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2013

Mathematical Contest in Modeling (MCM/ICM) Summary Sheet

(Attach a copy of this page to your solution paper.)

Two-layered Coupled Network Model of Earth's Health

Earth's health, closely related to our daily life, has aroused widely concerns. In this paper, we construct *a two-layered coupled network model*, which is based on the *mean-field dynamic Lotka-Volterra equations* and mechanism of *Reaction-Diffusion methods*, to analyze, forecast and evaluate the biological and environmental health condition of our planet.

In our two-layered coupled network model, we choose different regions, according to the global geography and climate distribution, as the nodes of the global network. Meanwhile, several representative elements of earth's condition, e.g. population, forests, air quality, biodiversity and so forth, are selected to be the nodes of the elementary network, and we describe the interaction of all elements based on the *mean-field dynamic Lotka-Volterra equations*. Furthermore, we build and prove the propagation equations among the elements of different regions, to analyze and forecast properties and structures of global network.

Using the data provided by *Yale University*, we first fit the parameters of our model by *Ordinary Least Square method (OLS)*. And then, the future trend of every element in each region can be predicted. Moreover, we propose a method to seek the tipping point, an irreversible critical change point definded by James Hansen - the father of global warming research. Additionally, taking the region where United States belongs for instance (the following analysis are all used the data of this region), we find that only two of the elements, air quality and biodiversity, may have tipping points, and their value is 38.76 and 59.86, respectively. Hence, suggestions could be provided for government and dicision makers.

Next, paying attention on the human effects, sensitivity analysis of impact factors, which describe the influence of population on other elements, are draw out. According to the results obtained, we find that population has huge effects on remaining elements.

Meanwhile, we analyze the elementary network structure. Firstly, according to the absolute value of parameter, we rank all elements and reserve primary links. Centrality of every element can be obtained via network topology analysis, and the result shows that forests element is the central point of elementary network. Secondly, we propose a new network with weights and a conception "Influence degree" which equals to the average absolute value of weight sum of its *out-degree*, to evaluate influence capacity of nodes. The final results demonstrate that population plays the most important role in network, while forests element ranks fourth. Therefore, we draw the conclusion that the central point in the ecology network is different from the one with huge influence.

At last, taking uncertainties into account, we optimize our model by joining disturbance which obeys the *normal distribution*. Compared with our original model, we analyze the predictive results obtained by improved model.

Finally, we discuss the strengths and weaknesses of our model in detail.

Key words: two-layered coupled network model , mean-field dynamic L-V equations, Reaction-Diffusion methods, sensitivity analysis, network structure

Content

1 Introduction

Earth's environmental and biological health plays a vital role in survival and evolution of species, quality of life and prosperity. And the interactions between environmental change and human societies have a long, complex history spanning many millennia, but these have changed fundamentally in the last century. Human activities are now so pervasive and profound that they are altering the Earth in ways which threaten the life-supporting system upon which humans depend[1].

Interested in the biological and environmental health condition of our planet, several general methods have been proposed to forecast. Most models are based on biological forecasting methods, which depend on projecting recent trends of a range of health measures to evaluate the impact of future socioeconomic development pathways on biodiversity and ecosystem services[2]. Others, phenomenological models of species distributions are applied to focus on examining the potential role of the synthetic discipline of evolutionary community ecology and further predict how climatic changes may alter presently observed geographic ranges[3,4,5].

The above approaches are useful in predicting states and trends of certain health measures at vastly different temporal scales, however insufficiently, and still exist two major inadequacies: (a) Most studies focus on local change of biological and environmental systems and ignore complex global change; (b) They overlook complex relationships and cross-effects in myriad environmental and biological systems. Therefore, there is an urgent need for even more full-scale and scientific models.

Aiming at improving the above two major inadequacies, our paper addresses to provide a better predictive global model based on existing biological forecasting models, with the following two specific quantitative modeling challenges:

- Embracing the complexity of Earth's interrelated systems and the effects of local conditions on the global system, together with significant human effects, the global model we propose should be able to improve bio-forecasting.
- Identify the tipping points with several factors by global model and effective preparations can be done to prevent or limit that.

To further present our solutions, we arrange our paper as follows:

i. In section 2, we construct a two-layered coupled network to analyze, forecast and evaluate the biological and environmental health condition of our planet. And detailed description of global network and elementary network models are also given out.

ii. In section 3, we construct differential equations to describe our network model. We apply mean-field dynamic Lotka-Volterra method to describe the interaction of elements, and propagation equations to describe propagation of elements among different regions.

iii. In section 4, we apply our model to solve a series of problem, such as prediction of Earth's health and identification of tipping point. And sensitivity analysis is given out, as well as the study of network structure.

iv. In section 5, taking uncertainties into account, we optimize our model by joining disturbance.

v. At last, we discuss the strengths and weaknesses of our model in detail.

2 Construct a Two-layered Coupled Network

Resulting in different types of networks, global networks can be captured from various information, where each node represents an Earth's health condition, which could be nations, continents, oceans, habitats, or any combination of them or others, and a link could represent nodal or environmental influences, or the flow or propagation of physical elements (such as pollution) over time.

