The Petroleum Armageddon

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Summary

We describe the depletion of petroleum, a vital nonrenewable resource, over the next few decades. Petroleum is a fossil fuel that fuels our industries, heats our buildings, and powers our automobiles; plastics and fertilizers are also derived from oil. The production of nonrenewable resources that cannot be returned to the environment is generally considered to follow a bell-shaped curve; as interest and demand increase, the rate of production likewise increases until the world is producing at capacity, whence the rate decreases as the resource is slowly exhausted.

Our model assumes that production and consumption are equivalent; this does not account for stockpiles of oil or the delay caused by shipping and distribution. We also assume that total discoveries of new reserves and total production follow logistic curves; this is heavily supported by professional opinion and by our data.

Our approach includes four major functions of time: total production, total known oil, total remaining oil, and total demand. "Total" means cumulative; the derivatives of these functions are the production, discovery, and demand of oil at any time (except for total remaining oil). The equations include parameters for production, discovery, and demand, which allow our functions to follow historical data. The function for demand is based heavily on total production, in accordance with the law of supply and demand, which in turn depends on total known oil (as a carrying capacity) and on demand.

By varying the parameters, the model is flexible enough to provide for technological advance, economic limit/incentive, natural or manmade disaster, and increase or decrease in demand. The model also includes a management policy for future production, involving government limits on production to enable

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production at a nearly constant rate well into the 22nd century, at which point a decent alternative should be available. Policies for increasing the security of the oil supply, decreasing the impact on the environment, and developing an alternative to oil are suggested as part of this management policy.

Strengths of the model include its ability to adjust to virtually any factors influencing production, even when those factors overlap. Prominent among its weaknesses is its dependence on the assumptions. Another weakness is the possibility of a change in total recoverable oil, which would severely affect all four functions, although the model could easily be adjusted.

Introduction

The United States per capita GNP (gross national product) rose by a factor of 7.5 between 1870 and 1980. The fuel for such unparalleled growth was petroleum. Nonrenewable fuels now supply almost 90% of the energy produced domestically. But petroleum is a nonrenewable resource, with a limited supply. And we have used almost half of the world's total supply, demand is increasing, and world production will soon peak.

In 1956, near the height of the growth rate of the U.S. oil industry, geologist M.K. Hubbert drew a bell-shaped curve to depict production of oil in the U.S. over the coming decades. With remarkable accuracy, even despite a large unforeseen find in Alaska, Hubbert correctly predicted domestic oil production would peak in 1970. In 1989, the United States imported more fuel than it produced domestically; currently, it gets 60% of its fuel from imports. However, the lack of recent discoveries is even more disconcerting. We have already located over 1,600 billion barrels of oil in the world, whether already produced or still underground. If we accept the preferred estimate of 1,800 billion barrels as the total amount of oil recoverable for a profit, then we have only 200 billion barrels left to discover, which will only add about 20% to our current reserves.

Many people believe that once oil prices rise high enough due to scarcity, it will be profitable for oil companies to harvest reserves that are not yet economically viable However, there is something else more prevalent here than the price in dollars: the price in energy. In Hubbert's own words, "When the energy cost of recovering a barrel of oil becomes greater than the energy content of the oil, production will cease no matter what the monetary price may be" [Ecosystems 2005].

The fundamental question is, How long will our oil supply last? World production is predicted to peak between 2000 and 2020. The world supply can be expected to run out between 40 and 60 years from now.

Our model addresses depletion of oil over a long horizon by using historical data from 1930 onward. This model is flexible enough to account for almost any economic, political, and natural factors.

Historical Data

Figure 1 shows oil discovered in the last 70 years, together with a logistic curve fitted by least-squares.

