2016 Math League Summer Tournament
Math League Press, P.O. Box 17, Tenafly, New Jersey 07670-0017

July, 2016 – Grades 8 & 9

Team Questions – Time Limit 1 Hour

Each question is worth 10 points. Calculators are PROHIBITED.

1. If x, y, and z are the lengths of the sides of $\triangle T$, and if $3(x^2+y^2+z^2)-(x+y+z)^2=0$, what is the least possible degree-measure of an angle of $\triangle T$?

2. If A is a constant and if the 3 roots of $x^3-6x^2+Ax+4=0$ are in arithmetic progression, what is the largest of the 3 roots?

3. If S is the set of all points (a,b) in the plane for which $|ax+b|\le 1$ for all x in the interval $0\le x\le 1$, what is the perimeter of the smallest region in the plane that contains every point in S?

4. What is the sum of all the positive integers less than 2000 which are relatively prime to 2000? [Note: Two integers are *relatively prime* if they have no common divisor greater than 1.]

5. There are n 4-element subsets of {1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15}. If I found the sum of the elements in each of these n subsets, what would be the sum of these n sums?

6. In how many different ways can 6 flags (each of a different color) be placed on 5 flagpoles so that each flag is on exactly 1 flagpole? [Note: 1 or more flagpoles may have no flags, and 2 flags that are on the same pole, but in different orders, count as 2 different ways.]

7. If a, b, c, and d are positive integers for which $a+b+c+d=2016$, how many of the fractions $\dfrac{a+b\pi}{c+d\pi}$ are integers?

8. If $x=\sqrt[3]{4}+\sqrt[3]{2}+1$, then $\dfrac{3x+1}{x^3}=a\sqrt[3]{4}+b\sqrt[3]{2}+c$, where a, b, and c are integers. What is the ordered triple (a,b,c)?

9. How many subsets of {1, 2, 3, 4, 5, 6, 7, 8, 9, 10}, including the empty set, contain no pair of consecutive integers?

10. In quadrilateral $ABCD$, $\overline{AB}$ is parallel to $\overline{CD}$. If the quadrilateral's diagonals intersect at E, the area of ΔABE is 3, and the area of ΔCDE is a positive integer ≤ 2016, what is the greatest possible integral value of the area of ΔADE?