Moreover, health measure should also be taken into account, which could be any element of Earth's condition, including demographic, biological, enviromental, social, political, physical, and/or chemical conditions, etc.

With a large amount of factors and aspects need to be considered, a simple network does not necessarily reflect well the state of the entire environmental and biological system. A complex network[6,7] with a multi-layer structure, whose layers have a certain interactions and dependencies, seems to be more reasonable and comprehensive.

Therefore, we come up with ***a two-layered coupled network***, *global network* and *elementary network*, which belongs to complex network, to reflect the global condition.

2.1 Global Network

As mentioned above, the nodes we chosen for our network must reflect the Earth's health conditions. Meanwhile, attention is paid to the biological responses on all scales from local to global. According to geography and climate, we first divide the whole world into several regions, which is shown in Figure 1 .

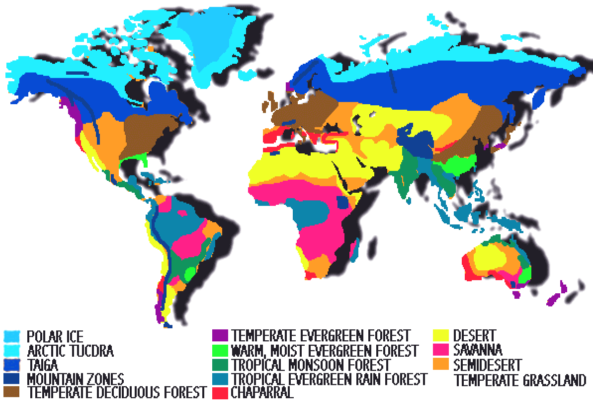

Figure 1 Global geography and climate

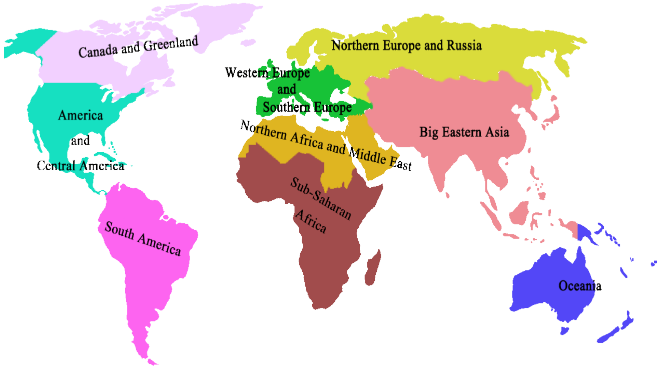

Figure 2 Divide Earth into pieces

Figure 2 shows the planimetric map of Earth with different colors. Apparently, the whole world are divided into nine regions, each of which has obvious difference in geography and climate and is signed by a certain color.

Furthermore, the regions we divided are regarded as the nodes in our global network. And the links between each two nodes, the red lines shown in Figure 3, represent the flow or propagation of physical elements over time. Actually, there must be a great amount of lines which are connected in pairs; however, when considering the fact that the farther the distance of each two nodes is, the weaker propagation effect is, we only adopt the direct connection for every node, namely, the links between an arbitary node and its neighbouring nodes. Figure 3 shows the nodes chosen and the direct connecting of each node in the network, of course, global network.

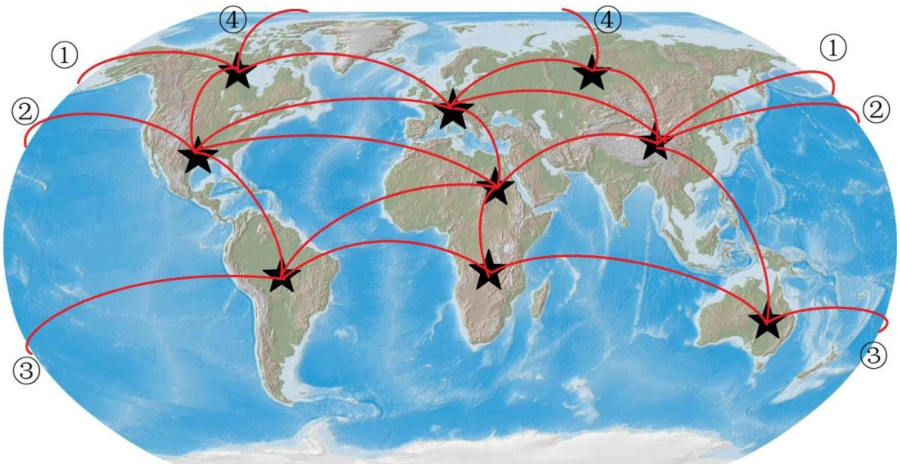

Figure 3 Global network with nine nodes and direct links

The map showed above is planimetric, however, the links belong to spherical connection. Namely, some links could not be well draw. Thus, we think that the links with a same number are one link, just as line① and line② in Figure 3.

For the convenience of data search, groups of representive countries (including territory, territory sea, and exclusive econimic zone) in each region, which occupy more than 70% of their own region area, is chosen to be our investigation.

2.2 Elementary Network

Our two-layered coupled network consists of global network, which has been constructed in the above section, and elemementary network, the subnetwork we will continue in this section.

