Better Living through Math: An Analysis of Healthcare Systems

Denis Aleshin
Bryce Lampe
Parousia Rockstroh

Harvey Mudd College
Claremont, CA

Advisor: Darryl Yong

Summary

Compelled by the great disparities among healthcare systems across the globe, we create a mathematical model to predict key areas for improvement in stunted healthcare systems. We first establish a framework for discussing and comparing healthcare systems; using data taken from the World Health Organization, we use this framework to rank the systems of the U.S., Sweden, and Nigeria. Our rankings agree with previous studies.

Using a probabilistic model incorporating economic factors, we investigate the effects of various changes to the U.S. system and develop a strategy to improve its rank. Our results indicate that the U.S. should place more emphasis on the prevention of illness, and it should shift toward a more-centralized system so as to make care more accessible to lower- and middle-class individuals.

Introduction

While the U.S. has historically spent more per capita on healthcare than most other countries, the U.S. has seen little improvement in healthcare, and even the U.S. Congress admits that the system is far from the best [1993]. Although healthcare is a significant voting issue, Americans remain confused as to what the remedy for their healthcare should be [Hitti 2008]. Additionally, recent problems such as medical tourism—traveling

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to foreign countries for healthcare—have reinforced the apparent need for reform [Kher 2006], but uncertainty remains as to what reforms should be implemented.

We provide a guideline for improving U.S. healthcare. We offer a framework for comparing and predicting various aspects of healthcare systems. We define important terms and identify metrics for measuring quality. We use the combined metrics to rank the healthcare systems of the U.S., Nigeria, and Sweden; these rankings agree with previous literature and support the effectiveness of our metrics.

We present a predictive model for a healthcare system that can account for different economic classes. Tests run with this model suggest that putting more emphasis on prevention of illness and shifting toward more-centralized healthcare would greatly benefit the U.S.

Defining Healthcare

What is Healthcare?

Healthcare is the utilization of medical knowledge with the intent of maintaining or restoring an individual's health of body or mind. A healthcare system is a network of facilities and workers with the purpose of administering healthcare to a country's population.

Quality of Healthcare

The quality of a healthcare system should reflect how proficient it is at keeping individuals healthy. However, what is considered healthy can change over time, so we define our terms to accommodate changes in medical opinions over time.

The Organization for Economic Co-operation and Development (OECD), a large organization concerned with improving international living standards, defines quality of a national healthcare system as:

> The degree to which health services for individuals and populations increase the likelihood of desired health outcomes and are consistent with current professional knowledge. [2004]

A health outcome is a measurable statistic associated with some feature of the overall health of a nation. We take desirable health outcomes to be universal, and we classify a health outcome as desirable or undesirable depending on the current consensus of the medical community. For example, an increase in a population's average lifespan should always be desired over a decrease, and fewer smokers in a population should always be desired over more smokers [Peto and Lopez 2000].

Metrics for Assessing Quality

We define a *metric* for the quality of a country's healthcare system as a measurement of something that is capable of impacting a health outcome. A *desirable* metric is associated with a desirable health outcome (e.g., average access to medical care, frequency of contraceptive use, frequency of immunizations), and vice versa for an *undesirable* metric (e.g. occurrence of diseases, waiting times for doctors, unaffordable costs).

Due to the large differences in how healthcare is provided throughout the world, some metrics—especially those impacted by culture or geographic conditions—might be inappropriate for comparisons between nations. That is, for a metric to be an effective measure of quality, it should measure something that is impacted directly by health systems and it should be influenced by as few outside factors as possible.

Quality Criteria

The OECD has identified three primary components of success of any healthcare system:

- promotion of good health,
- prevention of illness, and
- treatment and diagnosis of illness [Kelley and Hurst 2006].

Additionally, the OECD has compiled a list of metrics that best measure the quality of each of these components [2004]. We use a slightly modified version of the OECD's description for a healthcare system; we consider a system to consist of the following components:

Prevention. Since promotion of good health and prevention of illness primarily apply only to healthy populations, we treat these two components as one single component, measured by metrics suggested by the OECD for their prevention and promotion components [2004].

Accessibility. People are kept away from treatment or diagnosis by the lack of proximity of healthcare facilities, unavailability of staff, and the price of care [Feldstein 2006]. A healthcare system cannot be effective if it cannot be reached by its population. Metrics for this parameter should measure the system's ability to accommodate people's needs in these respects.

Treatment. This component is unchanged from the OECD definition; the quality of this component should be measured by metrics suggested by the OECD for their treatment and diagnosis component [2004].

Which Metrics to Use

Two common metrics for healthcare quality are life expectancy and infant mortality rate, but both are influenced by factors beyond the control of a reasonable healthcare system [O'Neill and O'Neill 2007]. Life expectancy can be considered more a measure of quality of *life* than quality of *healthcare*; it does not distinguish between treatable causes of death (e.g., disease) and other causes (e.g., war). Similarly, infant mortality rates are strongly influenced by cultural, social, and educational factors. Because of the outside forces, comparisons made with only these metrics are not reliable [O'Neill and O'Neill 2007].