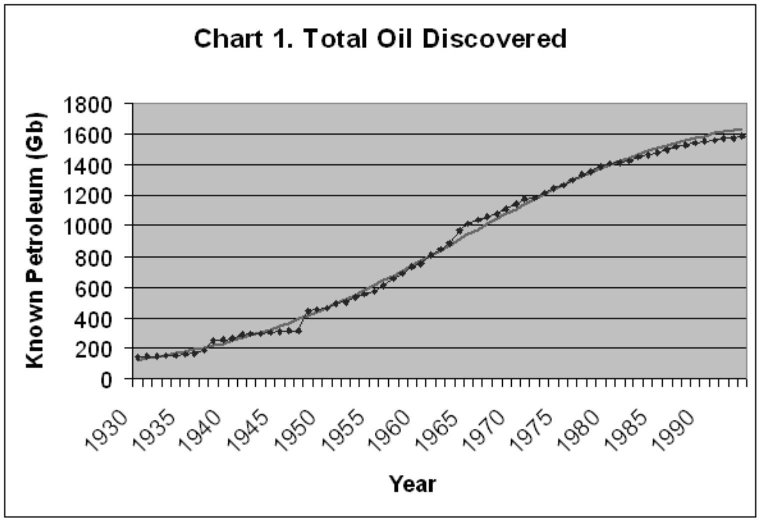

Figure 1. Total oil discovered with fitted logistic curve. Data source: Campbell [n.d.].

This logistic curve has a carrying capacity of 1,800 Gb (gigabarrels = billion barrels), the total of known oil (harvestable at a profit of both money and energy with modern technology). This number is one of the most disputed among scientists in this field; 1,800 Gb is approximately the median estimate [Campbell 1997].

The logistic curve models cumulative production; its derivative models actual production, or the rate of harvest. The derivative of a logistic curve is a normal distribution (bell curve). **Figure 2** shows a bell curve fit by least-squares to production data. The rise from expected levels in the early 1970s is due to the OPEC price increase. The decline from the peak at 1979 is due to concern over oil supplies following the Iranian revolution. In the data set, the post-WWII economic boom and the early-1980s recession are clearly visible.

Assumptions

- Production and consumption are identical, that is, there is no delay between production and consumption.
- The demand function must obey the economic laws of supply and demand: that supply and price are inversely proportional, and that demand and price are directly proportional.
- The model year 0 ($t = 0$) is 2000.
- Oil cannot be artificially produced.

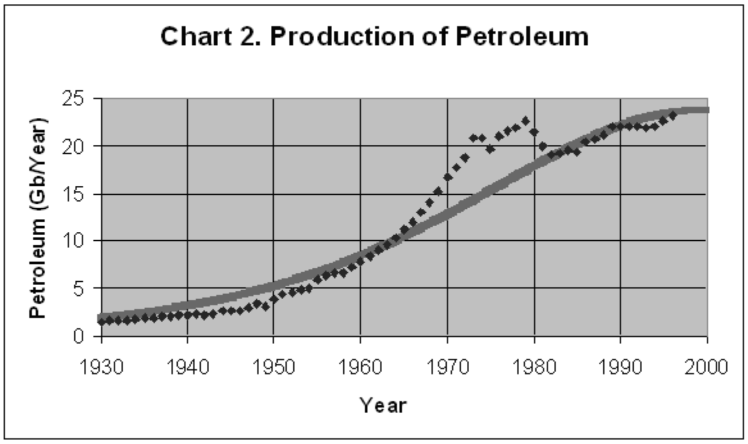

Figure 2. Oil production with fitted bell curve. Data source: Ramirez [1999].

- Harvesting oil will follow the bell curve based on past data, and discovery the logistic curve indicated, although actual past production and discoveries do not fit the data exactly, nor can future figures be expected to. According to our model, the midpoint of cumulative oil production will occur in early 2006, with peak oil production per year of 28 Gb/year. By the end of 2072, 99% of the total oil will have been produced.

Model

Our model has four main functions:

- $S(t)$, the cumulative amount of oil discovered by time t;
- $H(t)$, the cumulative amount of oil harvested by time t;
- $D(t)$, the cumulative amount of oil demanded by time t; and
- $M(t)$, the total amount of untapped oil at time t.

Both $S(t)$ and $H(t)$ correspond to data and both $D(t)$ and $M(t)$ depend on $H(t)$.

We model the discovery function, $S(t)$, as growing logistically toward a carrying capacity M_0 and also depends on demand:

$$S' = \frac{dS}{dt} = kDS\left(1 - \frac{S}{M_0}\right),$$

where k is a constant.

The total amount of oil ever harvested by time t, $H(t)$, also follows a logistic curve. It too should increase with demand.