In order to evaluate and forecast the health condition, the elements of Earth's condition are indispensible, which seems to be various and hard to be defined.

Luckly, the *2012 Environmental Performance Index (EPI)*[8] ranks 132 countries on 22 performance indicators in the following 10 policy categories. Moreover, tracking performance and progress on above categories and indicators, the EPI's methodology facilitates cross-country comparisons among economic and regional peer groups.

With the help of the EPI data, we finally choose seven representive elements as our study objects, *air quality (effects on human health), water (effects on human health), biodiversity and habitat (biodiversity, for short), forests, human population density, climate change, and energy consumption*, respectively.

Clearly, we can know the meaning of the above seven elements just as their names imply. And their values have been regularized by *the Yale Center for Environmental Law and Policy (YCELP)* and *the Columbia University Center for International Earth Science Information Network (CIESIN)* in the same rule.

Therefore, the elements we choosen are regarded as the nodes of the elementary network, namely, there are seven nodes in subnetwork, and the links must be the relationship and interaction of each two elements.

Hence, according to the analysis of global and elementary network, we complete our final two-layered coupled network and its structure as showed Figure 4.

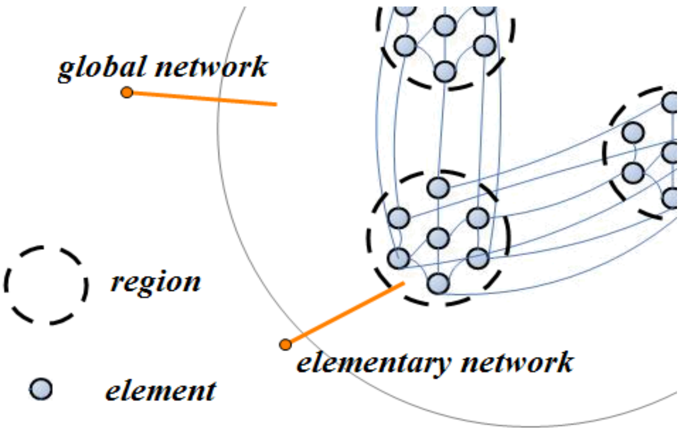

Figure 4 A two-layered coupled network

3 Network Model Based on Differential Equations

In order to forecast the biological and environmental health conditions of our planet, we has constructed a two-layered coupled network under a full-scale view, which concludes global network and elemetary network. Besides, the elements chosen above are good embodiments of the health level of our Earth. Therefore, we consider that the Earth's change can be measured via observing and studying the change of elements' values.

Generally speaking, the change of elements' values has internal reason and exterior reason which are planed to be explained in this section.

3.1 Internal Reason : Changes in Elementary Network

Internal reason, means that the change of elements' values happens in elementary network. Namely, the relationship and interaction of each two elements. Here, we apply the *Lotka-Volterra equations* to describe this phenomenon.

3.1.1 Mechanism of *Lotka–Volterra Equations*

The Lotka–Volterra equations[9], also known as the predator - prey equations, are a pair of first-order, non-linear, differential equations frequently used to describe the dynamics of biological systems in which two species interact, one predator and its prey.

We assume that $u_1(t),\ u_2(t)$ denote the quantity or density of two species at time t, respectively. Starting with their relative growth rates, we should also take self-development in their own species and interaction between them two into consideration. Thus, Lotka–Volterra equations can be described below :

$$\begin{cases} \dfrac{du_1}{dt} = u_1\left(a_1 + b_1 u_1 + c_1 u_2\right), \\ \dfrac{du_2}{dt} = u_2\left(a_2 + b_2 u_1 + c_2 u_2\right), \end{cases} \tag{1}$$

Where : $a_1,\ a_2$ denote the unconstrained natural growth rate of these two species, respectively ; $b_1,\ b_2$ denote the impact factor of species u_1, which influences the natural growth rate of these two species, respectively ; similarily, $c_1,\ c_2$ denote the impact factor of species u_2.

3.1.2 Interaction among Elements

As for our problem, the mechanism of *Lotka–Volterra equations* could also applied well in changes of every element. For seven given elements, there are seven equations for a certain node of global network. Here, we take human population density element for instance to detailedly descripte it..

Defined "natural" as non-human traditionally, ecology has changed a lot nowadays. Human, as a strong and powerful group, impacts a wide range of fields. The product aggregates human activity in recent decades[1]: increased emission of greenhouse gases, damage to stratospheric ozone, depletion of aquifers and large-scale destruction of rainforest. These impair the productivity (soil, forest, oceans, biodiversity) or stability (climate, sea-level, ultraviolet filtration) of Earth's natural systems.

Human effects have increased significantly in our current biosphere situation, thus, to some extent, it is evitable to choose human population density as an element. Obviously, here, human population density we paid attention on has huge impacts on itself and the other element.