We follow guidelines of the OECD, which has concluded that an effective metric is best characterized by three things:

> First, it [must] capture an important performance aspect [of the healthcare system]. Second, it [must] be scientifically sound. And third, it [must be] potentially feasible. [2004]

Data for Metrics

The World Health Organization (WHO) offers an abundance of statistics relating to healthcare, which are widely believed to be accurate and unbiased. We rely on the WHO as the primary source for health outcomes associated with our metrics.

Our Metrics

We choose metrics based on the recommendations of OECD [2004] and the availability of data in the WHO database [2008]. We group them by component of health, as set out earlier.

Prevention

Obesity. This metric reflects the emphasis that a healthcare system places on healthy dietary habits as well as the public's desire to adopt those habits. Data for this metric are readily available from the WHO as "Adults aged $>$ 15 years who are obese."

Prevalence of contraceptives. Contraceptives prevent both unwanted pregnancies and the spread of sexually-transmitted diseases. The majority of abortions are performed due to unwanted pregnancies; abortions have substantial long-term consequences in women, both psychologically and medically OECD [2004]. This metric responds to measures taken by a healthcare system to reduce risks of unprotected sex. Data are available from WHO as "contraceptive prevalence rate."

Smoking. Reducing smoking has traditionally been the responsibility of healthcare systems. This metric is a measure of how susceptible the public is to beneficial influence from the healthcare system [OECD 2004]. Data are available from WHO as "prevalence of current tobacco use among adults aged > 15 years."

Immunizations. These metrics quantify how proficient a healthcare system is at preventing and controlling communicable diseases [OECD 2004]. WHO offers data for diphtheria, measles, tetanus, hepatitis B, toxoid, and pertussis immunizations in one-year-olds [WHO 2008]. We take an additional data set for polio immunizations from Earth Trends [n.d.].

Low birth weight. This metric is an indicator of the prenatal care that at-risk mothers receive. It reflects a healthcare system's ability to identify risk factors in patients as well as its capacity for preventing those factors from causing serious harm [OECD 2004]. Data are available from WHO as "low birth weight, newborns."

Accessibility

Abundance of medical personnel. This indicates the availability of professionals capable of administering care to the population. The WHO provides several data sets for this metric, including the proportions of physicians, nurses, midwives, dentists, and pharmacists in the population.

Abundance of medical facilities. This metric measures the proximity to healthcare systems. Data for this metric is limited; the WHO provides data only for "medical beds per 100,000 population."

Affordability for individuals. This metric measures how much money individuals pay for care. Data for this metric are not directly available from WHO but instead we derive them from its "private spending" and "out of pocket spending" statistics.

Treatment

Success of treatments. This metric should reflect a healthcare system's level of care. The OECD suggests using the readmission rates for patients who have suffered congestive heart failure [2004], but these data are not widely available. Hence, we resort to using the "tuberculosis detection rate" and "tuberculosis treatment success" data provided by the WHO as an alternative.

Meta-Metrics

It would be convenient to combine all the metrics in a meaningful way; we propose an algorithm for computing what we call *meta-metrics*. Begin by selecting a healthcare component; for each of the metrics corresponding to this component, do the following:

- Determine the maximum and minimum values of the metric for a large sample of countries; if a large sample not available, then the metric cannot be used reliably.
- Scale each country's datum linearly into the interval $[0, 1]$, where the minimum value is mapped to 0 and the maximum value to 1.
- If the metric is undesirable (e.g., prevalence of obesity), subtract the scaled values from 1 to transform the metric into a desirable metric (e.g., lack of obesity).

Then calculate the average value of all metrics associated with a country and define this number to be the country's meta-metric value for the chosen healthcare component.

A meta-metric represent how well a country performs, on average, relative to the rest of the world for a given healthcare component. A value close to 1 signifies that the component delivers care of the highest quality currently available; a value near 0 signifies that the country delivers some of the poorest quality care. Because of their compactness, meta-metrics are easy to use for comparisons between existing and potential healthcare systems.

Comparing Healthcare Systems

United States

The U.S. is the only developed country that does not employ universal coverage [Torrens 1978]. Instead, healthcare is different for every person, and consists of a loose association of coverage plans provided by private sources, the government and employers. The average middle-class person is usually covered by some sort of insurance and employs a private physician in sole charge of managing the individual's healthcare. Physicians exercise substantial influence on the U.S. system, because of their position in healthcare administration, as well as general tendencies of policy to favor private medical practice. This influence leads to the question of whether or not physicians or the federal government should control healthcare. More pressing issues are also troubling the U.S., as the increasing health budget is yielding little advance in the overall quality of care [Torrens 1978].

