The Coming Oil Crisis

Wei Deling
Chen Jie
Xu Hui
East China University of Science and Technology
Shanghai City, China

Advisor: Ni Zhongxin

Summary

We model depletion of oil, a typical vital nonrenewable resource. Based on the theory of supply and demand, we establish a differential equation system that includes demand, supply, and price, and derive explicit formulas for the three variables. We modify the model to reflect exponetially increasing worldwide oil demand.

We fit the modified model to worldwide oil demand data 1970–2003. We conclude that all oil will be used up in 2032 without countermeasures. We then take economic, demographic, political, and environmental factors into account.

To meet the needs of people today without compromising those of future generations, we establish a criterion of rational oil allocation between generations and model optimal oil allocation under this criterion, with an illustration.

We provide a strategy for oil exploitation to reduce the possibility of disasters in the short term.

Finally, according to marginal utility replacement rules, we study the trade-off between oil and its alternatives. Since our model is based on demand-supply theory and the intrinsic law of nonrenewable resources, it can be applied to general nonrenewable resources.

Task 1: Modeling the Depletion of Oil

Under the following assumptions, no restriction is made to protect oil, so it will be exhausted in the fastest way.

The UMAP Journal 26 (2) (2005) 127–146.

Assumptions

- Oil refining capacity is adequate.
- All undiscovered oil is available when necessary—as long as there is demand, there is supply, until all the oil on Earth is completely used up.

Notations

- $U(t)$: Oil undiscovered in year t.
- $R(t)$: Oil discovered but has not been used (reserves) in year t.
- $D(t)$: Worldwide oil demand in year t (in thousands of barrels per day (*bpd*))
- $S(t)$: Worldwide oil supply in year t.
- $P(t)$: Oil price in year t.
- P_0: Equilibrium price of oil.

Modeling

From the above definitions, $U(t) + R(t)$ is the total remaining oil on Earth in year t, and $\sum_{i=t}^{n} D(i)$ is the total demand from year t through year n.

To learn when the total remaining oil will be used up, we find n such that

$$\sum_{i=1}^{n} D(i) \le U(t) + R(t) < \sum_{i=1}^{n+1} D(i); \qquad \textbf{(1)}$$

then oil will be depleted between year n and year $n+1$.

Data

- Estimated undiscovered oil worldwide in 1997 was 180 billion barrels, that is, $U(1997) = 180$ ($\times 10^9$ bbl) [Campbell 1997].
- Worldwide oil reserve in 1997 was 1,018.5 billion barrels, that is, $R(1997) = 1{,}018.5$ ($\times 10^9$ bbl) [Energy Information Administration 2004].
- The worldwide oil demand from 1980 to 2003, $D(i)$ ($i = 1980, \ldots, 2003$), is shown in **Table 1** [Energy Information Administration 2004].[1]

To predict future demand, we consider the following system of first-order linear ordinary differential equations that express "supply-demand" principles:

[1] EDITOR'S NOTE: Subsequent to the contest, the EIA revised the 2003 demand to 79,892 bpd and posted the 2004 demand as 82,631 bpd.

Table 1.
World-wide oil demand, 1970–2003 (thousands of barrels/day (bpd)).
Source: Energy Information Administration [2004].

1970	46,808	1980	63,108	1990	66,443	2000	76,954
1971	49,416	1981	60,944	1991	67,061	2001	78,105
1972	53,094	1982	59,543	1992	67,273	2002	78,439
1973	57,237	1983	58,779	1993	67,372	2003	79,813
1974	56,677	1984	59,822	1994	68,679		
1975	56,198	1985	60,087	1995	69,955		
1976	59,673	1986	61,825	1996	71,522		
1977	61,826	1987	63,104	1997	73,292		
1978	64,158	1988	64,963	1998	73,932		
1979	65,220	1989	66,092	1999	75,826		

$$\frac{dS}{dt} = a\tilde{P}, \tag{2}$$

$$\frac{d\tilde{P}}{dt} = -b(S - D), \tag{3}$$

$$\frac{dD}{dt} = -c\tilde{P}, \tag{4}$$

where $\tilde{P} = P(t) - P_0$ and a, b, c are positive constants.

Eq. **(2)** means that if the oil price is greater than its equilibrium price, the output will increase accordingly, and vice versa. Eq. **(3)** says that if oil supply exceeds demand, the price will decline. Eq. **(4)** indicates when the price goes up or down, demand of shrinks or expands accordingly.

After careful calculation, we get the solution of the system:

$$\tilde{P}(t) = k\sin(\omega t + \phi), \tag{5}$$

$$S(t) = S_0 - \frac{ak}{\omega}\cos(\omega t + \phi), \tag{6}$$

$$D(t) = D_0 + \frac{ck}{\omega}\cos(\omega t + \phi), \tag{7}$$

where

- $k = \sqrt{\tilde{c}_1^2 + \tilde{c}_2^2}$,
- $\phi = \arctan(\tilde{c}_1/\tilde{c}_2)$,
- $\omega = \sqrt{b(a+c)}$, and
- $S_0 = D_0$, $\tilde{c}_1$, and $\tilde{c}_2$ are parameters to be determined.

We are particularly interested in **(7)**. It implies that oil demand is periodic. However, as time passes, the world population is expanding exponentially, and the demand of oil increases accordingly. Therefore, we modify **(7)** to reflect this intrinsic tendency to increase. We add to the right-hand side of **(7)** an exponential term $k_1 \exp\big(k_2(t - t_0)\big)$, where k_1, k_2, t_0 are constants), getting

$$D(t) = a_1 + a_2\cos(a_3 t + a_4) + a_5\exp(a_6 t). \tag{8}$$

Fitting **(8)** to the data in **Table 1**, we get the curve in **Figure 1**, for the function

$$D(t) = 31950 + 556.7\cos(1.605t - 3159.659) + 1.239 \times 10^{-16}\exp(0.02366t).$$