Focusing on elementary network of an arbitary in global network, we assume that $X_P^i(t)$ represents the density of human population density of node i at time t. Thus, the density changes of population density can be obtained below :

$$\begin{aligned}\left(\frac{d\left[X_P^i(t)\right]}{dt}\right)_{IN} &= X_P^i(t)\cdot\left[\gamma_P^i+\alpha_{PP}^i X_P^i(t)+\alpha_{PW}^i X_W^i(t)+\alpha_{PB}^i X_B^i(t)\right.\\ &\quad\left.+\alpha_{PF}^i X_F^i(t)+\alpha_{PA}^i X_A^i(t)+\alpha_{PC}^i X_C^i(t)+\alpha_{PE}^i X_E^i(t)\right],\end{aligned} \tag{2}$$

Where : A,W,B,F,P,C,E is short for seven elements, air quality, water, biodiversity, forests, population density, climate change, and energy consumption, respectively ; γ_P^i denotes the unconstrained natural growth rate of population density; α_{PP}^i denotes impact factor of population density which has an effect on itself, common sense tells us that large population density will restrict its own growth; and α_{Pm}^i $(m=A,W,B,F,C,E)$ denote the impact factors of other six elements which influence the natural growth rate of population density.

Similarly, other six elements also have the above equation. For concise writing, we give out general formula below :

$$\left(\frac{d\left[X_k^i(t)\right]}{dt}\right)_{IN} = X_k^i(t)\left[\gamma_k^i+\sum_m \alpha_{km}^i X_m^i(t)\right], \tag{3}$$

$$k,m=A,W,B,F,P,C,E.$$

3.2 Exterior Reason : Changes in Global Network

Exterior reason, means that the change of elements' values happens in global network. Namely, the flow or propagation of physical elements.

3.2.1 Apply the Mechanism of *Reaction-Diffusion Model*

In order to investigate the propagation of physical elements among nodes, we detail this phenomenon by applying the mechanism of *two-dimensional reaction-diffusion model*[10] *(R-D equation)*, which is used to express propagation effect via seeking divergence of the regional density gradient with a diffusion coefficient.

Being regarded as flow which has the ability to move from higher density to lower density, the propagation effect can be vivid described by divergence. However, the nodes are discrete; thus, we also need to deviate the expression of propagation between discrete points.

Regarded as known information, the distance between a node and its neighbors, which is easy to obtain, have something to do with propagation. Furthermore, a detail detailed proof is provided to seek the relationship in next section.

3.2.2 A Proof Based on A Point Source Flow

A scalar field $f\left(\rho,\theta\right)$, and its gradient, whose magitude is the maximum rate of change of f per unit length of the coordinate space at the given point, could be described below in polar coordinate system:

$$V=\nabla\left[f\left(\rho,\theta\right)\right]=\frac{\partial f}{\partial\rho}\vec{e}_{\rho}+\frac{1}{\rho}\frac{\partial f}{\partial\theta}\vec{e}_{\theta} \tag{4}$$

Therefore, this scalar field $f\left(\rho,\theta\right)$ will transfer into a vector field V after derivating gradient. For a source flow[11], all the streamlines are straight lines emanating from a central point o, as shown in Figure 5. Obviously, we see that the components in the radial and tangential directions are $\partial f/\partial\rho$ and $\partial f/\partial\theta$, respectively, where $\partial f/\partial\theta=0$.

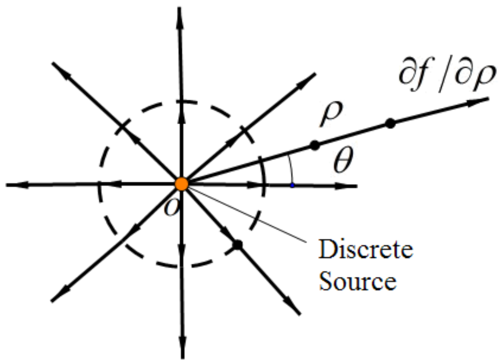

Figure 5 A point source flow

According to the mechanism of *reaction-diffusion model*, the divergence of every point in this vector field V can be obtained easily :

$$
\begin{aligned}
divV = div(\nabla f) &= div\left(\frac{\partial f}{\partial \rho}\vec{e}_{\rho}\right) \\
&= \frac{1}{\rho}\frac{\partial}{\partial \rho}\left(\rho \cdot \frac{\partial f}{\partial \rho}\right) = \frac{1}{\rho}\frac{\partial f}{\partial \rho} + \frac{\partial^2 f}{\partial \rho^2}
\end{aligned} \tag{5}
$$

Where : $div(\bullet)$ denotes divergence while $\nabla(\bullet)$ denotes gredient.

The second part in above equation, that is, $\partial^2 f/\partial\rho^2$, has little impacts on the final value. Here, for the convenience of calculations, we overlook this part.

$$div(\nabla f) \propto \frac{1}{\rho} \tag{6}$$

And final conclusion could be draw that divergence of gradient of a scalar field is inversely proportional to the radial distance.

3.2.3 Propagation Equations

According to the conclusion drawed above, together with the mechanism of *reaction-diffusion model (R-D equation)*, we know that propagation of elements between discrete points is inversely proportional to their distance .

Assume that the propagation of element k happens from node j to node i, obviously, node j is adjacent to node i. And the change of element k can be described as:

$$\delta = \beta_k^i \cdot \frac{X_k^i(t) - X_k^j(t)}{R_{ij}}, \qquad k = A, W, B, F, P, C, E. \tag{7}$$

Where : R_{ij} denotes the distance from node j to node i, and β_k^i denotes the propagation factor of node i.