To test the effectiveness of the meta-metrics, we compare several countries for which there is a clear ranking of healthcare already established. Based on "financial fairness," the WHO ranked the healthcare systems of Sweden, the U.S., and Nigeria as 12th, 54th, and 180th in the world [2000b].

Meta-metric values, calculated from the metrics and processes described earlier, are given in **Table 1**.

Table 1.
Meta-metrics.

Meta-metric	U.S.	Sweden	Nigeria
Prevention	.68	.79	.54
Accessibility	.61	.80	.23
Treatment	.52	.38	.37

Sweden

Sweden operates a nationalized healthcare system that every citizen contributes to based on a proportion of income. As a result, the OECD asserts that citizens enjoy roughly equal benefits, regardless of economic status [Tengstam 1975]. The system is heavily regulated and is run by the National Board of Health and Welfare, which is responsible for supervising medical care in both the public and private sectors. In addition, this Board is in charge of certifying physicians, nurses, and midwives, and also supervises and reviews the decisions of the County Councils, where most of the responsibility for funding and maintaining healthcare falls [Tengstam 1975]. Anderson [1972] suggests that in many ways the Swedish system is superior to that of the United States because of Sweden's longstanding commitment to, and enforcement of, universal healthcare.

Sweden's average world ranking for healthcare trumps the U.S. in all areas but treatment. However, the treatment meta-metric is calculated using a weak tuberculosis metric.

Nigeria

Nigeria operates a three-tiered health system comprised of a national healthcare system financed by all citizens; government health insurance that is provided for government employees; and firms that contract with private healthcare providers. However, a significant number of Nigerians do not enjoy all the benefits of this system. Like many other African countries, the roots of the Nigerian healthcare system can be traced back to a British colonial era. During this period, the health system was equipped to provide care for only a small portion of the population; the system was never adequately adapted to handle the region's growing population [World Bank 1994]. An additional hindrance in the system is an incredible disparity of wealth between upper- and lower-class citizens [World Bank 1994]. Examples of failures in the health system abound. In one case, a 1985 outbreak of yellow fever devastated a small town (killing more than 1,000 people) despite the fact that a vaccine has been available since 1930 [Vogel 1993].

Compared to the U.S. and Sweden, Nigeria's meta-metrics place it at the bottom.

Strengths and Limitations of Meta-Metrics

Our meta-metrics demonstrate the following advantages:

Flexibility. Additional metrics can be easily incorporated into the meta-metrics.

Relevance. Meta-metrics convey the average performance of a country's healthcare system relative to the rest of the world.

Accuracy. The WHO and our meta-metrics both rank the U.S., Sweden, and Nigeria in the same relative order.

These meta-metrics also demonstrate the following disadvantages:

Data is not concurrent. Data sets reported by the WHO can often be several years older than other data sets.

Demanding. Data are required from a large number of countries in order to determine the worldwide maximum and minimum values for metrics.

Simplicity. It may be wiser to weight the metrics in the calculation of meta-metrics instead of taking just their mean.

A Model for a Healthcare System

Assumptions

We assume that for a given nation:

Wealth is not distributed equally. This is especially true for the U.S. [Wolff 2004], which is the focus of most of our attention.

WHO data for that nation is recent and reliable. This assumption is not entirely valid, since some statistics from the WHO that we use date back to 2000. However, this should be less of a problem as data become more widely and frequently reported.

The healthcare system operates in a consistent way. This is not at all true, but for the sake of simplicity we must assume that the system is predictable.

Meta-metrics accurately reflect the performance of the health system. Our results for the U.S., Sweden, and Nigeria support this assumption for all but the treatment meta-metric.

Certain meta-metrics scale with income. Measures taken by a healthcare system to prevent illness affect all people equally [Torrens 1978]. To account for economic factors, we assume that accessibility and treatment scale with wealth. That is, an individual with more money has an easier time finding care and paying for treatment. This is a gross oversimplification, but it allows the model to convey more information.

Definition of the Model

Let A, T, and P be the country's accessibility, treatment, and prevention meta-metrics. We treat them as probabilities of certain events occurring within the healthcare system:

P**:** the probability that an individual will be in good health;

A**:** the probability that an individual will have access to affordable healthcare, should they need it; and

T**:** the probability that a sick individual will be correctly diagnosed and properly treated.

We model a healthcare system as the stochastic process pictured in **Figure 1**, and we repeatedly apply this process to track the flow of healthy individuals through the system.