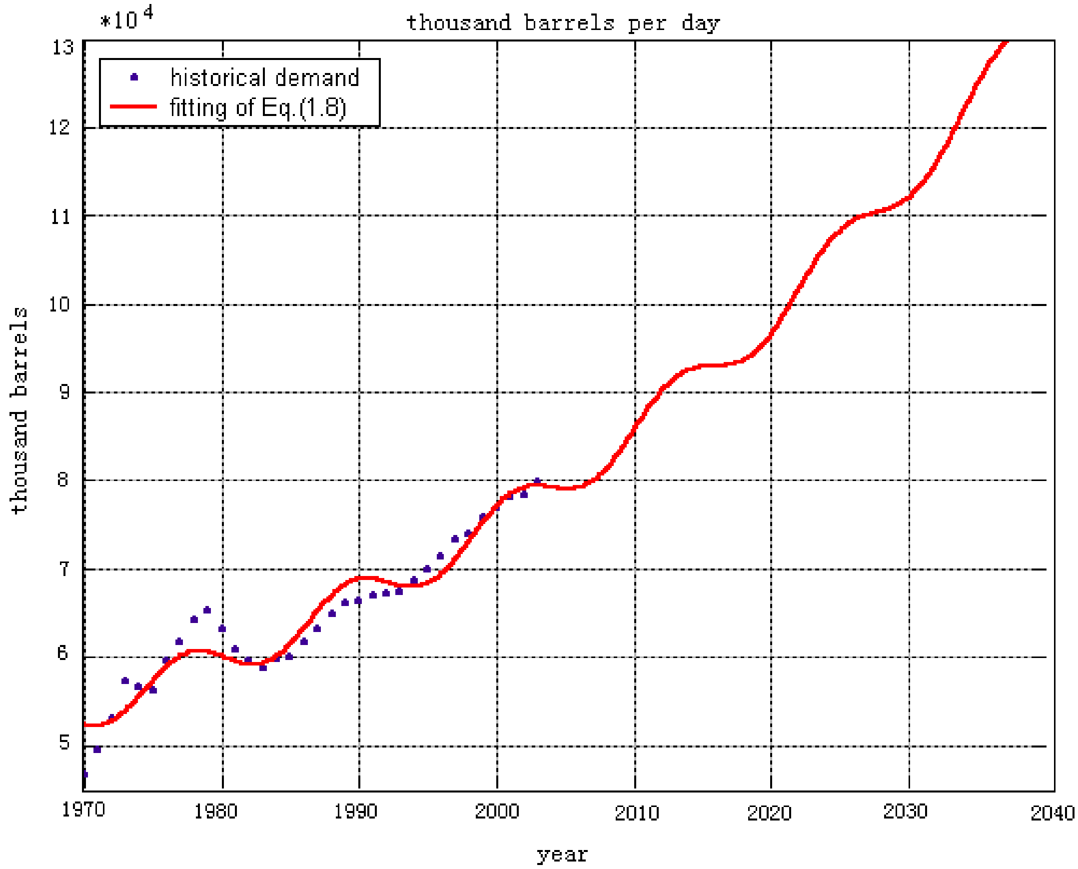


Figure 1. Data and fitted curve for oil demand per day; vertical scale is in 10^7 bpd.

With the passage of time, the third term $[a_5\exp(a_6t)]$ on the right-hand side of **(8)** will play a more important role and the second term $[a_2\cos(a_3t + a_4)]$ can be neglected. Thus, for the sake of convenience, we reduce **(8)** to

$$D(t) = a_1 + a_5\exp(a_6t).$$

For comparison, we also do linear fitting plus an unvarying-demand case in which future demand is the same as in 2003. Fitting to **Table 1** gives for $t \geq 2004$:

Exponential fit	$D(t) = 29820 + 2.265 \times 10^{-15}\exp(0.02223t)$
Linear fit	$D(t) = 771.2t - 1.467 \times 10^6$

The predicted demand is shown in **Figure 2** as average daily demand.

All oil on Earth will be used up by 2032 and 2033 according to the exponential and linear fits, and by 2037 if future demand remains at the level of 2003.

With an increase in oil demand, its price will accordingly rise. As shown by the broken lines in **Figure 2**, the rising price will lead to a decline in demand; we discuss this phenomenon in detail later.

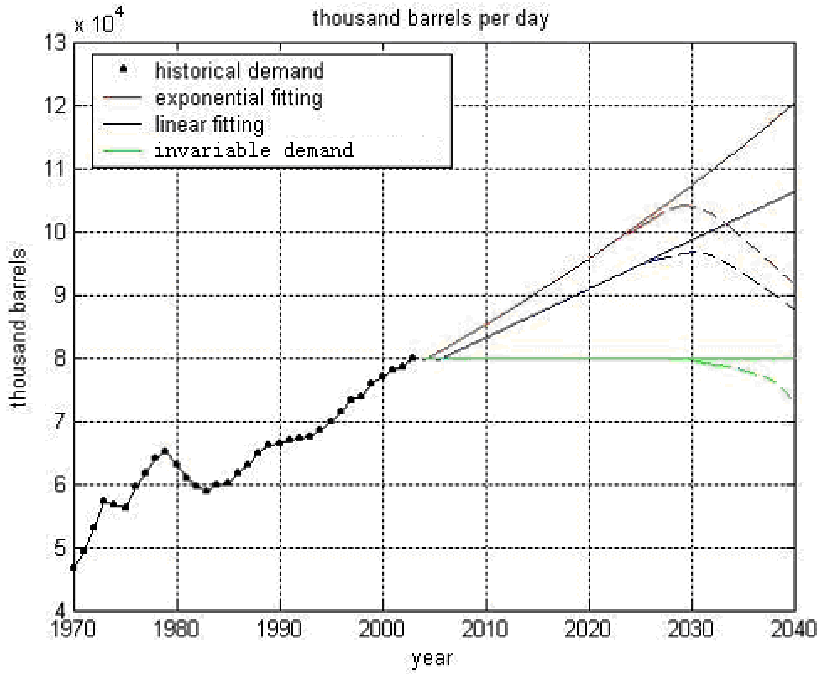


Figure 2. Estimated future oil demand, for several scenarios; vertical scale is in 10^7 bbl/d.