Apparently, all neighboring nodes of node i will contribute to the propagation of element k. The whole change depends on the number of the neighboring nodes.

For seven given elements and nine nodes, there are several equations and in order to concise writing, we give out general formula below :

$$\left(\frac{d\left[X_k^i(t)\right]}{dt}\right)_{EX} = \sum_j \left[\beta_k^i \cdot \frac{X_k^i(t) - X_k^j(t)}{R_{ij}}\right], \tag{8}$$

$$k = A, W, B, F, P, C, E.$$

Where : node j must be neighboring node of node i.

However, for human population density, its values have not been regularized as other six elements by the rule proposed by *YCELP* and *CIESIN*. Hence, the larger population density is, the less the values are. Namely, the propagation of population density has somewhat difference with other six elements.

3.3 Simultaneous Solution and Parametric Fitting

Actually, the global network and elementary network of our two-layered coupled network are always interacting with each other, and even, they are closely connected and have feedbacks. Therefore, we need to proceed simultaneous solution.

According to the above analysis and solution, we find that the change of elements' values equels to the sum change caused by both internal reason and exterior reason. Therefore, our final ***two-layered coupled network model*** can be described as follows :

$$\begin{aligned}\frac{d\left[X_k^i(t)\right]}{dt} &= \left(\frac{d\left[X_k^i(t)\right]}{dt}\right)_{IN} + \left(\frac{d\left[X_k^i(t)\right]}{dt}\right)_{EX} \\ &= X_k^i(t)\left[\gamma_k^i + \sum_m \alpha_{km}^i X_m^i(t)\right] + \sum_j \left[\beta_k^i \cdot \frac{X_k^i(t) - X_k^j(t)}{R_{ij}}\right] \end{aligned} \tag{9}$$

$$k, m = A, W, B, F, P, C, E.$$

Using the given elements data of ten years, we proceed parametric fitting with the parameter equations we constructed above by *Ordinary Least Square method (OLS)*.

4 Applications and Analysis

After parametric fitting and the value of all parameters are determined, our model is final completed. And next, we focus on solving relative problems and together giving out analysis by applying our model.

4.1 Predict Earth's Health Change and Indentify Tipping Point

The prediction of the global health change is our main task, and the tipping point is an important and dispensible indictor to manifest this change.

4.1.1 Predict Health Change

According to our constructed model and the data searched on, we first predict the value and trend of every element in next fifty years. There are nine regions, thus, for concise writing, we choose two representive regions, where United States and Sub-Saharan Africa belong to, respectively, as our study objects. Here, for concise writing, we denote these two regions as U-region and S-region.

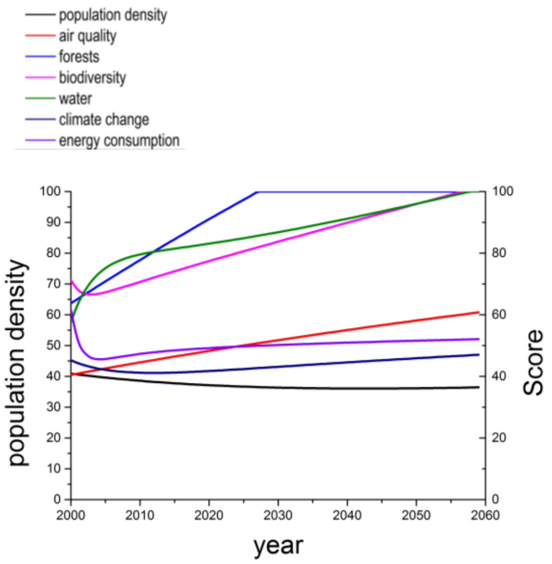

Figure 6 Elements' trends of U-region

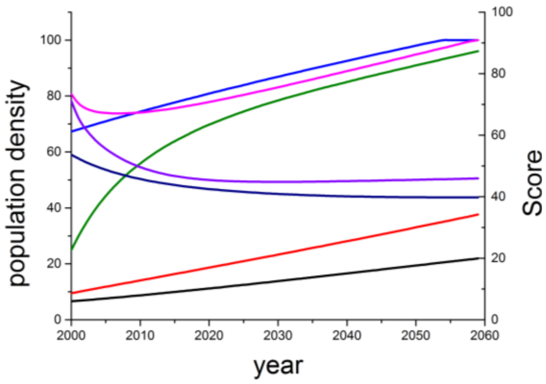

Figure 7 Elements' trends of S-region

Figure 6 and Figure 7 show the trends of every element in U-region and S-region in next fifty years, respectively. Obiviously, in U-region, forests, water, and biodiversity will be sharply increased while remaining four elements almost stay unchangeable. Namely, under reasonable policies, the ecosystem in U-region is health. On the other hand, S-region is a undeveloped region, however, with the growth of population, the abundant natural resources are demanded to improve the quality of life. Hence, excessive exploitation results in deterioration of climate , althoughn the value of some elements increase.