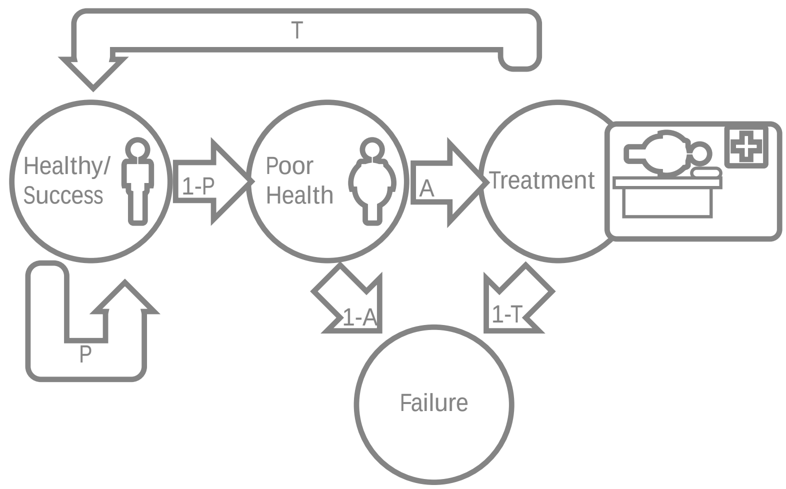


Figure 1. Model of the healthcare process, with four states and probabilities of transitions among them.

If at some time n we have a population of H_n healthy individuals, then we expect $H_n(1-P)$ of those people to fall into poor health in the next time interval. Of those who fall ill, a proportion AT of them will find access to treatment and become healthy. Hence, we predict the number of healthy individuals after $n+1$ units of time to be

$$H_{n+1} = H_n - H_n(1-P) + H_n(1-P)AT.$$

For an initial healthy population H_0, this simplifies to

$$H_n = H_0(P + AT - APT)^n. \tag{1}$$

Retention of the Model

To quantify the efficiency of a healthcare system, we consider how many iterations n of the healthcare process are required before H_n falls below some threshold $H_{\min}$. Hence, we substitute $H_n = H_{\min}$ into **(1)** and solve for n to find the *retention* R:

$$R = \frac{\ln H_{\min}}{\ln H_0 + \ln(P + AT - APT)}. \tag{2}$$

The retention R measures how long the modeled system can operate, starting from a healthy population, before an overwhelming majority of the population is no longer healthy. A larger retention value indicates a more effective system. For all calculations of R, we take H_0 and $H_{\min}$ to be 100 and 1.

Economic Weighting

One of the primary discriminatory factors of healthcare in the U.S. is economic status; we would like to take this into consideration. To do so, we consider three economic classes:

- Group 1: Those who control the lowest quartile of wealth.
- Group 2: Those in the middle quartiles for wealth.
- Group 3: Those in the upper quartile for wealth,

We adjust the parameters A and T based on the wealth of a group.

Since our meta-metrics describe the average performance of the system, our model—without the economic weightings presented in this section—describes the effect of the system on the "average person," a person of median wealth (hence in Group 2). Analogously, we treat the median person in the lower quartile as a representative of Group 1 and the median person in the upper quartile as representative of Group 3.

We adjust the probabilities A and T for Group 1 by a factor of C_*, the ratio of the median wealth of an individual in the lowest quartile to that of the average person.

Since wealth in the U.S. is so unevenly distributed, comparing the median individual in the upper quartile to the average person would be misleading. Instead, we adjust the probabilities A and T for Group 3 by a factor of C^*, which now represents the wealth of the median individual in the upper quartile with respect to the richest person in Group 3. This gives us a weight based on how the wealth is distributed in the upper quartile.

Simply put, these factors give us a sense of the economic disparity between the groups; quality of accessibly and treatment scales with wealth, and C^* and C_* and appropriate scaling factors. We calculate their values in the **Appendix**.

Let A_i denote the accessibility of the healthcare system for an individual in Group i, and let T_i represent the successful treatment of an individual in group i. We weight each of these probabilities as follows:

$$A_i = \begin{cases} AC_*, & \text{if } i = 1; \\ A, & \text{if } i = 2; \\ A + (1 - A)C^*, & \text{if } i = 3. \end{cases}$$

We will assume that treatment scales in the same way, so we have:

$$T_i = \begin{cases} TC_*, & \text{if } i = 1; \\ T, & \text{if } i = 2; \\ T + (1 - T)C^*, & \text{if } i = 3. \end{cases}$$

The rationale for these weightings is that A and T increase as wealth increases.

By considering healthcare with respect to the actual distribution of wealth, we add a great deal of richness to the model. Adjusted meta-metrics are given in **Table 2**.

Table 2.

Adjusted probabilities for economic classes.

	Class		
	Upper	Middle	Lower
Accessibility A	.87	.61	.11
Treatment T	.82	.52	.09

Analysis of U.S. Healthcare

Applying the data from **Table 1** to **(2)** gives retention values shown in **Table 3**. These values show that the model preserves the earlier rankings.

Table 3.

Retention values.

Sweden	29.6
United States	18.9
Nigeria	8.5

Our interests lie in using this model to make predictive judgments about changes to the U.S. system. To identify the areas in which the retention of the U.S. system is most susceptible to change, we vary one meta-metric while holding the others constant. As seen in **Figure 2**, the slope is much larger for prevention; retention responds more to changes in prevention than in other components. Interestingly, although the U.S. and Sweden have different values for their prevention meta-metrics, they respond essentially identically to changes in prevention.

