2015-2016 Math League Contests, Grades 6– 8

Second-Round, Jan - Feb 2016

Instructions:

1. This second-round contest consists of two parts. Part 1 is math questions. Part 2 is English essay.
2. This document contains 12 pages in total, including this page.
3. For all the questions below, login to your account at http://www.mathleague.cn/, and enter your answers. Answers written on this sheet or any other place will NOT be credited.
4. In Part 1, you are asked to read a math subject, Chances and Probabilities, and supplementary materials. Then you have 20 questions to work on. You will need to give precise, unambiguous answers to Questions 1-19.
5. Question 20 is Projects and Papers, which means you need to do your research and write a paper for it. There is no word limit on your paper, but it doesn't necessary mean the longer the better. The best paper is precise and succinct. Please don't feel frustrated at all if you can't write a paper, as the topic is very hard for a middle school student, even for teachers. Please don't get discouraged even if you can't finish all Questions 1-19, as they are not trivial questions and it requires a lot reading and thinking. Students who can work out a few questions should be commended.
6. The more questions you answered correctly, the more credit you will get.
7. You can seek help by reading books, searching the Internet, asking an expert, and etc. But you can't delegate this to someone else and turn in whatever he/she wrote for you. To make it clear, the purpose of the second-round contest is to test your ability to read and research. You need to be the one who understand the topics and solve the problems. You will be caught if it is not the case during the interview.
8. For Part 1, you can write in either English or Chinese.
9. In Part 2, you are asked to write an essay regarding to one topic. You have to write in English in Part 2.
10. If you have any questions regarding the contest, please contact us at once at INFO@LTHOUGHTS.COM
11. Submission of your answers:
    a) For all the questions below, login to your account at http://www.mathleague.cn/, and enter your answers. Answers written on this sheet or any other place will NOT be credited.
    b) You need to submit your answers no later than 12:00AM, Feb 7, 2016, Beijing Time. Later submission will not be accepted.

ALL ANSWERS MUST BE EXACT UNLESS SPECIFIED. PROBABILITY MUST BE EXPRESSED IN EITHER DECIMALS OR FRACTIONS.

Part 1 –Chances and Probabilities

The following is an excerpt from some math book, Chances and Probabilities, (see separate document).

Supplementary Materials,(see separate document).

In this document, although the answers are unique, the solutions are not. Searching for alternate solutions is highly recommended and encouraged. Remember, this test does not measure how fast you can solve these problems. Rather, it measures the efforts you put into the problems: the willingness and dedication to spend a long time on solving a particular difficult question.

Question 1:(6 pts)

A student rolls a die and randomly picks an integer from 1 to 4. Let X be the outcome of the die, and Y be the number he picks. Let W = X + Y.

Die / Integer	1	2	3	4	5	6
1	W = 2	W = 3	W = 4	W = 5	W = 6	W = 7
2	W = 3	W = 4	W = 5	W = 6	W = 7	W = 8
3	W = 4	W = 5	W = 6	W = 7	W = 8	W = 9
4	W = 5	W = 6	W = 7	W = 8	W = 9	W = 10

The probability of each colored rectangle is 1/24.

(a) (2 pts)What is the sample space of W?[Note: Please list your answers in ascending order. There will be 10 or fewer answers. If you have fewer than 10 answers, please leave the trailing spaces blank.]

Ans: S = {2, 3, 4, 5, 6, 7, 8, 9, 10}. The sample space of W is the set of possible values for W.

(b) (2 pts) What is the size of the sample space of W?

Ans: N = 9. The size of the sample space is the number of elements in the sample space.

(c) (2 pts) What is the probability assignment of the sample space of W? In other words, what is the probability of *each* event in W?[Note: Please list your answers in the ascending order of the W-values. i.e. first enter the probability for the smallest W-value, then enter the probability for the second smallest W-value, and so on.There will be 10 or fewer answers. If you have fewer than 10 answers, please leave the trailing spaces blank.]

Ans: As the table suggests,

P(W = 2) = 1/24

P(W = 3) = 2/24 = 1/12

P(W = 4) = 3/24 = 1/8

P(W = 5) = 4/24 = 1/6

P(W = 6) = 4/24 = 1/6

P(W = 7) = 4/24 = 1/6

P(W = 8) = 3/24 = 1/8

P(W = 9) = 2/24 = 1/12

P(W = 10) = 1/24

Question 2: (8 pts)

The Multiplication Rule:

If there are a_1 ways to do Task #1, a_2 ways to do Task #2, …, and a_n ways to do Task #N, then there are $a_1 * a_2 * \ldots * a_n$ ways to do all the tasks.

A computer password consists of four letters (A through Z) followed by a single digit (0 through 9). Assume that the passwords are not case sensitive (i.e., that an upper case letter is the same as a lowercase letter).

(a) (2 pts) How many different passwords are possible?

Ans: 26*26*26*26*10 = 4569760.

(b) (2 pts) How many different passwords end in 1?

Ans: 26*26*26*26*1 = 456976.

(c) (2 pts) How many different passwords do not start with Z?