4.1.2 Identify Tipping Point

Tipping point[12,13,14] is a critical point, and drives Earth toward an irreversible change in the biosphere. Namely, before reaching this point, efficient measures could be taken to control the situation ; however, once passing the critical point, an irreversible change appears.

Take air quality for instance. For the purpose to seek the tipping point of air quality, we first alter the initial value of air quality from high to low, when keeping the initial value of other six elements constant. And then, the trend of air quality in next fifty years should be given out by our model, to judge whether it is an irreversible change. When the value of air quality keeps reduce together with time and seems to be no chance to recover, at this moment, this value is the tipping point we looked for.

Using the date of all elements in U-region, we look for the tipping points of these elements, according to above method. And we find that only two of the seven elements, air quality and biodiversity, have tipping points.

When reaching tipping point, the trends of these two elements are showed below, compared with the original change :

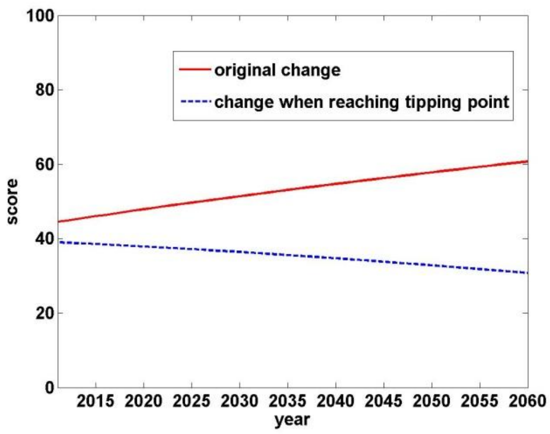

Figure 8 Tipping point (air quality)

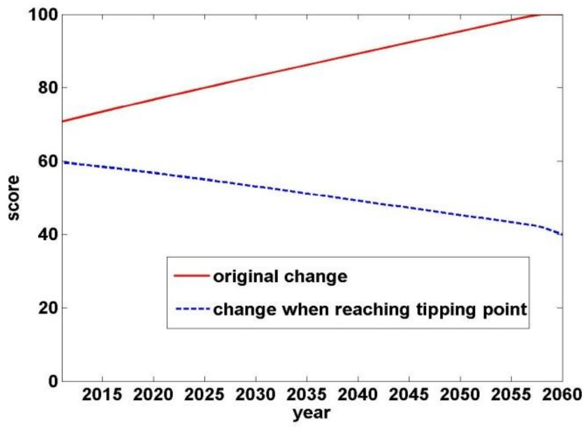

Figure 9 Tipping point (biodiversity)

From Figure 8 and Figure 9, we can see that the trends of air quality and biodiversity will keep declined, once reaching their own tipping points. And the tipping point of air quality and biodiversity are 38.76 and 59.86, respectively. Common sense tells us, this phenomenon is detrimental to global health. Therefore, we suggest that government and dicision makers in U-region need to pay close attention on air quality and biodiversity, and efficient measures should be taken to prevent the tipping points.

4.2 Sensitivity Analysis and Importance of Human Effects

As known to all, human, a strong and powerful group, has impacted a wide range of field. And even, human effects have increased significantly in our current biosphere situation. In this section, laying stress on the importance of human effects, we proceed sensitivity analysis of our model. Namely, we will study the influence of human population density to other six elements.

For the purpose of concise writing, we choose two representive impact factors for detailed destription.

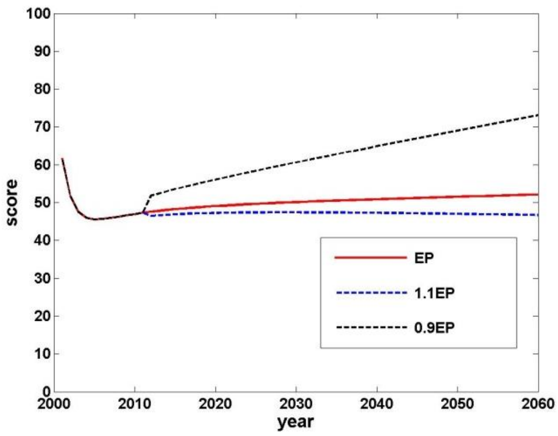

Figure 10 Sensitivity analysis of factor EP

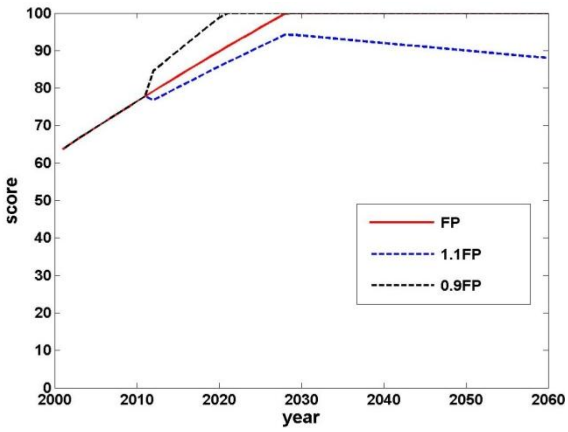

Figure 11 Sensitivity analysis of factor FP

P.S. FP, and EP are short for the impact factors of population density which influence the nature growth rates of forests and energy consumption, respectively.