Sensitivity Analysis

It is difficult to get an accurate value for $U(t)$, undiscovered oil on Earth in year t. But we can estimate and vary the estimate to see whether the variation vastly changes n. Varying $U(1997)$ by $\pm 10\%$, for each demand model, the change in n is less than one year.

Task 2: Other Factors

We modify the exponential model to include other factors.

Assumptions

- Annual demand for oil reflects annual consumption.
- We do not take into account interactions between factors.
- We ignore small fluctuations in the future consumption of oil.

Economic Factors

We use GDP as the measure of economy. **Table 2** shows recent data for world total GDP and the corresponding oil consumption.

The correlation between world GDP and oil consumption is .9930, with linear regression equation

$$\text{consumption} = 1183 \times \text{GDP} + 38140. \tag{9}$$

Table 2.
World total GDP ($\$10^8$) and world oil consumption (10^3 bpd), 1995–2003.

Year	1995	1996	1997	1998	1999	2000	2001	2002	2003
GDP	27.134	28.247	29.433	30.257	31.377	32.85	33.64	34.6487	36
Oil	69955	71522	73292	73932	75826	76954	78105	78439	79813

Using **(9)**, we predict cumulative consumption. We take 2001 as the starting point, when total remaining oil (undiscovered plus known reserves) was $U(2001) + R(2001) = 1.1178 \times 10^{12}$ bbl. We calculate the time to oil exhaustion under different cases: GDP growing at 10%, 5%, 3%, and 1% (**Figure 3**).

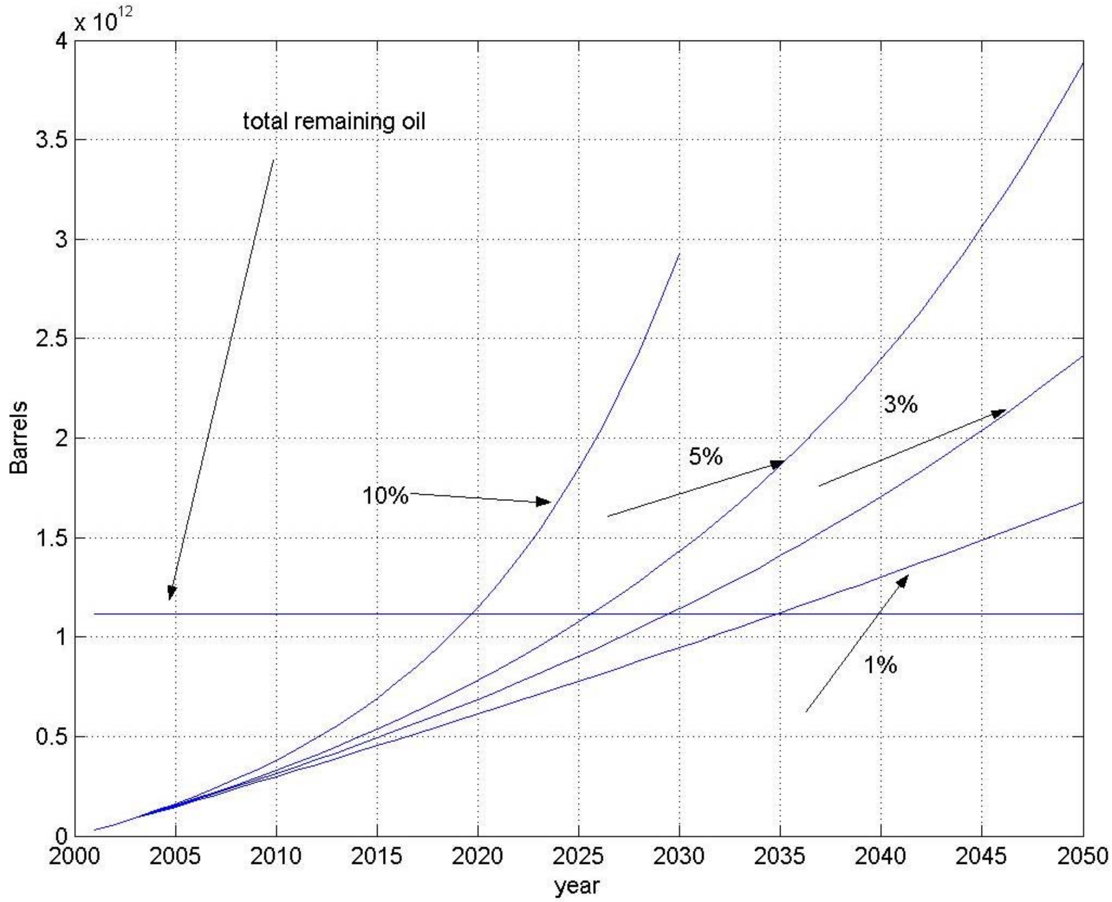


Figure 3. Cumulative oil consumption under some rates of GDP growth; vertical scale is in 10^{12} bbl.

The horizontal line denotes the total remaining oil in 2001. The x-axis coordinate of the intersection of the horizontal line and a curve denotes oil exhaustion time. The faster the GDP growth rate, the larger the oil consumption and the sooner the time of exhaustion. For 10%, oil will be depleted in 2020; for 5%, in 2026; for 3%, in 2029; and finally, for 1%, oil will be used up in 2035.

Demographic Influence

We resort to a logistic model to predict the world population $x(t)$:

$$x(t) = \frac{k}{1 + \left(\frac{k}{x_0} - 1\right) e^{-(t-t_0)r}},$$

where

- t is time, with initial time $t_0 = 1980$;
- $x(t)$ is the population, in billions of people, with $x_0 = x(1980) = 4.4585$;
- k is the environment capacity—the maximum population that the Earth can accommodate—in billions of people, and we take $k = 10$; and
- r is the intrinsic growth rate of the population, determined from data.

We use population data from 1980, 1990, and 2000 to fit the equation and get

$$x(t) = \frac{10}{1 + \left(\frac{10}{4.4585} - 1\right) e^{-0.0327(t-1980)}}, \tag{10}$$

Using **(10)**, we predict the future population, as shown in **Table 3**.