The carrying capacity to which $H(t)$ levels off is $S(t)$, since oil harvested cannot exceed oil discovered. Thus, we have the following differential equation:

$$H' = \frac{dH}{dt} = bDH\left(1 - \frac{H}{S}\right),$$

where b is a constant.

Total world oil, $M(t)$, is given by

$$M(t) = M_0 - H(t).$$

However, considering natural disasters and outside manipulations, expressing $M(t)$ as follows is more relevant and more practical:

$$\frac{dM}{dt} = M' = -\frac{dH}{dt} = -bDH\left(1 - \frac{H}{S}\right).$$

By the basic economic laws of supply and demand, supply and price are inversely proportional, and price and demand are directly proportional. Transitively, supply and demand are inversely proportional, or $D(t) = c/H(t)$ for some constant of proportionality c. From this relationship, we get

$$D' = \frac{dD}{dt} = \frac{c}{bDH\left(1 - \frac{H}{S}\right)}.$$

We used these four functions and the improved Euler's method to create a spreadsheet to project estimates from known initial values. The tangent at the initial value is calculated; then, the tangent at a point some distance h along the x-axis from the initial value is calculated using the differential equation. As $h \to 0$, the estimate becomes increasingly accurate.

Figure 3 illustrates the depletion and cumulative discovery, harvest, and demand of oil. For many purposes, the derivatives of these functions are more relevant: At time t, H' is production rate, D' is demand rate, and S' is discovery rate. The interplay between these rates is illustrated in **Figure 4**.

Production noticeably lags behind discovery but follows a similar bell curve. Due to the sensitivity of the demand function , it takes a very low production to cause a perceptible demand increase.

To customize the model, we add several more factors.

- We implement a limiting factor $L(t)$ for H' in the simple linear form $L(t) = mt + r$, where m is the limit of H' and r is a constant. WHen $H'(t) > L(t)$, we use the the value of $L(t)$ instead of $H'(t)$. Doing so allows the model to simulate governmental or other external restrictions on the rate of harvesting.

- We make the constant b in the differential equation for H' more flexible by dividing it into two different factors: b and a second harvesting constant. The new harvesting constant comes into effect at a certain starting time. This implementation allows the model to be modified easily to simulate the effects of a future technological innovation or other external change in harvesting rate. The difference between the limiting factor and the harvest constant is that while the limiting factor caps the rate of harvesting, the harvest constant sets no such limit but simply changes the rate of harvesting at a certain time.