Ans: 25*26*26*26*10 = 4394000.

(d) (2 pts) How many different passwords have no Z's in them?

Ans: 25*25*25*25*10 = 3906250.

Question 3: (12 pts)

Permutation VS Combination:

A **permutation** is an arrangement of objects in specific order. The order of the arrangement is important.

A **combination** is a way of selecting items from a collection, such that (unlike permutations) the order of selection does not matter.

(a) (2 pts) There are 6 different positions in an office. The employer is selecting among 9 people to fill in these spots. Each position is held by only one person. How many different choices does he have?

Ans: Since the positions are different, for a selection of 6 people, it matters which person is at which position. Thus this problem is about permutation (The order matters).

$_9P_6$ = 9*8*7*6*5*4 = 60480.

(b) (2 pts) There are 12 players joining the chess tournament. If each player must play each of the other 11 players once, how many games are there in the tournament?

Ans: Since each player plays each of the other 11 players only once, "Player 1 VS Player 2" and "Player 2 VS Player 1" are the same. Thus this problem is about combination (The order does not matter).

We are looking for the number of games—the number of ways to select 2 players:

$_{12}C_2 = \binom{12}{2} = \frac{12*11}{2*1} = 66$.

(c) (2 pts) In a class of 20 students, 4 students are absent today. How many different combinations of absent students are possible?

Ans: We are choosing 4 out of the 20 students. The order does not matter—this problem is about combination.

$_{20}C_4 = \binom{20}{4} = \frac{20*19*18*17}{4*3*2*1} = 4845$.

(d) (2 pts) In a math contest, the top 3 scorers have rewards. The first, second, and third finishers will win a \$300 gift-card, \$200 gift-card, and \$100 gift-card respectively. If there are 50 students participating in this contest, how many different ways can the winners be selected? (Assume that there is no tie for each of the top 3 places.)

Ans: Being the #1 scorer and being the #2 scorer make a difference. The order matters—this problem is about permutation.

$_{50}P_3$ = 50*49*48 = 117600.

(e) There are 10 athletes entered in an Olympic events.

(i) (2 pts) In how many ways can one pick the winners of the gold, silver, and bronze medals?

Ans: Receiving a gold medal and receiving a silver medal make a difference. The order matters—this problem is about permutation.

${}_{10}P_3$ = 10*9*8 = 720.

(ii) (2 pts) In how many ways can one pick the seven athletes who will not earn any medals?

Ans: We are picking 7 people from 10 people. The order does not matter—this problem is about combination.

${}_{10}C_7 = \binom{10}{7} = \frac{10*9*8*7*6*5*4}{7*6*5*4*3*2*1} = 120$.

Question 4: (12 pts)

Suppose that three events—A, B, and C—are defined on a sample space S. Use the union, intersection, and complement operations to represent each of the following events:

(a) (2 pts) None of the three events occur.

Ans: $A^C \cap B^C \cap C^C$.

The intersection of [A does not occur], [B does not occur], and [C does not occur].

(b) (2 pts) All of the three events occur.

Ans: $A \cap B \cap C$.

The intersection of [A occurs], [B occurs], and [C occurs].

(c) (2 pts) Only event A occurs.

Ans: $A \cap B^C \cap C^C$.

The intersection of [A occurs], [B does not occur], and [C does not occur].

(d) (2 pts) Exactly one of the three events occur.

Ans: $(A \cap B^C \cap C^C) \cup (A^C \cap B \cap C^C) \cup (A^C \cap B^C \cap C)$.

The union of [only A occurs], [only B occurs], and [only C occurs]. This is an extension based on part (c).

(e) (2 pts) Exactly two of the three events occur.

Ans: $(A^C \cap B \cap C) \cup (A \cap B^C \cap C) \cup (A \cap B \cap C^C)$

The union of [only A does not occur], [only B does not occur], and [only C does not occur].

(f) (2 pts) At least one of the three events occur.

Ans: $A \cup B \cup C$

The union of [A occurs], [B occurs], and [C occurs].

Question 5: (10 pts)

The following is the breakdown of 100 marbles.

	Blue	Clear	Green	Total
Large	5	15	2	22
Medium	25	5	3	33
Small	30	10	5	45
Total	60	30	10	100

(a) (2 pts) If you choose one marble at random, what is P(Blue ∪ Small)?

Ans: $\frac{5+25+30+10+5}{100} = \frac{3}{4}$.

(b)(2 pts) If you choose one marble at random, what is P(Clear | Small)?

Ans: P(Clear | Small) = $\frac{P(Clear \cap Small)}{P(Small)} = \frac{\#(Clear \cap Small)}{\#(Small)} = \frac{10}{45} = \frac{2}{9}$. This is by the definition of conditional probability.

(c) (2 pts) If you choose four marbles with replacement, what is P(at least one is clear)? [Hint: Use the idea of the complement event.]

Ans: P(at least one is clear) = 1 – P(at least one is clear^C) = 1 – P(none is clear) = 1 – $(70/100)^4$= 7599/10000.