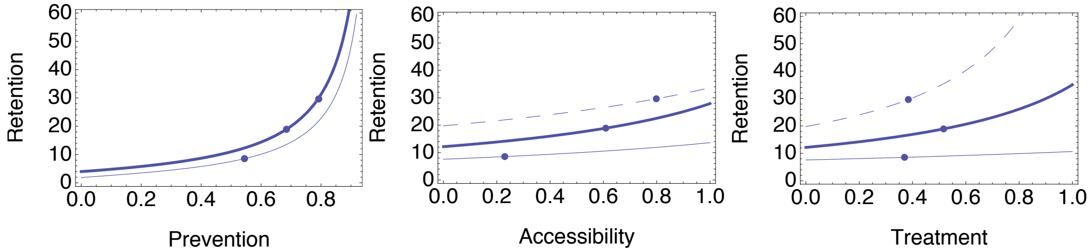


Figure 2. Variations in meta-metrics by nation. In each case, the remaining two meta-metrics are held constant at the values given in **Table 1**. Current values are depicted as dots. Sweden is represented by a dashed line, the U.S. by a bold line, and Nigeria a thin line. Although their prevention meta-metrics differ, the U.S. and Sweden effectively share the same prevention curve.

By considering the impact of economic status on accessibility and treatment, we can gain even more insight. **Figure 3** shows how retention reacts to changes in meta-metrics by economic class: The economic levels in the U.S. react very differently to changes in meta-metrics. This result agrees with out hypothesis that a person's economic status plays a large role in determining the quality of healthcare that the individual receives.

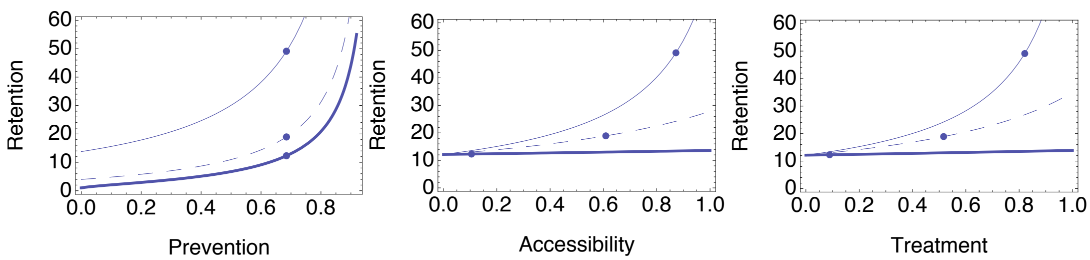


Figure 3. Variations in meta-metrics by U.S. economic class. In each case, the remaining two meta-metrics are held constant at the values given in **Table 2**; for P, we use the value given in for the U.S. in **Table 1**. Adjusted meta-metric values are depicted as dots. The middle class is represented by a dashed line, the lower class by bold line, and the upper class by a thin line.

Strengths

Our model exhibits the following positive characteristics:

Extendability. Our model is a very comprehensive assessment of the interaction of the healthcare system with the population. The advantage of using a stochastic process in creating this interaction is that we can always extend it to be more complex. For example, if we gained access to reliable data for readmission for failed treatments, we could add this pathway into our model and obtain an even more accurate simulation of the healthcare process.

Predictive power. Our model is capable of accurately predicting areas in which national healthcare is lacking relative to other countries, and it can be used to provide insight into the most effective way to change its standing.

Agreement with reality. As discussed later, the results from our model correspond to the current state of U.S. healthcare. Further testing could strengthen this claim.

Economic associations. A large problem with U.S. healthcare is that it varies greatly among individuals, especially by wealth. By incorporating the relationship between availability and treatment into our model, we can more efficiently identify problem areas.

Limitations

Our model also shows the following drawbacks:

Possible failure. It is possible for the model to fail if a country dominates all metrics used in calculating the prevention meta-metric; the model would predict infinite retention. If this occurs, then additional metrics should be considered in the calculation of the prevention meta-metric.

Oversimplification. Our probabilistic model is rather simple, although it produces surprisingly relevant results.

Unconfirmed. Meta-metrics have only been verified to agree with past rankings for a selection of three countries. The accuracy of the model depends directly on the effectiveness of the meta-metrics.

Limited. Our definition for healthcare includes mental health, although our data primarily correspond to physical problems.

Demanding. The model depends on meta-metrics, which in turn require large amounts of worldwide data.

Major Suggestions for the U.S.

Our model predicts that the quickest way to improve the world standing of the U.S. healthcare system is to enhance preventive measures. Lack of spending on the prevention component of the system partially explains the current dilemmas facing the U.S.—namely, the lack of response from increased spending on healthcare [O'Neill and O'Neill 2007] and the growing obesity problem [Wang and Beydoun 2007]. Additionally, it is likely that these inadequate preventive measures are causing more and more Americans to fall into poor health unnecessarily, thereby placing more strain on the system.