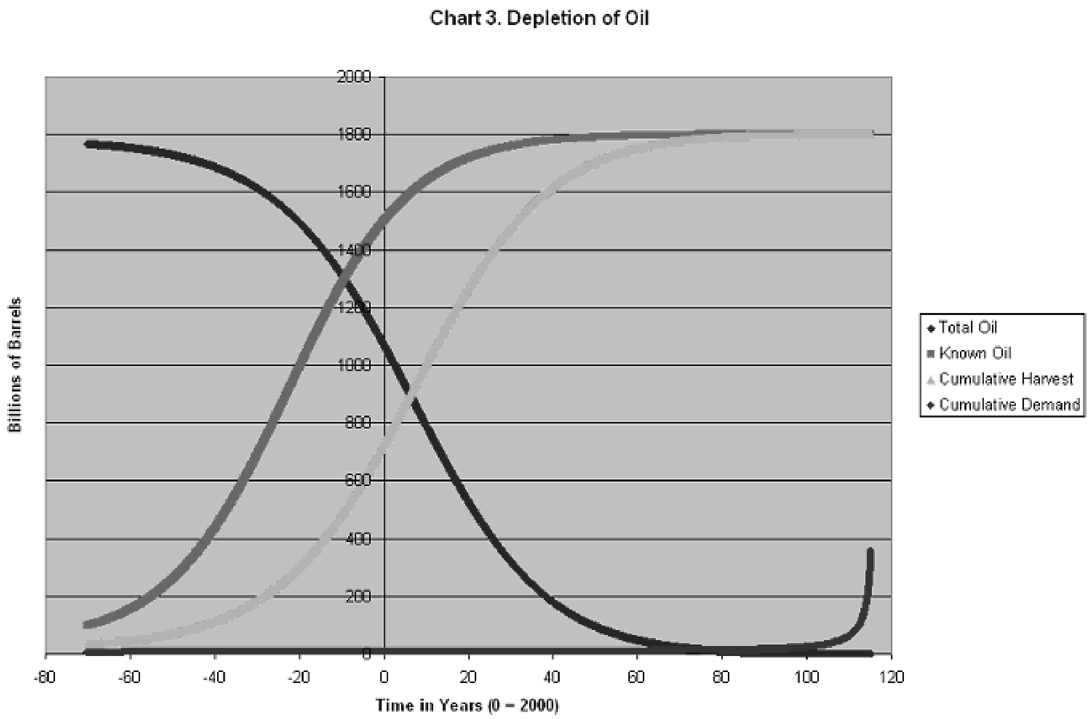

Figure 3. Depletion of oil (year 0 = 2000).

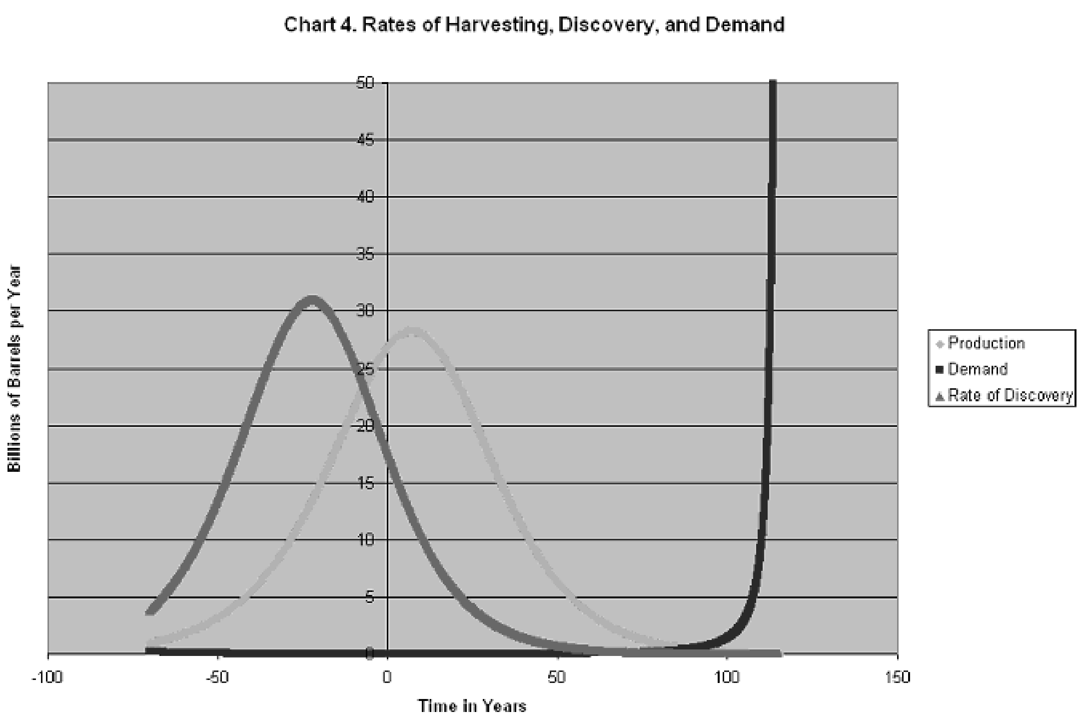

Figure 4. Rates of harvesting, demand, and discovery (year 0 = 2000).

Manipulations

To apply this model to hypothetical real-world situations, we manipulate the customization parameters. First, we imagine a moderate limit of 12 Gb/year on the consumption rate, beginning in 2010. **Figure 5** shows the result.