Table 3.
World population estimated from the logistic model.

1980	1990	2000	2010	2020	2030	2040	2050	2060
4.4585	5.2736	6.0744	6.8212	7.4849	8.0495	8.5126	8.8811	9.1560

Similarly, we obtain a relationship between consumption and total population. The correlation coefficient is .9877, with linear regression

$$\text{consumption} = 1443 \times \text{population} - 11170. \tag{11}$$

The time of oil exhaustion, based on the logistic growth of the population, is 2033.

Political Influence

Here, we mainly discuss the influence of wars.

$$D(t) = 9.14 \times 10^{-11} \times e^{0.01718t}.$$

The annual rate of growth of consumption is

$$r = \frac{D(t+1)}{D(t)} - 1 = e^{0.01718} - 1 = 1.73\%.$$

Figure 4 shows the annual growth rate for oil consumption during the past decades, plus a horizontal line at r. The growth rate declined sharply in 1974, 1980, and 1990, coinciding with the fourth Middle East War (1973), the Iran-Iraq War (1980), and the Gulf War (1990), all in the Middle East, the center of oil production. So wars strongly impact the price of oil, and consequently demand.

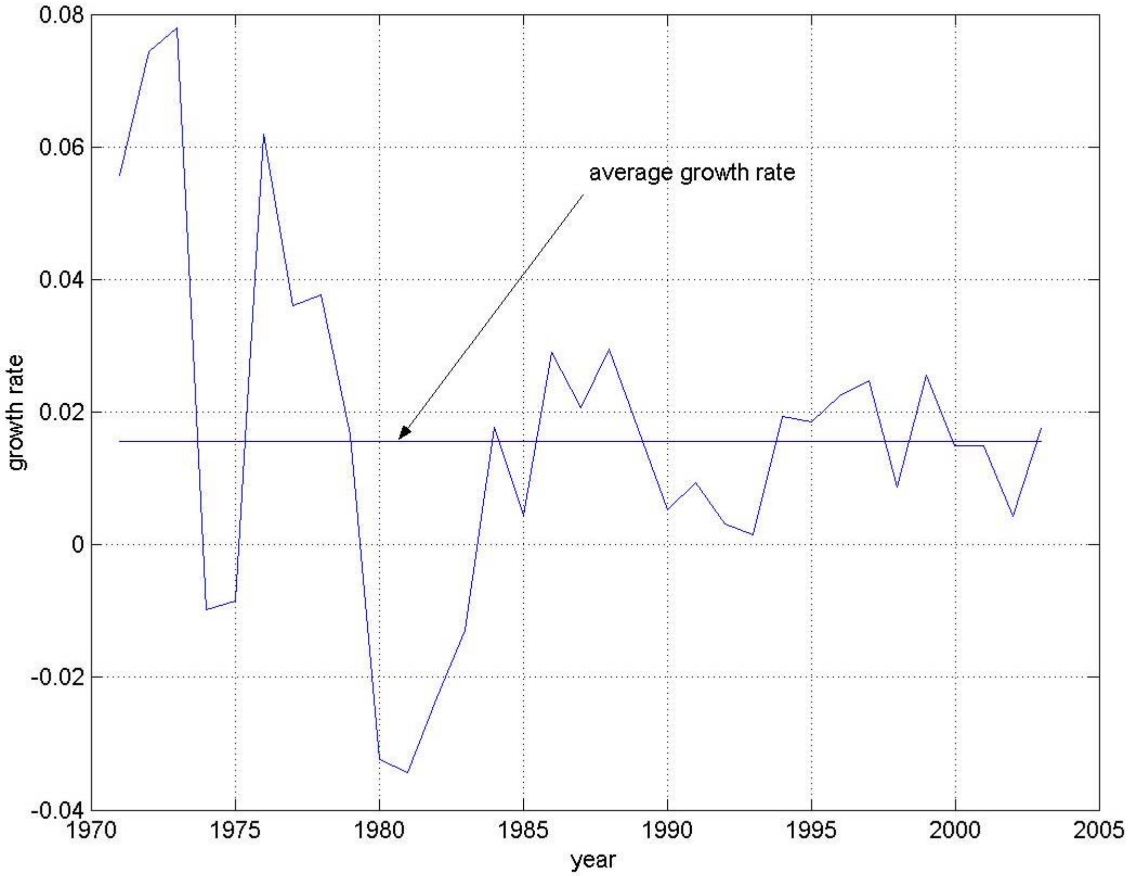


Figure 4. Historic growth of oil consumption.

Environmental Influence

The use of oil inevitably leads to environmental pollution. To protect the environment against excessive pollution, governments can adopt measures to limit the use of oil, thus curbing oil demand. We take the amount of carbon dioxide discharged by oil consumption as the scale to measure environment pollution; world data are shown in **Table 4**.

Table 4.
World carbon dioxide emissions from the consumption of oil (10^6 metric tons).

1993	1994	1995	1996	1997	1998	1999	2000	2001
9220	9284	9388	9586	9691	9766	9939	10138	10292

The correlation between oil consumption and carbon dioxide emission from oil consumption is 0.9937, with the regression

$$\text{consumption} = 10.09 \times (\text{CO}_2 \text{ from oil}) - 25320. \qquad \textbf{(12)}$$

Using **(12)**, we determine the amount of consumption to allow under different controlled annual emission growth rates. **Figure 5** shows the results and the corresponding dates for exhaustion of oil, for emission growth rates of 1% and 3%.

