

```
clear;clc;
D=2;u=6;k=0.015;L0=5000;Ct0=0;
t=0.2;h=1;
n=300;m=500;
for i=1:1
        C(1,i)=L0;
```

```
end
for i=2:m
        C(1,i)=Ct0;
end
for j=2:n
        C(j,1)=0;
end
alpha=-D/h^2;beta=1/t+2*D/h^2+k/2;gama=-D/h^2;
a=alpha*ones(1,n);b=beta*ones(1,n);c=gama*ones(1,n-1);
a(n)=alpha-gama;b(n)=beta+2*gama;
d=ones(1,n);
for j=1:m-1
  d(1)=C(2,j)*(1/t-u/h)+C(1,j)*(u/h-k/2)-alpha*C(1,j+1);
for i=2:n
  d(i)=C(i,j)*(1/t-u/h)+C(i-1,j)*(u/h-k/2);
end
x=thomas(a,b,c,d);
C(:,j+1)=x;
end
for i=1:5
C(1,i)=L0;
end
for i=6:10
        C(1,i)=Ct0;
end
for i=1:n
        value(i)=max(C(i,:));
end
nn=1:n;
for i=1:n
        trans(i)=C(i,j);
end
x=1:n;
x=x';
y=x;
z=trans;
z=z';
r = sqrt(x.^2+y.^2);
p = length(z);
alpha = linspace(-pi,pi,p);
xx = r * cos(alpha);
yy = r * sin(alpha);
zz = z * ones(1,p);
mesh(xx,yy,zz);
```