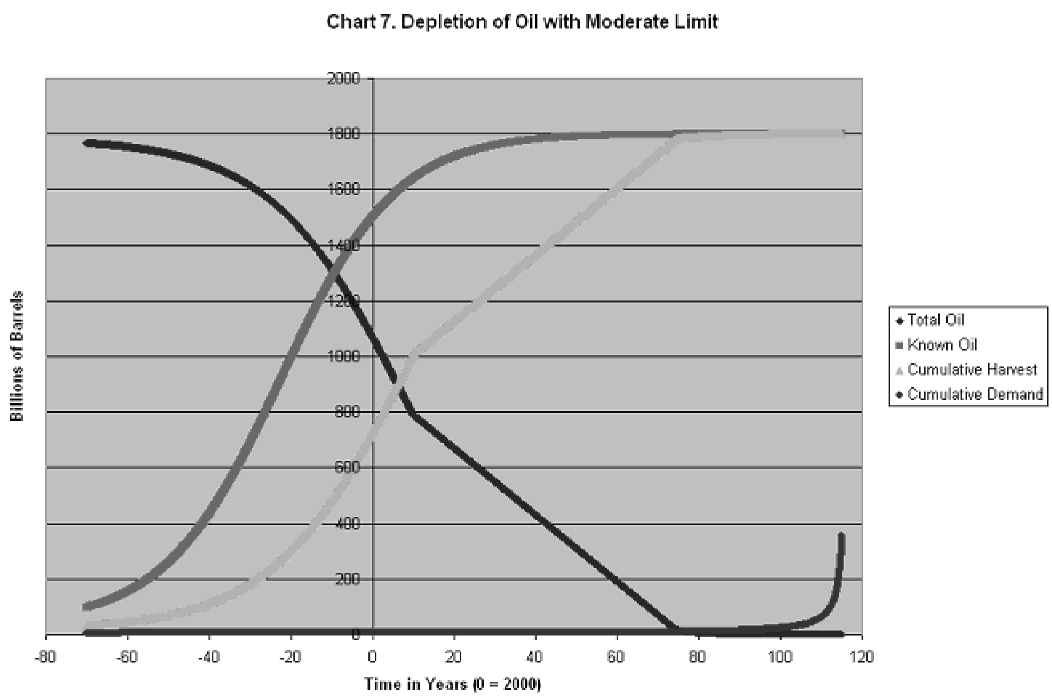

Figure 5. Depletion of oil with moderate annual limit of 12 Gb/yr.

Such a worldwide limit would be difficult to implement. Eventually, no matter how production is limited, the oil supply will run out (unless of course production is completely halted); in this scenario, the oil is depleted just about as quickly without a limit. All that can be manipulated is short-term versus long-term satisfaction of demand. A harsh limitation on harvesting would satisfy less of the demand for a longer period of time, while a less restrictive limitation would satisfy more of the demand but for a shorter period of time. Additionally, the sharp drops in rate of production in 2010 and 2075 would damage the world economy and deprive a large percentage of the population of the oil it needs. Thus, the problem of oil depletion cannot be mitigated, only manipulated.

An alternative, but less effective, policy would be a 60% downgrade in efficiency of oil-harvesting methods or technologies that occurs or is imposed suddenly in 2010. Mathematically, we decrease the harvest constant, b, by a factor of 0.4. Such a restriction would conserve oil for a longer period of time while causing a sharp drop in current production; however, the effect would be more gradual, producing an economic recession rather than economic collapse. A corresponding increase in efficiency of oil harvesting, due to technological innovation, would accelerate depletion.

To simulate a natural disaster, we make manual adjustments to $S(t)$ and $M(t)$. **Figure 6** illustrates the effects of a disaster in 2010 that destroys 400 billion barrels of known but unharvested oil (imagine wiping out a very productive oil field). After the natural disaster, the supply approaches a new carrying capacity of $M_0 - 400$.