We therefore suggest reallocation of funding to place more emphasis on promoting health and preventing illness. **Figure 2** indicates that these changes could quickly increase the quality of U.S. healthcare to be more on a par with Sweden's system.

Additionally, a common criticism of U.S. healthcare is the large inequities in affordability and quality of treatment between the upper, lower, and middle classes [Wolff 2004]. When simple economic factors are combined with our model, **Figure 3** shows that the lower and middle classes experience little to no sensitivity to changes in the system's accessibility or treatment components. At the same time, however, the upper class gains significantly more retention from increases in both of these meta-metrics. Hence, the model suggests that money spent on improving the accessibility component of the system has had a minimal impact on a majority of the population.

Thus, additional reform of U.S. healthcare is needed to make the system more accessible to the lower and middle classes. Sweden has had great success with its highly-regulated universal healthcare system. Therefore, we also suggest that the U.S. grant more control of healthcare to the government so it can enact and enforce stricter regulations on the preventive care provided by private practitioners.

Conclusion

We have researched the motivations and goals of healthcare. Based on quality, relevance and availability, we selected a set of health outcomes that we grouped into metrics, and further organized into logical groups of meta-metrics. Applying these meta-metrics to compare the healthcare systems of the U.S., Nigeria, and Sweden confirmed their validity when considered alongside previous work.

We then used those meta-metrics to construct a stochastic model to generalize healthcare and defined the concept of *retention* to compare different health systems. Furthermore, we incorporated economic factors into our model in order to distinguish between different income classes. By analyzing the influence of each metric on retention, we identified problems in the U.S. healthcare system. In light of these problems, the U.S. system should be restructured to improve promotion of health, and government control should be increased in order to provide more-accessible healthcare for the lower and middle classes.

Future work should seek additional health outcome statistics to increase the accuracy of our metrics, especially for the treatment component of healthcare. Additionally, meta-metric values could be computed for additional countries to further investigate their potential for describing the quality of healthcare.

Appendix

We compute the economic weights C_* and C^* by studying the distribution of wealth within the U.S. Economists often study the distribution of wealth in a country by constructing a *Lorenz curve*, which is the approach we take here.

The Lorenz Curve

The Lorenz curve was described in 1905 by Max O. Lorenz to display the distribution of wealth or assets in a society. A Lorenz curve is obtained by plotting on the x-axis the percentage of people and on the y-axis the corresponding percentage of wealth. Thus, a point at (x, y) on the Lorenz curve indicates the percentage y of total wealth that the bottom $x\%$ of people have. **Figure A1** shows the approximate Lorenz curve for wealth in the U.S. in 2001. The thin line with slope 1 is the line of perfect equality, which corresponds to an equal distribution of wealth among individuals in a society.

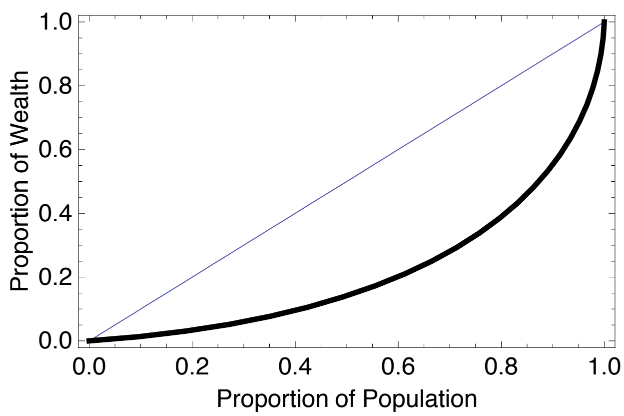


Figure A1. A Lorenz curve for wealth in the U.S. (bold) approximated using data from 2001, along with the line of perfect equality (thin).

A Lorenz curve has properties useful in approximating it:

- It begins at $(0, 0)$ and ends at $(1, 1)$,
- cannot rise above the line of perfect equality,
- is increasing, and
- is convex.

The Lorenz Curve for the U.S.

We approximate the Lorenz curve for the U.S. using data from 2001. We display in **Table A1** the data from Wolff [2004].

Table A1.
Financial wealth distribution by household in the U.S. in 2001, according to Wolff [2004, 30].

	Top 1%	Next 19%	Bottom 80%
% wealth	39.7%	51.5%	8.8%

We also know that 0% percent of the population have 0% of the wealth, and the collective population has all the wealth. This give us the boundary conditions $(0, 0)$ and $(1, 1)$.

We approximate the Lorenz curve using a Bézier spline fit algorithm because of its ability to generate a smooth curve with relatively few data points. The Bézier fit also guarantees that the curve will be convex, as we would expect a Lorenz curve to be. The disadvantage is that the curve does not pass through all the data points.