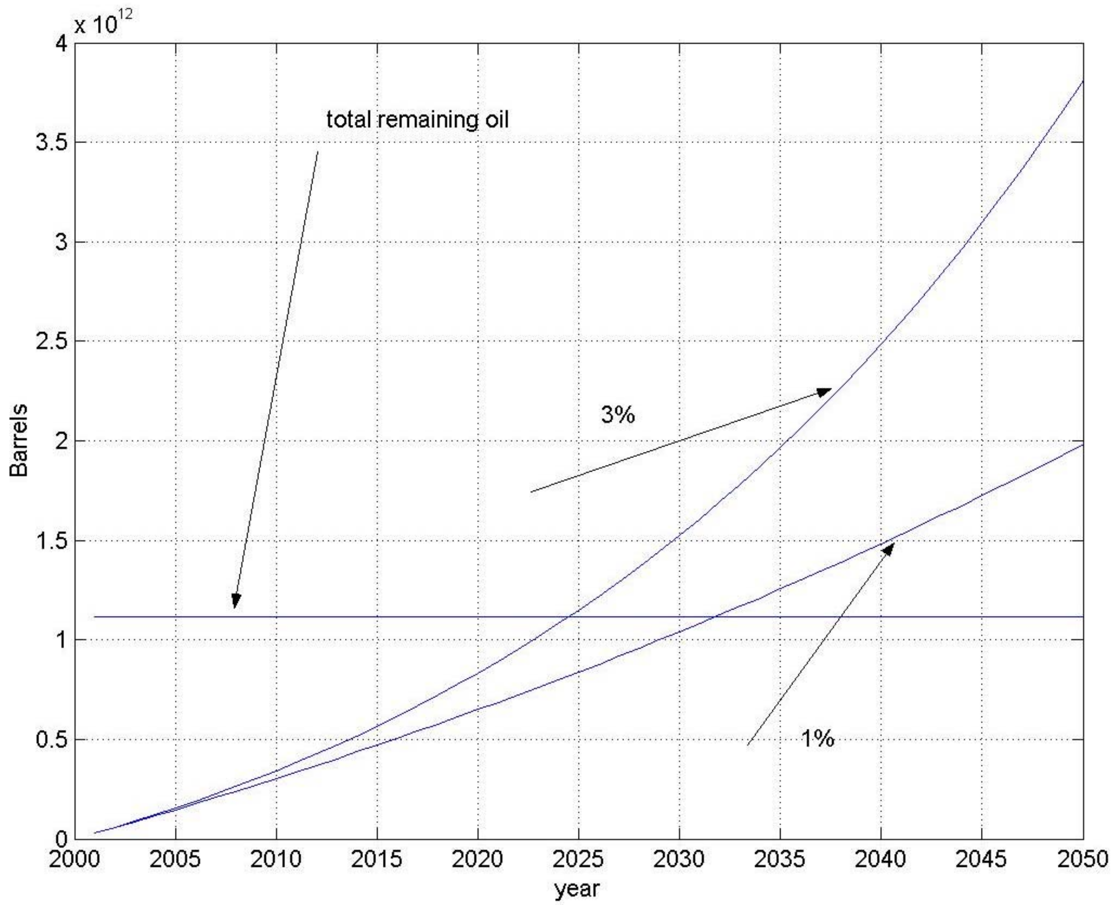


Figure 5. Cumulative oil consumption under various rates of CO_2 emission growth rates.

Limitations

The above models are based on the assumption that all the other factors are fixed when modeling for a specific factor. But this cannot be true in reality, because one factor may interact with others. Thus, interactions should be taken into account in further study.

Task 3: Sustainable Use

To prevent excessive consumption and rapid depletion, and to take into account our offspring's interests, we should allocate consumption rationally between generations.

Assumptions

- Annual demand for oil truly reflects oil consumption.
- Oil consumption in year t cannot be far less than that in year $t-1$.
- We must provide a rational consumption allocation between generations.
- A generation consists of n years.

Allocation of Oil Between Generations

The total remaining oil in year t is $U(t)+R(t)$, so

$$U(t)+R(t)=m_1+m_2,$$

where m_1 is the amount of oil for people today and m_2 is the amount of oil left for offspring. We define *the degree of rational consumption allocation for oil* as

$$\eta=\frac{m_1}{m_2}\times 100\% \qquad (0\le\eta\le\infty).$$

If the value of η is too high, the amount of oil for contemporary human beings is too small to meet their needs.

Modeling the Rational Consumption Allocation

We expect that future oil demand will not undergo a sharp decline, and we want oil to be allocated among generations fairly. Meanwhile, we want the resource to be used in the most efficient way.

We model an interval of n years, i.e., one generation. We have the following linear programming optimization problem:

$$\max\sum_{i=1}^{n} c_i d_i \tag{13}$$

such that

$$\frac{r}{\sum_{i=1}^{n} d_i}\ge\eta',$$

$$d_i\ge\alpha d_{i-1},\qquad i=1,2,\ldots,n,$$

$$d_i\ge 0,\qquad i=1,2,\ldots,n,$$

where

- c_i is the *utilization rate* of oil (crude oil available divided by refinery capacity) at year i;
- η' = degree of rational consumption allocation of oil between generations;
- d_i is oil consumption in year i (with d_0 oil consumption at the initial time);
- r is the total remaining oil in the first year of one generation;
- α is a set percentage such that the oil consumption in a given year must not be less than α times the consumption in the previous year, with α close to 1.

The objective is maximum utilization over n years. The first constraint assures a rate of allocation between generations, while the second assures that oil consumption in year i is not less than α times the consumption in year $i-1$. When η', c_i, r, α are given, we can obtain the optimal consumption allocation of oil over n years by solving the linear programming problem **(14)**. As for estimating c_i, we believe that the utilization rate should increase as time passes but should always be smaller than 1. Thus, we should have

$$c_i = 1 - a_1 e^{a_2 t}, \tag{14}$$

where a_1 and a_2 are constants determined by fitting historical data.

We give an illustration. We set $n = 20$, $\alpha = 1$, $\eta' = 1.67$, $r = 1.0 \times 10^{12}$, and year 2004 as the base year, so that d_0 = oil consumption in 2004. **Figure 6** shows oil consumption under optimal allocation from 2005 on. Oil consumption under optimal allocation is far less than under exponential growth. The optimal consumption varies smoothly until the late phase of the prediction, when it jumps sharply. This may be because we chose an inappropriate η'-value. However, choosing the η'-value is rather difficult, because it should incorporate many factors such as population, price, specific economic environment, etc.

Implementation

- We could levy a relatively heavy tax on oil compared to other resources.
- We could encourage the development of alternatives to oil.