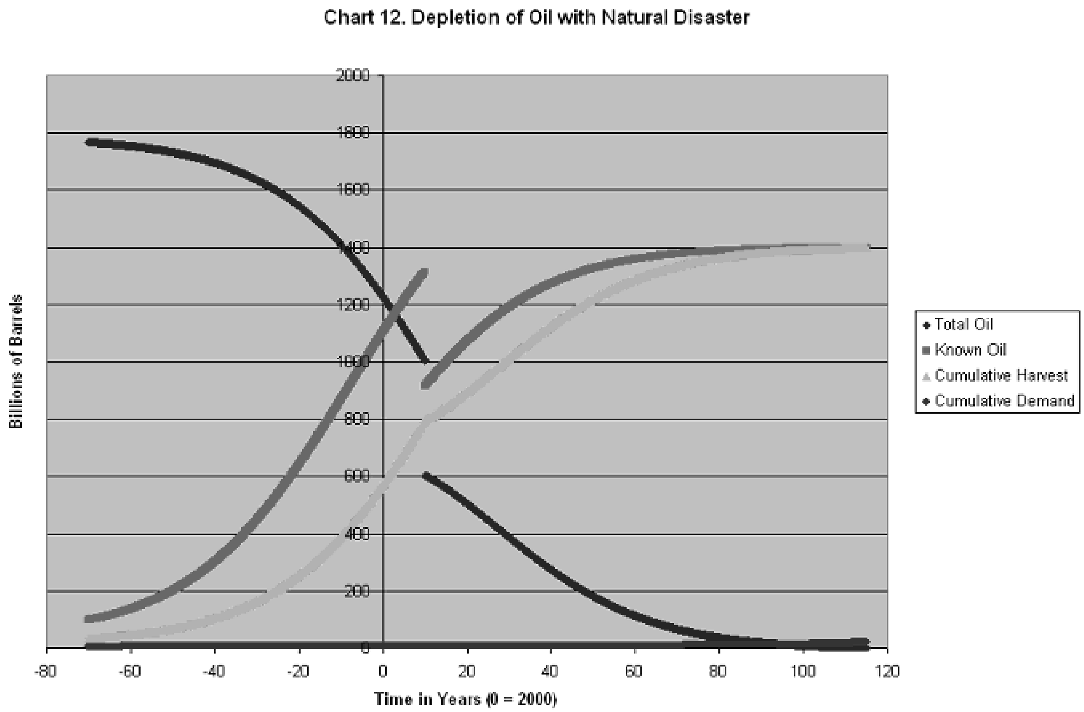

Figure 6. Depletion of oil with natural disaster occurring in 2010 to unharvested oil.

Theft, terrorism, or any oil-wasting (like an oil spill) would have a similar effect but on already-harvested oil rather than unharvested oil. Thus, H would decrease by the same amount as did S in the previous example, but $M(t)$ would not change. This kind of disaster would not cause the world's oil supply to deplete more rapidly, as the natural disaster did. In fact, it decreases the rate of depletion. However, it eventually reaches the same result.

For a future technological development necessitating more oil, that is, a sudden increase in demand, the model reflects the expected effect of shortening the horizon to oil exhaustion (**Figure 7**); the opposite is seen with a development, such as introduction of an oil substitute, that reduces demand.

Future Alternatives

We must develop another fuel source, or combination of sources, to replace oil. This fuel source need not be renewable; the U.S. has enough coal reserves to last for centuries. But there must ultimately be a switch to renewable fuel sources (such as nuclear, hydroelectric, solar, and wind). We assume that a