(d) (2 pts) If you choose 20 marbles with replacement, what is P(exactly 15 are small)? [Hint: Reread Example 15.24].[Note: Answers must be in decimal form, correctly rounded to 4 decimal places.]

Ans: For each pick, we know that P(small) = 45/100 and P(non-small) = 55/100.

The picks are independent, which means result of one pick does not affect the result of the other picks.

We want 15 small marbles and 5 non-small marbles, which means small marbles appear 15 times and non-small marbles appear 5 times.

In 20 marbles we selected, there are $\binom{20}{5}$ possibilities that exactly 15 marbles are small (and each of the rest is medium or large).

Putting everything together, we get $\binom{20}{5}(45/100)^{15}(55/100)^5$. Rounding to 4 decimal places, this is 0.0049.

(e) (2 pts) [Irrelevant to the marble problem]: A factory is manufacturing n identical products. If p is the probability that a randomly selected product is defective, what is the probability that exactly k products are defective?[Assume that n is a positive integer, $0 < p < 1$, and k is a nonnegative integer such that $k \leq n$. Write an expression that answers the question in terms of n, k, and p. Hint: This question has the same idea as Question 5(d).]

Ans: Same idea as Part (d). This problem is a generalization.

Using the same reasoning, we get the answer $\binom{n}{k} p^k (1-p)^{n-k}$.

Question 6: (6 pts)

Solve each of the following:

(a) (2 pts) If $P(A) = \frac{1}{2}$, $P(A|B) = \frac{4}{7}$, and$P(B|A) = \frac{2}{5}$. What is $P(A \cup B)$?

Ans: From the given information, we have:

$$P(A|B) = \frac{P(A \cap B)}{P(B)} = \frac{4}{7}. \quad (\#1)$$

$$P(B|A) = \frac{P(A \cap B)}{P(A)} = \frac{P(A \cap B)}{1/2} = \frac{2}{5}. \quad (\#2)$$

From #2, we get $P(A \cap B) = 1/5$.

From #1, we get $\frac{1/5}{P(B)} = \frac{4}{7}$, which implies $P(B) = 7/20$.

We have $P(A \cup B) = P(A) + P(B) - P(A \cap B) = 1/2 + 7/20 - 1/5 =$ 13/20.

(b) (2 pts) If $P(A)= 0.6$, $P(B)= 0.4$, and $P(A|B^C) = 0.4$. What is $P(A \cup B)$?

Ans: From the given information, we have:

$P(A|B^C) = \frac{P(A \cap B^C)}{P(B^C)} = \frac{P(A \cap B^C)}{1-P(B)} = \frac{P(A \cap B^C)}{1-0.4} = 0.4$, which implies $P(A \cap B^C) = 0.24$.

$P(A \cup B) = P(A \cap B^C) + P(B) = 0.24 + 0.4 =$ 0.64.

Note that "$P(A) = 0.6$" is an extraneous information.

(c) (2 pts) If A, B, and C are independent events, $P(A) = 0.5$, $P(B) = 0.3$, and $P(C) = 0.4$. What is $P(A \cup (B \cap C^C))$?

Ans: $P(A \cup (B \cap C^C)) = P(A) + P(B \cap C^C) - P(A \cap (B \cap C^C))$

$= P(A) + P(B)P(C^C) - P(A)P(B \cap C^C)$

$= P(A) + P(B)P(C^C) - P(A)P(B)P(C^C)$

$= P(A) + P(B)(1 - P(C)) - P(A)P(B)(1 - P(C))$

$= 0.5 + 0.3(1 - 0.4) - 0.5(0.3)(1 - 0.4)$

$=$ 0.59.

By definition, if A and B are two independent events, then $P(A \cap B) = P(A)P(B)$.

Use this definition to prove the following statements:

1. If A and B are two independent events, then $P(A \cap B^C) = P(A)P(B^C)$.
2. If A and B are two independent events, then $P(A^C \cap B^C) = P(A^C)P(B^C)$.

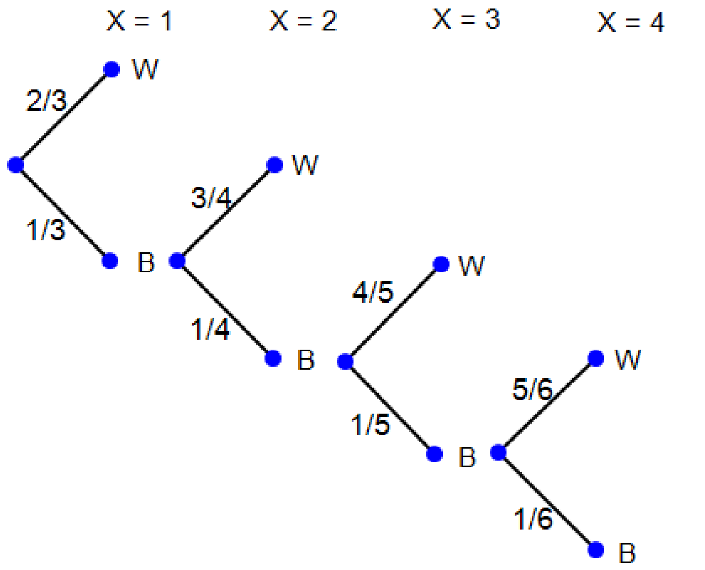
Question 7: (3 pts)

Steve, Dan, Tom, Jerry, and three other boys are standing in a line to take a group picture. If Steve must stand next to Dan, and Tom must stand next to Jerry, how many ways are there to arrange these seven boys in a line?