Task 4: The "Security" Policy for Oil

We believe that the problem of oil security arises mainly due to the different utilization rates among countries. If a country with low oil utilization is assigned a redundancy of oil, whereas a country with high utilization gets an insufficient, there will be great waste. We can establish a model to find the optimal distribution of oil among countries with different utilization rates.

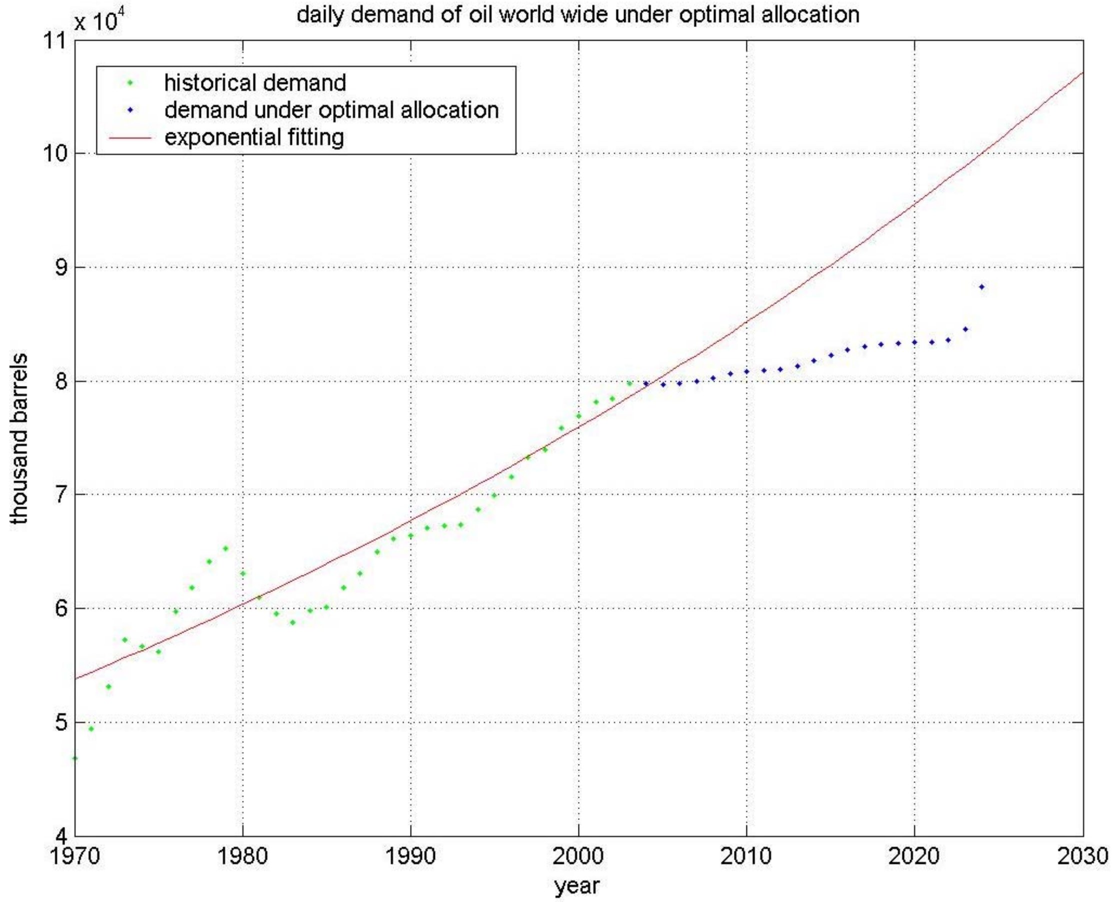


Figure 6. Oil consumption under optimal allocation (dotted line), compared with exponential growth (solid line).

Assumptions

- The annual oil consumption of the whole world is according to the optimal oil allocation model in Task 3.
- We do not take trade barriers into account, hence assume that reallocation of oil among countries is feasible.

Modeling

Suppose there are n main oil-consuming countries in the world. Given the year t, we can establish the linear programming as follows:

$$\max \sum_{i=1}^{n} l_i(t) x_i(t) \tag{15}$$

such that

$$\sum_{i=1}^{n} x_i(t) = d(t),$$
$$x_i(t) \geq \alpha_i(t) x_i(t-1), \quad i = 1, 2, \ldots, n,$$
$$x_i(t) \geq 0, \quad i = 1, 2, \ldots, n,$$

where

- $l_i(t)$ is the oil utilization rate of country i in year t;
- $x_i(t)$ is the oil use of country i in year t; ;
- $d(t)$ is worldwide oil consumption allocation in year t, which can be obtained using the model in Task 3; and
- $\alpha_i(t)$ is the the minimum ratio of $x_i(t)$ to $x_i(t-1)$, as a percentage.

The first constraint means that total oil consumption by all countries in a given year should equal total oil consumption under optimal oil allocation. The second constraint means that oil consumption of a country in a particular year is not less than a certain proportion of the previous year's consumption.

Limitations of the Model

In reality, countries would more likely consider their own interests, leading to trade barriers and making it impossible to get the optimal distribution.

Implementation

For countries with low utilization rates, we could levy a relatively heavy tax on oil or set a limit on annual oil consumption.

Task 5: Natural Disasters

An oil field occupies a large area, destroys vegetation in the vicinity, changes the components of the soil, and deteriorates the environment nearby, to the detriment of animals' habitat. Exploitation of the field influences the groundwater and causes desertification. And then there are oil spills, which often lead to the pollution of nearby waters. However, we mainly consider short-term effects of natural disasters on oil exploitation.

Assumptions

- All oil produced is consumed.
- The total amount of oil produced meets the needs of economic development.

Short-term Effects

Let n be the number of oil fields on Earth. For sustainable development of the economy, we must keep the total output of all oil fields the same as worldwide oil consumption under the optimal allocation of Task 3. Thus, we have

$$\sum_{i=1}^{n} x_i(t) = d(t),$$

where $x_i(t)$ is the output of the oil field i in year t, and $d(t)$ is the worldwide oil consumption under the optimal oil allocation of Task 3.