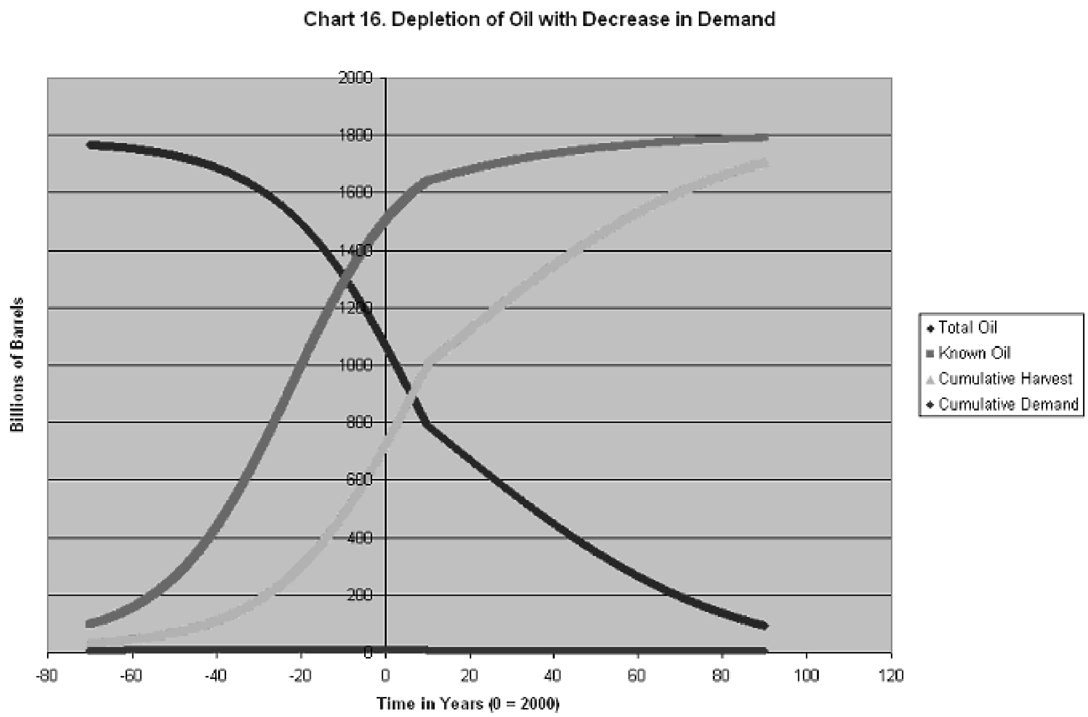

Figure 7. Depletion of oil with sudden increase in demand.

conglomeration of renewable and nonrenewable fuel sources ultimately completely replaces oil, long after the model's range.

We create a complex management policy to govern the harvesting of oil for the next century, starting in 2010. From then on, scientific alternatives to oil would be encouraged by any means necessary: government funding, taxes, etc. The world energy crisis would be given precedence above all other projects. Hoping that by the turn of the century this scientific development would be near completion, the model projects to conserve oil so that in 2100 no less than 10% of the initial world oil supply would remain: $0.1M_0$. With $M_0 = 1800$, the policy would provide that $H(100) = 1620$.

We manipulate the harvest constant rather than enforce a production limit. The total deficit is the same; the difference is when the deficit occurs and how quickly it grows. Imposing a rate limit fixes production until the rate limit exceeds the default H'; thus, short-term deficit and future deficit are equal. By decreasing the harvest constant, the short-term deficit is less than the future deficit, since production decreases over time. We assume a preference for short-term production over long-term production, since the sudden drop in oil production in 2010 caused by a limit would be much harder to cope with than a gradual drop caused by a decreased harvest constant.

We find that the optimal reduction of the harvest constant is by 70.5%, causing $H(100)$ to be as close as possible to 1620.

As soon as an alternative to oil is available and marketable, demand for oil will drop (we assume by a factor of 20). As long as production is not too low by this point, the limited oil supply will have sustained the demand. For example, **Figure 8** displays the result if in 2050 an alternative to oil is introduced and widely accepted, making the management policy obsolete.