Figure 10 and Figure 11 show the sensitivity analysis of the impact factors of population density which influence the nature growth rates of forests and energy consumption, respectively. In above two figures, the red solid line represents the original trend of element, the dashed lines represent adjusted trends of element ; black one matches expanding influence of impact factor, while blue one matches reducing influence of impact factor.

Apparently, the original influence of EP keeps almost constant. Namely, it is a steady parameter, and is also controlled well in our planet. Furthermore, reducing influence of EP seems to be subtle fluctuattion, while expanding influence of EP helps to improve the health condition.

Compared to EP, FP plays an important role in the future. Expanding influence of

FP facilitate the improvement of health condition, however, reducing influence of FP seems to be more complicated. In the early stage, the influence of FP is weaken, but, in next stage, FP has negative effects. Therefore, it is a miss to weaken the influence of FP.

4.3 Analyze the Network Structure

According to the value of parameters in our mean-field dynamic Lotka-Volterra model obtained by parameteric fitting, we judge the intension, positive and negative effects of the interaction of these parameters via comparison among them. Setting the one tenth of the average absolute value of intension as the limitation, we think that the parameters, whose value is smaller than this limitation, have little effects and could be neglected. Due to the different values of all parameters in different regions, an interaction network of all elements in different regions can be draw out.

Take U-region for instance. The structrue of the interaction network of U-regions can be obtained as follows :

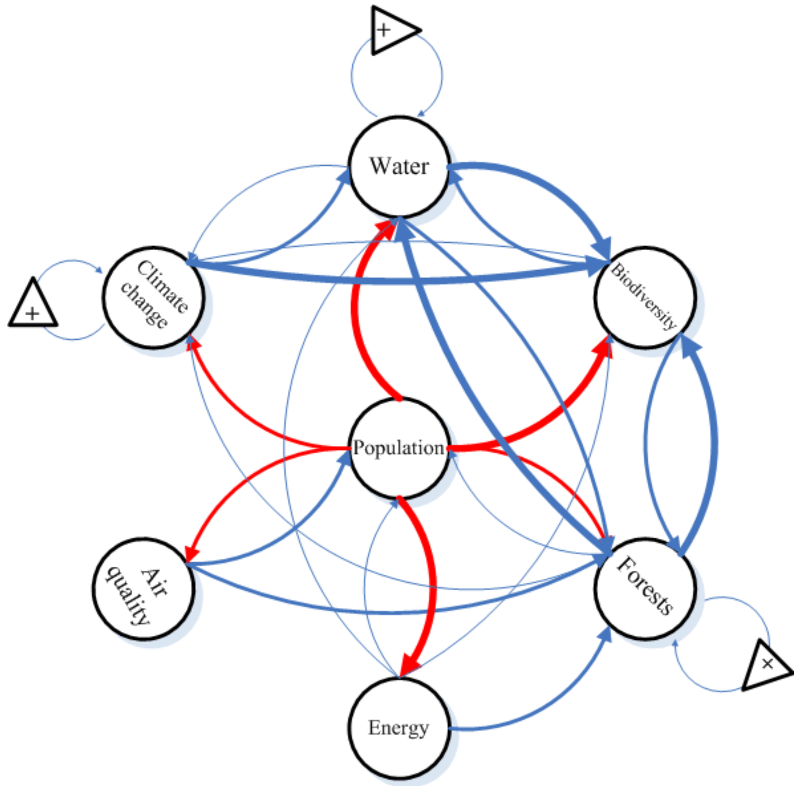

Figure 12 The interaction network of U-regions

In Figure 12, the thickness of links represents the magnitude of parameters, that is, impact factors, the capacity to influence other elements; the blue links and red ones represent positive interaction and negative interaction, respectively, while triangle represents self-feedback.

According to our interaction network above, we draw out network topology analysis and network analysis with weights.

General speaking, there are some metrics that are widely used to manifest the feature of networks. Here, we provide four basic metrics[15] :

"Out-Degree" *Out-Degree is simply the number of links which direct to one node.*

"In-Degree" *In-Degree is simply the number of links which one node direct to.*

"Local Centrality" *Local Centrality is calculated simply in terms of the number of links which directly connect to one node.*

"Global Centrality" *Global Centrality simply means the sum of geodesic of other nodes connecting to one node.*

In general, both *local centrality* and *global centrality* can find out the central point which locates in core position of a series of connections.

However, our network is a small one only with seven nodes and the distance of each two nodes is no more than two links. Thus, we apply *local centrality* to seek the central point of our network.

Additionally, as an interaction network with weights, the best way to evaluate the importance of a node, namely an element, is to weigh the influence of this node contributing to the whole network. Furthermore, the *out-degree* of a node represents the effects on other nodes, and its weight implies the intension of effects.

Therefore, we define a conception *"influence degree"* to describe the influence of a node to its network. And its value equals to the average absolute value of weight sum of its *out-degree* (considering that the weight can be positive or negative, thus, we use absolute value).