We believe that the susceptibility of an oil field to disasters is a function of its output in a given year and the ratio of its cumulative output to its initial oil reserve. Naturally, the more the oil produced, the greater the likelihood of disasters. We believe that the relationship is linear. And of course, a new oil field and an old one will have different effects on the environment. This difference is given by the ratio of the cumulative output to the initial oil reserve. We introduce a penalty function $e^{-a[1-\lambda_i(t)]}$ and set

$$p_i(t) = k_i x_i(t) e^{-a[1-\lambda_i(t)]},$$

where

- p_i is the susceptibility to disasters;
- k_i is the proportion coefficient, which is determined by the age and exploitation method of the oil field—a small value of k_i represents a young field with an advanced exploitation method;
- $\lambda_i(t)$ is the ratio for oil field i of its cumulative output until year t to its initial oil reserve.

We wish to minimize the total susceptibilities of different oil fields under the condition that worldwide oil demand is satisfied. That is:

$$\min \sum_{i=1}^{n} p_i(t) \qquad \text{such that} \qquad \sum_{i=1}^{n} x_i(t) = d(t).$$

In the solution, oil fields with small k_i will tend to have larger outputs, and vice versa. We also increase the value of n tentatively, i.e., increase the number of oil fields, and find that the total susceptibility decreases. This is mainly due to the fact that during early exploitation of an oil field the penalty function damps down the risk of disaster, thus favoring the development of new oil fields and decreasing the likelihood of disasters.

Implementation

We increase the output of old oil fields with small k_i (young fields with advanced methods of extraction) and reduce the output of those with large k_i

(older fields with obsolete methods of extraction). Also, if possible, we should exploit as many new oil fields as possible and decrease the exploitation of old oil fields, so as to control the susceptibilities to disaster.

Task 6: The Development of Alternatives

Even if we control the use of oil, its use can be prolonged only 4–5 years beyond an exhaustion horizon of 30 years. We urgently need an alternative to oil. For the sake of sustainable development, we must gradually accelerate consumption of the alternative as oil is depleted. The question is how to develop the alternative to keep the economy stable during the transition period.

Assumptions

- We consider a single kind of alternative.
- Oil and its alternative are interchangeable as energy resources.
- The measure of oil and its alternative is their contributions to GDP.
- The quantity of oil to produce a unit of energy does not change with time.

Analysis

Let the cost for oil to produce a unit of energy be C_1, and that of its alternative be C_2, with $C_1 \leq C_2$ (the cost for oil to produce a unit of energy is the lowest compared with any other resources [Vernon 1976]). But as the total amount of remaining oil declines, the price of oil will correspondingly increase. On the other hand, with advances in the technology for the alternative, its price will fall. The general tendency is shown in **Figure 7**.

With rising cost, oil consumption will decrease, while demand for the alternative will increase, until the day when oil is completely replaced. The question is, How fast should oil be replaced?

Modeling

From our model for optimal oil distribution among countries, we concluded that the consumption in future years will increase slowly. Let G be the value of GD, x the consumption of oil, y the consumption of the alternative, and t time. Let $G = G(x, y)$, so that

$$\frac{dG}{dt} = \frac{\partial G}{\partial x}\frac{dx}{dt} + \frac{\partial G}{\partial y}\frac{dy}{dt}. \qquad \textbf{(16)}$$

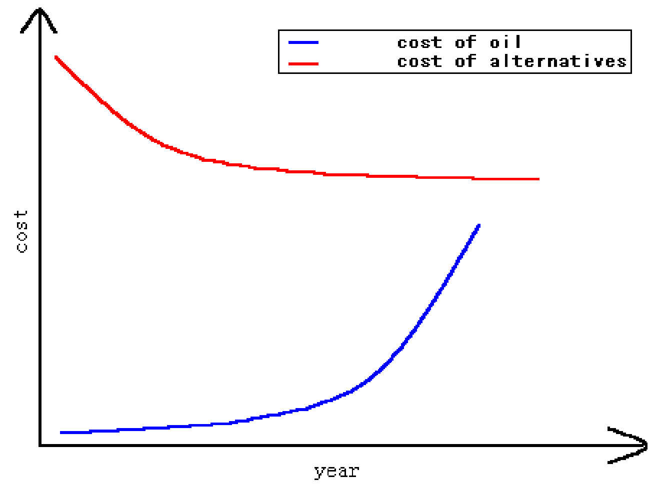


Figure 7. Trend of cost of oil (lower curve) and of its alternative (upper curve), per unit of energy.

We think that a smooth exponential decline of use of oil is reasonable, so we let

$$x(t) = x(t_0)e^{-b(t-t_0)} \qquad (t > t_0,\ b > 0),$$

so that

$$\frac{dx}{dt} = -x(t_0)be^{-b(t-t_0)}, \tag{17}$$

where t_0 is the year when oil demand begins to decline. Substituting **(17)** into **(16)**, we get:

$$\frac{dy}{dt} = \frac{\dfrac{dG}{dt} - x(t_0)be^{-b(t-t_0)}\dfrac{\partial G}{\partial x}}{\dfrac{\partial G}{\partial y}},$$

where

- dy/dt is the replacement rate,
- $\partial G/\partial x$ is the contribution rate of oil to GDP, and
- $\partial G/\partial y$ is the contribution rate of the alternative to GDP.

Knowing the other quantities, we can determine dy/dt and hence the consumption of the alternative to guarantee a stable economy.

We choose 2010 as t_0 and simulate the consumption of oil and the alternative. The result is shown in **Figure 8**.

Potential Oil Substitutes

Potential oil substitutes include solar energy, wind power, geothermal energy, hydroelectricity, and tides, as well as oil substitutes such as compressed natural gas, biofuels (biodiesel and ethanol), and gas hydrates.