Computation of C_* and C^*

We compute the weights C_* and C^* using the Lorenz curve, which we denote by $L(x)$. We define C_* to be the ratio of the cumulative wealth of the median person in the lowest quartile to the cumulative wealth of the average person. Thus, C_* is given by:

$$C_* = \frac{\int_0^{.125} L(x)\,dx}{\int_0^{.50} L(x)\,dx} \approx .17$$

Similarly, define C^* to be the ratio of the cumulative wealth of the median person of the upper quartile to total wealth. So C^* is given by:

$$C^* = \frac{\int_0^{.875} L(x)\,dx}{\int_0^{1} L(x)\,dx} \approx .63$$

The Gini Index of $L(x)$

The *Gini index* is a numerical measure of the distribution of wealth in a country, defined as

$$G = 2\int_0^1 [x - L(x)]\,dx = 1 - 2\int_0^1 L(x)\,dx$$

where $L(x)$ is a Lorenz curve. Thus, the Gini index is 1 minus twice the area below the Lorenz curve. Perfect equality in wealth corresponds to $G = 0$, perfect inequality to $G = 1$. Numerically integration of our function $L(x)$gives $\int_0^1 L(x)\,dx \approx .21$ and hence $G_{\text{USA},2004} \approx .579$.

Limitations of our Approximation

- Although we use data from 2001, the distribution of wealth does not change dramatically from year to year.
- We use only five data points, including the boundary conditions $(0,0)$ and $(1,1)$.
- The Bézier curve passes through the boundary points but not through the data points.

The Gini index for financial wealth of households in the U.S. in 2001 was .888 [Wolff 2004, 30], while our approximation is .579. We used scant data; moreover, the bottom 40% of households combined have negative financial wealth ("net worth minus net equity in owner-occupied housing" [Wolff 2004, 5]). Davies et al. [2008] have different data (**Table A2**) for household wealth, which they take more conventionally to include "non-financial assets [presumably including home equity], financial assets and liabilities" [2008, 2].

Table A2.

Wealth distribution for families in the U.S. in 2001, according to Davies et al. [2008, 4, Table 1].

	Top 1%	Top 5%	Top 10%	Bottom 50%
% wealth	32.7%	57.7%	69.8%	2.8%

Editor's Note: Calculation of the Gini Index from Available Data

The U.S. Census Bureau publishes wealth and income data *by quintiles*. The income data are published separately for families and for households [2005a; 2005b], while the wealth data are published for households only [2008a]. A household includes related family members plus any unrelated people who share the housing unit. The Bureau also publishes Gini indexes for income [2008b; 2007b; 2007a] calculated from the full Lorenz curves, together with other measures of inequality [n.d.].

The Gini index cannot be approximated from quintile data by using Simpson's rule for an integral, since Simpson's rule requires an even number of intervals. Using the trapezoid rule would underestimate the Gini coefficient because of the concavity of the Lorenz curve.

Gerber [2007] gives a simple method suitable to quintile data. For U.S. family income in 2000, the method gives a Gini index of .422, while the index given by the Census Bureau (based on the full Lorenz curve) is .433.

Further information about both the Lorenz curve and Further details about the Lorenz curve and the Gini index are given in a series of UMAP Modules by Schey [1979].

Editor's References

Gerber, Leon. 2007. A quintile rule for the Gini coefficient. *Mathematics Magazine* 80 (2) (April 2007): 133–135. Note: The URL for reference 3 of this paper should read `http://www.census.gov/hhes/www/income/histinc/f02.html` .

Schey, Harry M. 1979. The distribution of resources. UMAP Modules in Undergraduate Mathematics and Its Applications: Modules 60–62. Newton, MA: COMAP.

U.S. Census Bureau. 2005a. Historical income tables—Families. Table F-2. Share of aggregate income received by each fifth and top 5 perc [sic] of households (all races): 1967 to 2001. `http://www.census.gov/hhes/www/income/histinc/f02.html` .

________ . 2005b. Historical income tables—Families. Table F-2. Share of aggregate income received by each fifth and top 5 percent of families (all races): 1947 to 2001. `http://www.census.gov/hhes/www/income/histinc/f02.html` .

________ . 2007a. Historical income inequality tables. `http://www.census.gov/hhes/www/income/histinc/ineqtoc.html` .

________ . 2007b. Historical income tables—Households. Table H-4. Gini ratios for households, by race and Hispanic origin of householder: 1967 to 2006. `http://www.census.gov/hhes/www/income/histinc/h04.html` .

________ . 2008a. Net worth and the assets of households: 2002. `http://www.census.gov/prod/2008pubs/p70-115.pdf` .

________ . 2008b. Historical income tables—Families. Table F-4. Gini ratios for families, by race and Hispanic origin of householder: 1947 to 2006. `http://www.census.gov/hhes/www/income/histinc/f04.html` .

________ . n.d. Selected measures of household income dispersion: 1967 to 2005. `http://www.census.gov/hhes/www/income/histinc/p60no231_tablea3.pdf` .

References

Anderson, Odin. 1972. *Health Care: Can There Be Equity?* Chicago, IL: Wiley-Interscience.