Ans: Let's glue Steve and Dan as one person and glue Tom and Jerry as another. We have 3 boys, glued person #1, and glued person #2—a total of 5 people. To arrange 5 people in a line, there are 5! = 120 ways. Since each glued person can be arranged in 2 ways, the answer is 120*2*2 = 480.

Question 8: (3 pts)

A bucket has two white and one black marbles. You will continuously draw marbles from the bucket until you get a white, but if you draw a black you put it back with another white marble into the bucket. If we let X = number of draws, what is the formula for P(X = *k*)? [Assume that *k* is a positive integer.]



$P(X = 1) = \frac{2}{3}$

$P(X = 2) = \frac{1}{3} * \frac{3}{4} = \frac{3}{3*4}$

$P(X = 3) = \frac{1}{3} * \frac{1}{4} * \frac{4}{5} = \frac{4}{3*4*5}$

$P(X = 4) = \frac{1}{3} * \frac{1}{4} * \frac{1}{5} * \frac{5}{6} = \frac{5}{3*4*5*6}$

…

Generalizing, we get:

X	Numerator
1	2
2	3
3	4
4	5
…	…
k	$k+1$

X	Denominator
1	3
2	3*4
3	3*4*5
4	3*4*5*6
…	…
k	$\frac{(k+2)!}{2}$

$P(X = k) = \frac{k+1}{\frac{(k+2)!}{2}} = \frac{2(k+1)}{(k+2)!}$ or $\frac{2}{k!(k+2)}$.

Question 9: (3 pts)

Six 3-member families enter a raffle. The director selects 4 winners and each winner will receivean iPhone 500 as a prize. What is the probability that the 4 winners come from 4 different families?

Ans: We have 6 families. Since we want the winners to come from 4 different families, there are $\binom{6}{4}$ = 15 ways. Each of these four 3-member families has 1 person winning iPhone 500, which means there are 3 ways for each of these four families to win the prize. There are $15*3^4$ = 1215 ways to select 4 winners coming from 4 different families.

There are 18 participants and 4 awardees, so there are $\binom{18}{4}$= 3060 ways to select 4 winners out of the 18 people.

P(4 winners come from 4 different families) = 1215/3060 = 27/68.

Question 10: (5 pts)

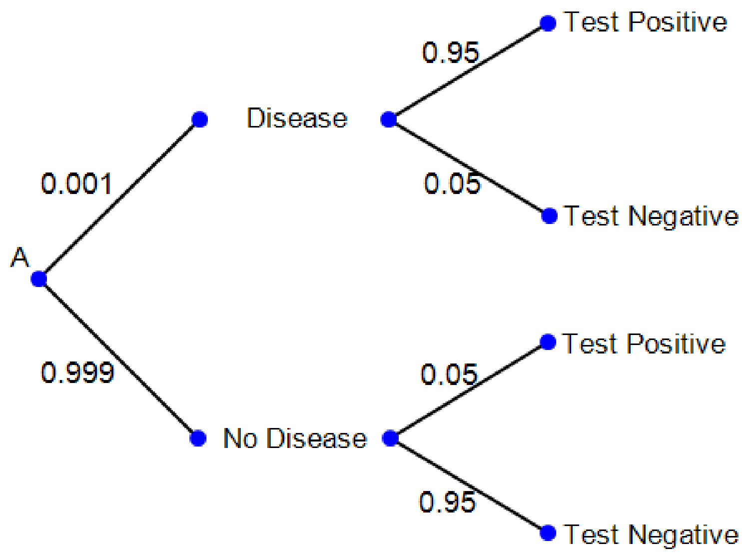
Suppose a test for a particular disease is 95% accurate, and suppose that only 0.1% of the individuals have this disease. If the test indicates that a person has that disease,

(a) (3 pts) what is the probability that the person actually has the disease? [Note: Answers must be in decimal form, correctly rounded to 4 decimal places.]

Ans: The given information implies that:

P(test positive | disease) = P(test negative | no disease) = 0.95; P(disease) = 0.001.

We want to find P(disease | test positive).



P(disease | test positive) = $\frac{P(disease \cap test\ Positive)}{P(test\ positive)} = \frac{0.001(0.95)}{0.001(0.95)+0.999(0.05)} \approx$ 0.0187.

(b) (2 pts) The answer to part (a) may be a little counterintuitive because it is much lower than you expect. Explain how this can happen.

Ans: The probability in Part (a) is so low because P(disease) = 0.001 is so low. Our expression that gets to the answer is $\frac{0.001(0.95)}{0.001(0.95)+0.999(0.05)}$. If we replace the "0.001" with higher numbers, the value of the expression will increase.

