2015-2016 年度美国“数学大联盟杯赛”(中国赛区)初赛

(八、九年级)

(初赛时间:2015 年 11 月 14 日,考试时间 90 分钟,总分 200 分)

学生诚信协议:考试期间,我确定没有就所涉及的问题或结论,与任何人、用任何方式交流或讨论,我确定以下的答案均为我个人独立完成的成果,否则愿接受本次成绩无效的处罚。

如果您同意遵守以上协议请在装订线内签名

选择题:每小题 5 分,答对加 5 分,答错不扣分,共 200 分,答案请填涂在答题卡上。

1. Jack and Jill are climbing stairs starting from the first floor at the same time and travelling at constant speeds. When Jack reaches the 11th floor, Jill reaches the 6th floor. Which floor will Jill be at when Jack reaches the 15th floor?

   A) 7 B) 8 C) 9 D) 10

2. Set $A = \{10, 20, 30, 40, 50\}$. The numbers in set B are each 100 more than the corresponding numbers in set A. What is the range of set B?

   A) 40 B) 100 C) 500 D) 540

3. The product of 12345679 and my number is a number with all its digits the same. My number could be

   A) 37 B) 60 C) 78 D) 81

4. The area of isosceles right triangle T is twice of that of square S. What is the ratio of the length of a leg of T to the length of a side of S?

   A) 1:4 B) 1:2 C) 2:1 D) 4:1

5. How many integers between 1 and 100 have exactly 5 divisors?

   A) 0 B) 1 C) 2 D) 3

6. Math books are for sale! If you buy two books, you can get the third one for half price. If each book costs \$6, what is the least that you can pay for 2016 math books at this sale?

   A) 9999 B) 10080 C) 11088 D) 12096

7. If p is a prime, what is the product of the positive divisors of p^{10}?

   A) p^{20} B) p^{40} C) p^{45} D) p^{55}

8. The sum of the first ? positive integers is 2016.

   A) 60 B) 61 C) 62 D) 63

9. 10 students in my class like mathematics, and 15 students in my class like English. If my class has 21 students, how many students like both mathematics and English?

   A) 4 B) 5 C) 6 D) 7

10. A regular polygon with each angle of measure 108° has ? sides.

    A) 5 B) 6 C) 7 D) 8

11. How many of the first 100 positive integers are double a prime?

    A) 15 B) 20 C) 25 D) 30

12. Cy can build a bridge in 3 years. Di can do the same task in 6 years. Working together, Cy and Di can build a bridge in __?__ years.

A) 1 B) 1.5 C) 2 D) 2.5

13. I made a list of 2016 numbers. The first four numbers in order are 2, 0, 1, and 6. I continued to write these same four numbers in the same order until I have written 2016 numbers. What is the last digit I wrote?

A) 2 B) 0 C) 1 D) 6

14. I am taking 10 tests this year. My average grade of the first 7 tests is 87. If my average grade on all 10 tests is a 90, what should my average be on the last 3 tests?

A) 90 B) 93 C) 95 D) 97

15. In the diagram at the right, any two adjacent points in the same column or same row are the same distance apart. How many different squares may be drawn whose vertices are 4 of the points in this diagram?