Davies, James B., Susanna Sandström, Anthony Shorrocks, and Edward N. Wolff. 2008. The world distribution of household wealth. Discussion Paper No. 2008/03. United Nations University UNU-WIDER World Institute for Development Economics Research. `http://www.wider.unu.edu/publications/`

working-papers/discussion-papers/2008/en_GB/dp2008-03/_files/78918010772127840/default/dp2008-03.pdf .

Earth Trends. n.d. Population, health and human well-being searchable database. http://earthtrends.wri.org/ .

Feldstein, Martin. 2006. Balancing the goals of health care provision. *National Bureau of Economic Research Working Paper Series*.

Hitti, Miranda. 2008. Poll: U.S. split on socialized medicine. *WebMD* (February 2008).

Kelley, Edward and Jeremy Hurst. 2006. Health care quality indicators project conceptual framework paper. *OECD Health Technical Papers* 23 (March 2006).

Kher, Unmesh. 2006. Outsourcing your heart. *Time* (May 2006).

O'Neill, June E., and Dave M. O'Neill. 2007. Health status, health care and inequality: Canada vs. the U.S. *National Bureau of Economic Research Working Paper Series* (September 2007).

Organization for Economic Co-operation and Development (OECD). 2002. *Measuring up. Improving Health System Performance in OECD Countries*. Paris, France: OECD Publications Service.

________. 2004. Selecting indicators for the quality of health promotion, prevention and primary care at the health systems level in OECD countries. *OECD Health Technical Papers* 16: 4 (December 2004.), pp. 9–10, 13-41.

Peto, Richard, and Alan Lopez. 2000. The future worldwide health effects of current smoking patterns. http://www.ctsu.ox.ac.uk/pressreleases/2000-08-02/the-future-worldwide-health-effects-of-current-smoking-patterns .

Tengstam, Anders. 1975. Patterns of health care and education in Sweden. *Centre for Educational Research and Innovation Papers*, pp. 1–40.

Torrens, Paul R. 1978. *The American Health Care System Issues and Problems*. St. Louis, MO: C.V. Mosby Company.

U.S. Congress. 1993. *International Health Statistics: What the Numbers Mean for the United States—Background Paper*. Washington, DC: U.S. Government Printing Office.

U.S. Senate. 2005. Saving dollars, saving lives: The importance of prevention in curing medicare. Hearing before the Special Committee on Aging (June 2005).

Vogel, Ronald J. 1993. *Financing Health Care in Sub-Saharan Africa*. Westport, CT: Greenwood Press.

Wang, Youfa, and May A. Beydoun. 2007. The obesity epidemic in the United States—gender, age, socioeconomic, racial/ethnic, and geographic characteristics: A systematic review and meta-regression analysis. *Epidemiologic Reviews* 29 (1) (January 2007): 6–28. http://epirev.oxfordjournals.org/cgi/reprint/mxm007v1 .

Wolff, Edward N. 2004. Changes in household wealth in the 1980s and 1990s in the U.S. Working Paper No. 407. Levy Economics Institute of Bard College. http://www.levy.org/pubs/wp407.pdf .

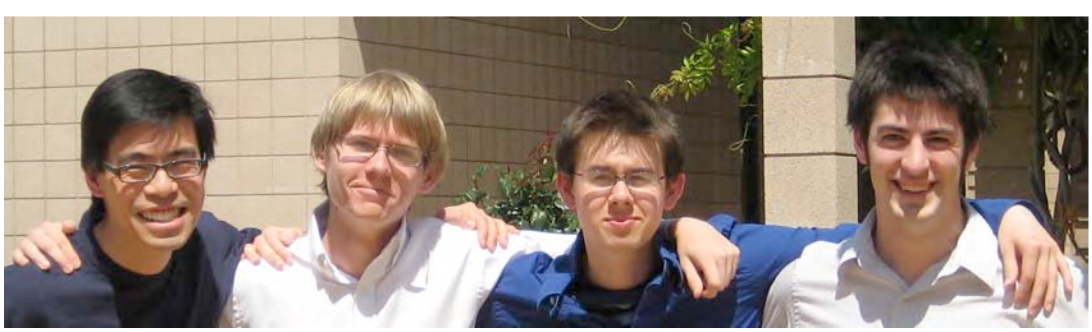
World Bank. 1994. *Better Health in Africa*. Washington, DC: World Bank Staff.

World Health Organization(WHO). 2000a. Annex Table 7: Fairness of financial contribution to health systems in all member states. *The World Health Report*.

________ . 2000b. How well do health systems perform? *World Health Report*, pp. 21–45.

________ . 2006. National health acounts. *WHO Statistical Information System*.

________ . 2008. The WHO agenda. http://www.who.int/about/agenda/en/index.html .



Advisor Darryl Yong, team members Denis Aleshin, Parousia Rockstroh, and Bryce Lampe.