Question 11: (11 pts)

More On Counting:

(a) (2 pts) In how many different ways can 10 people form a line?

Ans: ${}_{10}P_{10}$ = 10! = 3628800.

(b) (3 pts) In how many different ways can 10 people be seated around a circular table? [Hint: The answer to (b) is much less than the answer to (a). There are many different ways in which the same circle of 10 people can be broken up to form a line. How many?]

Ans: In a circular table, two seating arrangements whose only difference is rotation are considered to be the same.In a particular arrangement, there are 10 possible rotations. Thus the answer is 10! / 10 = 9! = 362880.

(c) (3 pts) In how many different ways can five boys and five girls get in line so that boys and girls alternate (boy, girl, boy, . . ., or girl, boy, girl, . . .)?

Ans: The first position can be occupied by 10 people. The second position must be filled by one of the 5 people of a different gender than the first person, and so on. The total number of ways that 5 boys and 5 girls can line up is 10*5*4*4*3*3*2*2*1*1 = $2(5!)^2 = 2(120)^2$ = 28800.

(d) (3pts) In how many different ways can five boys and five girls sit around a circular table so that boys and girls alternate?

Ans: In a circular table, two seating arrangements whose only difference is rotation are considered to be the same. In a particular arrangement, there are 10 possible rotations. Thus the answer is 28800 / 10 = 2880.

Question 12: (11 pts)

In this question, let the term "word" represent an arrangement of letters (they do not have to mean anything). How many different "words" can be formed using all the letters in

(a) (2 pts) the word PARSLEY?

Ans: This is the same as the number of ways to arrange 7 people in a line. Thus the answer is ${}_7P_7 = 7! = 5040$.

(b) (3 pts) the word PEPPER? [Hint: This question is very different (and harder) than the one in (a). What is the difference and what should you do?]

Ans: We have 6! = 720 ways to arrange all the letters to form 6-letter-words. However, some of these words are indistinguishable [= the same]. The three P's can only be arranged in 1 distinguishable [= different] way, but they can be arranged in 3! = 6 indistinguishable ways; the two E's can only be arranged in 1 distinguishable way, but they can be arranged in 2! = 2 indistinguishable ways.

In this problem, we want the number of distinguishable ways to arrange the 6 letters. Thus we need to divide out the repetitions from 720. The answer is $\frac{720}{6*2} = 60$.

(c) (3 pts) the word MISSISSIPPI?

Ans: Similar reasoning as Part (b)'s answer. The answer is $\frac{11!}{4!4!2!} = 34650$.

The answers of Parts (b) and (c) are based on the idea of multinomial numbers (multinomial coefficients). Here is a generalized concept:

Definitions of Multinomial Numbers:

Suppose there are*k kinds* of objects, and the objects within a kind are identical. Let n_1 be the number of objects of the 1st kind, n_2 be the number of objects of the 2nd kind …n_k be the number of objects of the kth kind. The number of ways to arrange all the objects in distinguishable ways is $\frac{(n_1+n_2+\cdots+n_k)!}{n_1!*n_2!\ldots n_k!}$.

The notations for multinomial numbers are:

$$(n_1, n_2, \ldots, n_k)! = \binom{n_1+n_2+\cdots+n_k}{n_1,n_2,\ldots,n_k} = \frac{(n_1+n_2+\cdots+n_k)!}{n_1!*n_2!\ldots n_k!}.$$

The combinations are known as the binomial numbers and are special types of multinomial numbers:

$$(r, n-r)! = \binom{n}{r,n-r} = \frac{n!}{r!(n-r)!} = \binom{n}{r} = {}_nC_r.$$

This implies that ${}_nC_r = {}_nC_{n-r}$. Try to show why this is true. Hint: One possible way is to use the definition of combinations. Another possible way to do this is to by analysis.

For more information about multinomial numbers, see the following links:

https://en.wikipedia.org/wiki/Multinomial_theorem

Written as multinomial numbers,

The answer to Part (b) is $\binom{6}{2,3,1} = \frac{6!}{2!3!1!} = 60$.

The answer to Part (c) is $\binom{11}{1,4,4,2} = \frac{11!}{1!4!4!2!} = 34650$.

The link above contains the problem in Part (c).

(d) (3 pts) The number of 10-letter "words" formed by REASSESSES is the same as the number of x-letter "words" formed by REDUCTIONS. What is the value of x?

Ans: The number of 10-letter words formed by REASSESSES is $\frac{10!}{3!5!}$. Refer to Part (c)'s solution for multinomial numbers.

The number of x-letters words formed by REDUCTIONS is ${}_{10}P_x = \frac{10!}{(10-x)!}$

Equating, $\frac{10!}{3!5!} = \frac{10!}{(10-x)!}$. We see that $3!5! = 6*5! = 6! = (10-x)!$.

Therefore, $6 = 10 - x$, which implies $x = 4$.

Question 13: (4 pts)

What is the largest 3-digit prime factor of $\binom{2000}{1000}$?[Note: You may refer to the prime table below.]