Table 1 Network topology analysis

Element	Centrality
Population density	9
Air quality	3
Forests	10
Biodiversity	6
Water	8
Climate change	7
Energy consumption	5

Table 2 Network analysis with weights

Element	Influence degree
Population density	7.93
Air quality	3.35
Forests	4.65
Biodiversity	2.56
Water	6.08
Climate change	5.14
Energy consumption	1.48

Table 1 and Table 2 show the results based on network topology analysis and network analysis with weights of our interaction network.

During network topology analysis (Table 1), we find that forests element is the central point which locates in core position of a series of connections. And population density comes off second more links while water follows by. However, when analyzing the network with weights (Table 2), we find that it is population density comes first on Influence degree which is far ahead and water comes off second best. Additionally, forests become the fourth.

Compared with the above two results, the rank seems to be different. Namely, to some extent, the central point in the ecology network is different from the one with huge influence.

5 Model Optimization - Joining Disturbance

Common sense tells us that there are a great amount of uncertainties in a realistic biological and environmental system. Thus, the development of the system seems to be hard to forecast. Here, in order to better descripte of our global coupled network, we join disturbance into every parameters in our model, whose distrubution obeys the *standard Gaussian distribution*[16], to manifest randomness.

Hence, our final ***two-layered coupled network model after joining disturbance*** can be obtained below :

$$\frac{d\left[X_k^i(t)\right]}{dt} = X_k^i(t)\left[\left(\gamma_k^i + \hat{\gamma}_k^i\right) + \sum_m \left(\alpha_{km}^i + \hat{\alpha}_{km}^i\right) X_m^i(t)\right.$$
$$\left. + \sum_j \left[\left(\beta_k^i + \hat{\beta}_k^i\right) \cdot \frac{X_k^i(t) - X_k^j(t)}{R_{ij}}\right]\right] \tag{10}$$
$$k, m = A, W, B, F, P, C, E.$$

Where : superscript Λ denotes the disturbance of parameters ; and node j must be neighboring node of node i .

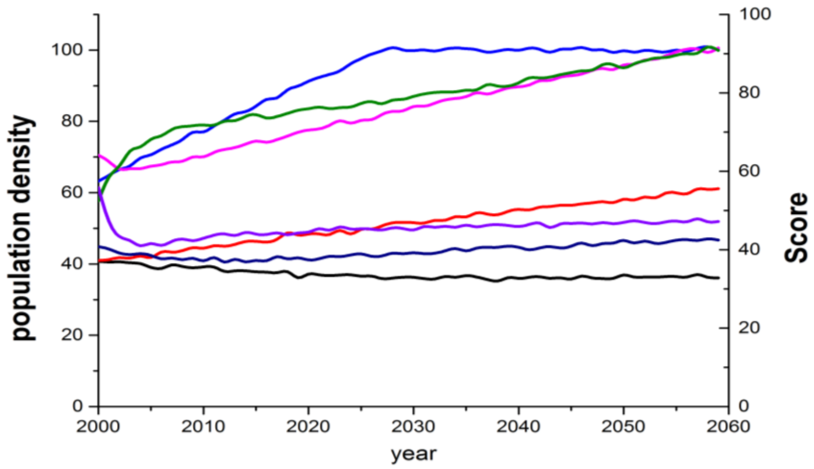

Figure 13 Trends of all elements after joining disturbance

Figure 13 shows the trends of all elements after joining disturbance. Compared with the orignal results, the results we obtained after joining disturbance seem to be more uncertain. And even, it is harder to predict the trend of elements' values.

6 Strengths and Weaknesses

6.1 Strengths

6.1.1 Global

Considering seven elements from different facts and nine regions divided from the whole world into our coupled network, our model is already comprehensive, to some extent.

Moreover, if more detailed data of regions we divided is easy to obtain, we could perfect our model by taking more elements into consideration, thus, complex global network chould be better described. Furthermore, if the data of all countries is provided, we can refine regions to have detailed description on Earth's change by dividing the Earth into countries.

6.1.2 Complexity

Our model manifests the complexity of Earth's interrelated systems, including multiple interactions, feedback loops, emergent behaviors, and impending state changes or tipping points. And it well describes the interaction and relationship of health elements and the behaviour of ecosystem.

6.1.3 Scientific Score

Our model and solution can give out a scientific and impersonal score based on the health level of regions and countries, and even, provide information and recommendations which are available for government and dicision makers.

Furthermore, according to these scientific scores, we draw out a rank which demonstrates the health condition of regional ecosystem and global ecosystem more vividly and visually.

6.1.4 Visualization

Due to the detailed description and wide consideration of our model, the results we obtained are also pretty plentiful. When the trend of every element varying with time is generated based on our dynamic model, we can observe the long-term influence of every element and its future trend intuitively.

Additionally, when paying attention on the interaction and relationship of elements and the propagation effect of Elements among different regions, we manifest their effects by analyzing structure of our two-layered coupled network.

6.2 Weaknesses

- Due to our complex model and two-layered coupled network, the inevitable phenomenon – calculated amount increases multiply – appears which has a large influence on efficiency.
- The analysis of interaction and relationship of elements is based on linear relation concluded by the data study, instead of its realistic mechanism. Therefore, the effects draw by our model may be somewhat discrepant with the reality.

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