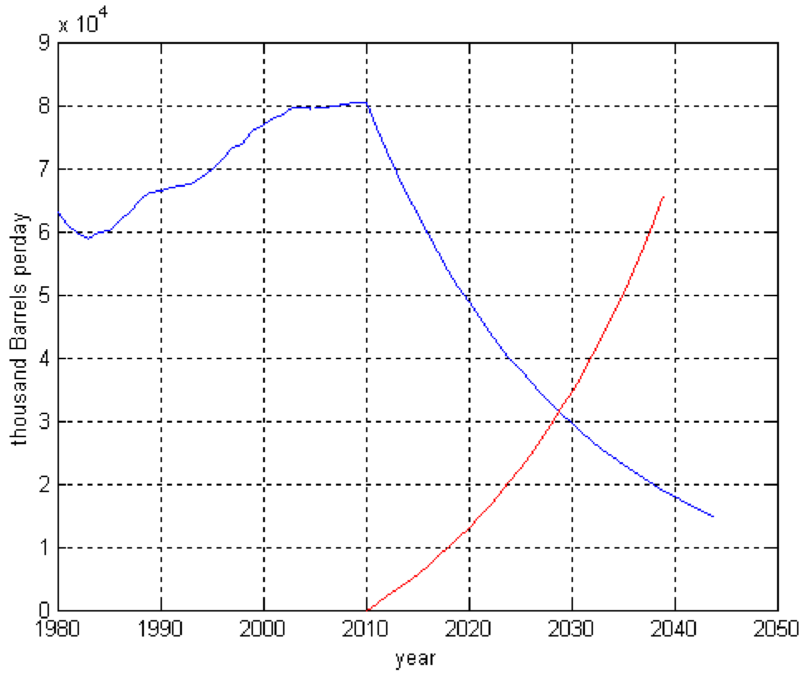


Figure 8. Consumption of oil (curve from upper left) and of its alternative (starting in 2010).

Biofuels are produced from agricultural crops that assimilate carbon dioxide from the atmosphere. The carbon dioxide released this year from burning these fuels will, in effect, be recaptured next year by crops grown to produce more biofuel. Also, biofuels contain no sulfur, aromatic hydrocarbons, or metals. Absence of sulfur means reduction of acid rain; lack of carcinogenic aromatics (benzene, toluene, xylene) means reduced impact on human health.

Gas hydrate is an ice-like crystalline solid; its basic unit is a gas molecule surrounded by a cage of water molecules. Gas hydrates are found in suboceanic sediments in the polar regions (shallow water) and in continental slope sediments (deep water).

Although these substitutes may provide some relief from the oil crisis, whether any of them—or all together—can solve the problem completely is unknown.

Conclusion

Considering the concurrent problems of population size and the adjustment of economies and lifestyles, the challenge of conversion to alternative energy resources is both urgent and formidable. A realistic appraisal should encourage people to prepare for the future. Delay in dealing with the issues will surely result in unpleasant surprises. Let us get on with the task of moving orderly into the post-petroleum paradigm.

References

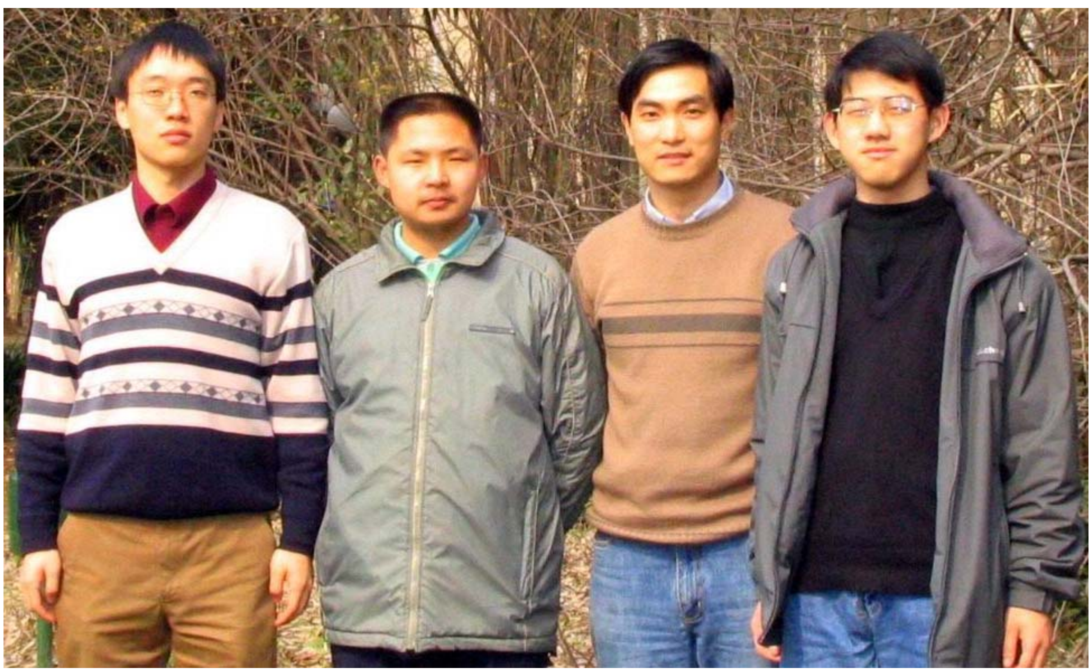
Campbell, C.J. 1997. *The Coming Oil Crisis*. Multi-science Publishing Co.

Energy Information Administration, U.S. Dept. of Energy. 2004. Table 4.6 World Oil Demand, 1970–2003. `http://www.eia.doe.gov/emeu/ipsr/t24.xls` .

Keeling, C.D., T.P. Whorf, and the Carbon Dioxide Research Group. 2002. Atmospheric CO2 concentrations (ppmv) derived from in situ air samples collected at Mauna Loa Observatory, Hawaii. `http://serc.carleton.edu/files/introgeo/interactive/examples/mlco2.doc` .

Ramirez, Vincent. 1999. Oil crises delay—A world price forecast. `http://www.betterworld.com/getreallist/article.php?story=20040214014558571` .

Vernon, Raymond. 1976. *The Oil Crisis*. Toronto, Canada: George J. McLeod Limited.



From left to right: Chen Jie, Xu Hui, advisor Ni Zhongxin, and Wei Deling.