Ans: $\binom{2000}{1000} = \frac{2000!}{1000!1000!}$.Let the prime be p, then $101 \leq p < 1000$. If $p > 500$, then the factor p appears twice in the denominator. Thus, we need p to appear three times in the numerator, or $3p < 2000$. According to the table below, the largest possible value for p is 661, which is our answer.

Prime Numbers between 1 and 1,000

	2	3	5	7	11	13	17	19	23
29	31	37	41	43	47	53	59	61	67
71	73	79	83	89	97	101	103	107	109
113	127	131	137	139	149	151	157	163	167
173	179	181	191	193	197	199	211	223	227
229	233	239	241	251	257	263	269	271	277
281	283	293	307	311	313	317	331	337	347
349	353	359	367	373	379	383	389	397	401
409	419	421	431	433	439	443	449	457	461
463	467	479	487	491	499	503	509	521	523
541	547	557	563	569	571	577	587	593	599
601	607	613	617	619	631	641	643	647	653
659	661	673	677	683	691	701	709	719	727
733	739	743	751	757	761	769	773	787	797
809	811	821	823	827	829	839	853	857	859
863	877	881	883	887	907	911	919	929	937
941	947	953	967	971	977	983	991	997	

Question 14: (4 pts)

In a takeout restaurant, there are 10 different food choices. A guest is given 4 boxes to take out the food he/she likes. Each box can only contain one type of food. The types of food in the boxes that the guest takes out are not necessarily different. How many ways are there to put the food into 4 boxes?

Divide the problem into cases. Different letters represent different dishes:

1. 4 different dishes (*ABCD*): There are $\binom{10}{4} = 210$ possibilities.
2. Exactly 2 boxes have the same dish (*AABC*): There are $\binom{10}{3}\binom{3}{1} = 360$ possibilities.
3. 2 pairs of the same dishes (*AABB*): There are $\binom{10}{2} = 45$ possibilities.
4. Exactly 3 boxes have the same dish (*AAAB*): There are $\binom{10}{2}\binom{2}{1} = 90$ possibilities.
5. All of the boxes contain the same dish (*AAAA*): There are $\binom{10}{1} = 10$ possibilities.

Adding the possibilities from these 5 cases, there are a total of 715 possibilities.

Question 15: (4 pts)

Reading from left to right, for how many integers greater than 9 is true that every digit after the first exceeds the digit it follows? [Note: As an example, one such integer is 24789.]

Ans: Each non-empty subset of {1, 2, 3, 4, 5, 6, 7, 8, 9} that contains 2 or more digits have one such ordering. The total number of non-empty subsets of a 9-element set is $2^9 - 1$. Since this total includes 9 one-digit subsets (which represent 9 one-digit numbers), we must subtract 9, making the answer $2^9 - 1 - 9 =$ 502.

Alternatively, we can look at the problem this way:

How many ways can we select 2-digit numbers that fit the criterion? For any selection of 2 different digits from {1, 2, 3, 4, 5, 6, 7, 8, 9}, there is only one way to arrange them in increasing order. Thus the question becomes: How many ways are there to select 2 different digits from the 9 digits? The answer is $\binom{9}{2}$.

Similarly, there are $\binom{9}{3}$ ways to select 3-digit numbers that fit the criterion, $\binom{9}{4}$ ways to select 4-digit numbers that fit the criterion, and so on.

Our final answer is $\binom{9}{2} + \binom{9}{3} + \cdots + \binom{9}{9} = 2^9 - \binom{9}{0} + \binom{9}{1} = 512 - 1 - 9 = 502$.

The gray highlighted step implies that $\binom{9}{0} + \binom{9}{1} + \cdots + \binom{9}{9} = 2^9$. The solution for Problem 19 (a) shows why it is true in a generalized way.

Question 16: (4 pts)

In a drawer Richard has 5 pairs of gloves. Each pair has a different color. On Day 1 Richard selects two individual gloves at random from the 10 gloves in the drawer. On Day 2Richard selects 2 of the remaining 8 gloves at random and on Day 3 two of the remaining 6 gloves at random. The probability that Day 3 is the first day Richard selects matching gloves is m/n, where m and n are relatively prime positive integers. What is$m+n$?

Ans: Let's count this problem backward. First, choose the pair which we pick on Wednesday in 5 ways.Then there are 8 gloves for us to pick from on Tuesday, and we don't want to pick a pair. Since there are 4 pairs, the number of ways to do this is $\binom{8}{2}-4$. Then, there are 6 gloves (2 pairs and 2 nonmatching gloves) for us to pick from on Monday. Since we don't want to pick a pair, the number of ways to do this is $\binom{6}{2}-2$.

Counting backward, we have$\binom{10}{2}$ ways to pick 2 gloves on Wednesday; $\binom{8}{2}$ ways to pick 2 gloves on Tuesday; $\binom{6}{2}$ ways to pick 2 gloves on Monday. Thus the answer is$\frac{5(\binom{8}{2}-4)(\binom{6}{2}-2)}{\binom{10}{2}\binom{8}{2}\binom{6}{2}}=\frac{26}{315}$; 26 + 315 = 341.