A) 12 B) 14 C) 18 D) 20

16. If $m = 2$, $a = 0$, $t = 1$, and $h = 6$, then $(m)(a)(t)(h) =$

A) 0 B) 2 C) 6 D) 12

17. If x is a real number and $x \neq 0$, which of the following must be negative?

A) $(x - 5)^2$ B) $|x - 5|$ C) $-x^2$ D) $-x^3$

18. $(5x + 4y + 2z) - (-7x + 4y + 5z) =$

A) $12x - 3z$ B) $-2x + 7z$

C) $-2x + 8y + 7z$ D) $-2x + 8y - 3z$

19. How many real solutions does $(t^2 - 4)(t^2 + 4) = 0$ have?

A) 1 B) 2 C) 3 D) 4

20. Which of the following is a factor of $x^3 - 27$?

A) $x + 3$ B) $x - 3$ C) $x^2 + 3$ D) $x^2 - 3$

21. $(4 - a)(1 - a) - 10 =$

A) $(a + 1)(a - 6)$ B) $(a + 1)(6 - a)$

C) $(1 - a)(a + 6)$ D) $(5 - a)(1 - a)$

22. $10! - 8! =$

A) 10×9 B) $10 \times 9 \times 8$ C) $2!$ D) $89 \times 8!$

23. $(x - 2016)^2 =$

A) $x^2 + 2016^2$ B) $x^2 - 2016x + 2016^2$

C) $(x + 2016)^2$ D) $(2016 - x)^2$

24. I read 1/2 of a book, then read 1/6 of the remaining pages. I still have 55 pages left to read. How many pages does the book have?

A) 132 B) 140 C) 144 D) 150

25. Steve's army has 26 soldiers and Dan's army has 24. Each of the two leaders sends s soldiers to slay off a ogre. The ratio of the remaining soldiers of Steve's army to Dan's army is 3:2. What is the value of s?

A) 4 B) 6 C) 10 D) 20

26. What are all values of x which satisfy $x^2 \le x$?

A) $x \ge 1$ B) $0 \le x \le 1$ C) $x \ge -1$ D) $x \le 0$

27. Which one of the following statements is true?

A) $-10^{10} = 10^{10}$ B) $5^7 + 5^8 = 5^{15}$

C) $\sqrt{x^2} = |x|$ D) $5^7 \times 7^8 = 35^{15}$

28. Jerry sells his math book for \$100 originally. After increasing the sales by p% then decreases the new sales by p%, the final price is \$99. Find the value of p. Assume that it is not negative.

A) 0 B) 5 C) 10 D) 20

29. $(2x^2)^2(2y^2)^3(2x)^4(2y)^5 =$

A) $2^{14}x^8y^{11}$ B) $2^{14}x^8y^{10}$ C) $2^{120}x^8y^{11}$ D) $2^{120}x^4y^{10}$

30. Cy's cycle collection consists of 601 unicycles, 400 bicycles, and some tricycles. If there are a total of 2016 wheels in Cy's collection, how many tricycles does he have?

A) 205 B) 335 C) 615 D) 1015

31. Dr. Math is tricking his students: "Think of a nonzero number. Square it, then subtract the original number. Then divide the current result by the original number, and finally subtract the original number. What's the final result?"

A) -4 B) -3 C) -2 D) -1

32. If $x \ne -3$ or -4, $\left(\frac{1}{x+3}+\frac{1}{x+4}\right)(x^2+7x+12) =$

A) 2 B) $x + 7$ C) $2x + 7$ D) $2x + 12$

33. Which of the following lines is perpendicular to the line $3x + 4y = 5$?

A) $4x + 3y = 2$ B) $44x + 33y = 0$

C) $21x + 28y = 1$ D) $16x - 12y = 6$

34. I began working on a speed exam at a rate of 10 questions per minute. After a while I slowed down and only did 7 questions per minute. I finished all 88 questions in 10 minutes. How many questions did I complete before I slowed down?

A) 50 B) 60 C) 70 D) 80

35. If a, b, and c are unequal primes, how many positive divisors does $a^8b^{13}c^{15}$ have?

A) 1560 B) 1561 C) 2016 D) 2017

36. A round-robin tournament (or all-play-all tournament) is a competition "in which each contestant meets all other contestants in turn". 6 teams compete in a round-robin tournament. The top 4 teams with the most credits will advance to the second round. For every match, the winning team gets one credit, and the losing team gets nothing. There is no tie for each game. In the end, 3 teams with the same highest credits, and the fourth team with the second highest credit, advanced to the second round. The fourth team is the only team with the second highest credit. How many credits did the fourth team get?

A) 0 B) 1 C) 2 D) 3