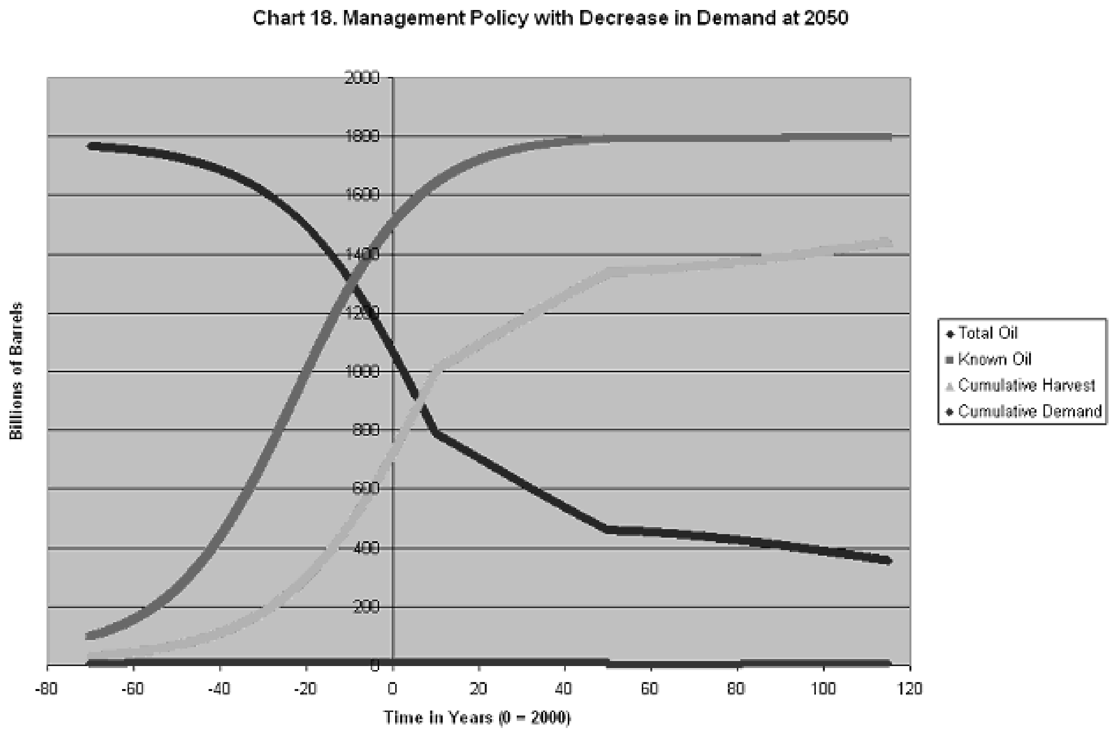

Figure 8. Effects of management policy with 95% decrease in demand in 2050.

Figure 9 shows the corresponding production scenario. Production drops suddenly by more than 75% at the beginning of this management policy until the appearance of an oil substitute. The drops caused by the change in the harvest rate constant and by the drop in demand are visible in 2010 and 2050.

The sudden drop in 2010 could be mitigated by severe conservation without reducing harvesting, so that at the time that production plummets there are large storehouses of unused petroleum that can appear on the market during the next few years and alleviate the economic crash and bankruptcies. Because it affects actual use instead of production, such a policy cannot be shown through the model, which is unable to distinguish between production and consumption. This sudden drop in production is necessary if any management policy is to be followed with any haste; even a gradual drop in production is bound to be accompanied by failing industries and economic recessions.

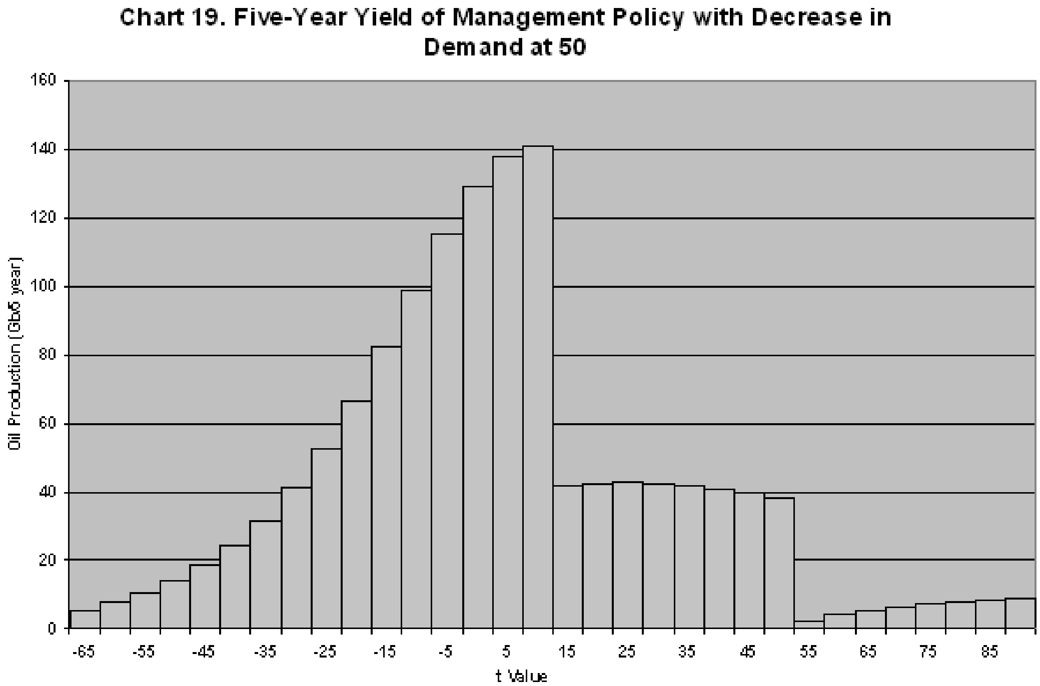

Figure 9. Production under management policy that decreases demand 95% in 2050.

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