The following alternate solution also works in the backward direction, but deals with the probability instead of the counting:

Let the fifth glove be arbitrary; the probability that the sixth glove matches in color is $\frac{1}{9}$.

Assuming this, then let the first glove be arbitrary; the probability that the second glove does not match is $\frac{6}{7}$.

The only "hard" part is the third and fourth glove. But that is simple casework. If the third glove's color matches the color of one of the first two gloves (which occurs with the probability $\frac{2}{6}=\frac{1}{3}$), then the fourth glove can be arbitrary. Otherwise (with probability $\frac{2}{3}$), the fourth glove can be chosen with probability $\frac{4}{5}$ (5 gloves left, 1 glove that can possibly match the third glove's color). The desired probability is thus $\frac{1}{9}*\frac{6}{7}*\left(\frac{1}{3}+\frac{2}{3}*\frac{4}{5}\right)=\frac{26}{315}$. The answer is 26 + 315 = 341.

The source of this problem comes from 2015 AIME Problems, except that the "socks" are changed to "gloves" and "Sandy" is changed to "Richard". AIME is the abbreviation for American Invitational Mathematics Examination—a popular and difficult exam in the USA—15 short answer questions in 180 minutes. For more information, please visit www.artofproblemsolving.com.

Question 17: (8 pts)

On each of the following, assume that $k \le n$. Write a formula in terms of$n$ and k as your answer.

(a) (4 pts) In how many ways can I place n indistinguishable objects into k distinguishable boxes so that each box contains at least 1 object?

Ans: Let n_1 be the number of objects in the 1st box. Let n_2 be the number of objects in the 2nd box, and so on. Let n_k be the number of objects in the kth box. We are seeking the number of positive solutions $(n_1, n_2, \ldots, n_k)$ that satisfy the equation

$n_1 + n_2 + \ldots + n_k = n.$

Line up the n objects. There are $n - 1$ spaces between them. Any choice of $k - 1$ of those spaces divides the n objects into k portions, which is equivalent to solving the above equation. The answer is $\binom{n-1}{k-1}$.

This problem is a generalization for 2015 Stanford-Math League Tournament, Grade 8 Individual Questions, #8. The idea of that contest problem is to distribute 15 indistinguishable objects [$5 bills] to 5 [distinguishable] people. Using the same idea, the answer is $\binom{15-1}{5-1} = \binom{14}{4} = 1001$.

(b) (4 pts) In how many ways can John select k out of the first n positive integers, disregarding the order in which these k integers are selected, so that no two of the selected integers are consecutive integers?

Ans:

Method I: Imagine that we are going to place n balls in a straight line to represent the first n positive integers. The k integers we will choose will be represented by black balls that must be stuck in among the $n - k$ we don't choose, represented by red balls. Each time a black ball is placed, it must be in one of the $n - k + 1$ spots: the $n - k - 1$ that are between red balls, and the 2 that are at the ends of the straight line. Each possible positioning of the k black balls into these $n - k + 1$ spots corresponds to choosing k of the first n positive integers with no consecutive. There are $\binom{n-k+1}{k}$ ways of doing so.

Method II: Let $a_1, a_2, \ldots, a_k$ be k integers (no two consecutive) that are chosen at random from S = $\{1, 2, \ldots, n\}$, with $a_1 < a_2 < \ldots < a_k$. Create a new set of numbers $A_1 = a_1$, $A_2 = a_2 - 1$, $A_3 = a_3 - 2$, $\ldots, A_k = a_k - k + 1$, so that $A_1, A_2, A_3, \ldots$, and A_k are k of the first $n - k + 1$ positive integers. There is a one to one correspondence between each selection of k integers (no two consecutive) from set S and the corresponding set $\{A_1, A_2, A_3, \ldots, A_k\}$ from $\{1, 2, 3, \ldots, n - k + 1\}$. Therefore, there are $\binom{n-k+1}{k}$ ways to choose k out of the first n positive integers such that no two are consecutive.

This problem is a generalization for US High School Math League 11/11/2014 Question #2-6. In that problem, we have n = 20 and k = 6. Thus the answer is $\binom{20-6+1}{6} = \binom{15}{6} =$ 5005.

Question 18: (4 pts)

If we roll a fair die 5 times and the outcomes are a, b, c, d, and e respectively, the probability that $a \le b \le c \le d \le e$ is m/n, where m and n are relatively prime positive integers. What is $m + n$?

Ans: The main difficulty is that the given inequality is not a strict one. It would be much easier to count the number of solutions if equality could be converted to inequality. Here's one way: We know that $1 \le a \le b \le c \le d \le e \le 6$. If $A = a$, $B = b + 1$, $C = c + 2$, $D = d + 3$, and $E = e + 4$, it follows that $1 \le A < B < C < D < E \le 10$. Every 5-tuple ($A$, B, C, D, E) gives rises to a solution (A, B– 1, C– 2, D– 3, E– 4) = (a, b, c, d, e) whose coordinates satisfy the given inequality. For example, if we choose (A, B, C, D, E) to be (2, 4, 7, 8, 10), then (a, b, c, d, e) = (2, 3, 5, 5, 6) on the outcomes of the die. Any choice of 5 different integers from 1 to 10 can only be ordered one way in increasing order. The number of such ordered 5-tuples (A, B, C, D, E) is $\binom{10}{5} = 252$. Finally, the total possible number of ordered 5-tuples (a, b, c, d, e) is 6^5. The desired probability is $\frac{252}{6^5} = \frac{7}{216}$; 7 + 216 = 223.

This problem is an extension of US High School Math League 12/11/2012 Question #3-6. The original question asks for rolling a die four times instead of five times. Now try to generalize:

If we roll a fair die n times and the outcomes are $a_1, a_2, \ldots, a_n$ respectively, what is the probability that $a_1 \le a_2 \le \ldots \le a_n$?

Question 19: (12 pts)

In terms of combinatorics, the entries in the Pascal's Triangle can be written as follows:

Row	
0	$\binom{0}{0}=1$
1	$\binom{1}{0}=1 \quad \binom{1}{1}=1$
2	$\binom{2}{0}=1 \quad \binom{2}{1}=2 \quad \binom{2}{2}=1$
3	$\binom{3}{0}=1 \quad \binom{3}{1}=3 \quad \binom{3}{2}=3 \quad \binom{3}{3}=1$
4	$\binom{4}{0}=1 \quad \binom{4}{1}=4 \quad \binom{4}{2}=6 \quad \binom{4}{3}=4 \quad \binom{4}{4}=1$
5	$\binom{5}{0}=1 \quad \binom{5}{1}=5 \quad \binom{5}{2}=10 \quad \binom{5}{3}=10 \quad \binom{5}{4}=5 \quad \binom{5}{5}=1$

(a) (4 pts) As observed, the sum of the *n*th row is 2^n. Using the idea of sets, subsets, and elements, explain why $\binom{n}{0} + \binom{n}{1} + \binom{n}{2} + \ldots + \binom{n}{n-1} + \binom{n}{n} = 2^n$. (Hint: If set A has *n* elements, how many subsets does A have? Why?)

Ans: If set A has *n* elements, $\binom{n}{0}$ represents the number of A's subsets with 0 elements; $\binom{n}{1}$ represents the number of A's subsets with 1 element, and so on. Together, $\binom{n}{0} + \binom{n}{1} + \binom{n}{2} + \ldots + \binom{n}{n-1} + \binom{n}{n}$ represents the total number of subsets that A has.

In each subset of A, each of the *n* elements has two possibilities: it appears or it doesn't. The combination of the existence of each element determines a unique subset of A. According to the multiplication rule, the total number of subsets of A is 2*2*2… (the product of *n* 2's), or 2^n.

Therefore, $\binom{n}{0} + \binom{n}{1} + \binom{n}{2} + \ldots + \binom{n}{n-1} + \binom{n}{n} = 2^n$. □

Refer to Question 5(e), we see that $P(X = k) = \binom{n}{k} p^k (1-p)^{n-k}$. Intuitively, we see that $\sum_{i=0}^{n} P(X = i) = P(X = 0) + P(X = 1) + \ldots + P(X = n) = 1$. In terms of mathematical definition, can you show why $\sum_{i=0}^{n} P(X = i)$, which is the same as $\sum_{i=0}^{n} \binom{n}{i} p^i (1-p)^{n-i}$, is equal to 1?

(b) (3 pts) Expand each of the following.

$(x + y)^0 = 1$, assuming that x + y is not 0.

$(x + y)^1 = x + y$

$(x + y)^2 = x^2 + 2xy + y^2$

$(x + y)^3 = x^3 + 3x^2y + 3xy^2 + y^3$

$(x + y)^4 = x^4 + 4x^3y + 6x^2y^2 + 4xy^3 + y^4$

$(x + y)^5 = x^5 + 5x^4y + 10x^3y^2 + 10x^2y^3 + 5xy^4 + y^5$

What is notable in the coefficients and exponents?

Ans: The coefficients of the expanded $(x + y)^n$ are the entries of the *n*th row of the Pascal's Triangle.

The exponents of each term sum up to *n*.

(c)(2 pts) What is the formula for expanding $(x + y)^n$?

Ans: $(x + y)^n = \sum_{i=0}^{n} \binom{n}{i} x^i y^{n-i}$

or

$(x + y)^n = \sum_{i=0}^{n} \binom{n}{i} x^{n-i} y^i$

or

$(x + y)^n = \binom{n}{0} x^n + \binom{n}{1} x^{n-1} y + \binom{n}{2} x^{n-2} y^2 + \cdots + \binom{n}{n} y^n.$

The answer to this question is known as the Binomial Theorem. Numbers such as $\binom{n}{1}$ are known as the Binomial Numbers.Using the Binomial Theorem, we are able to expand binomials raised to a nonnegative integer power. Try to expand trinomials or even 4-term-nomials and (more than 4)-term-nomials using the Multinomial Numbers.

(d) (3 pts) Withoutusing the concept of factorials or the Pascal's Triangle, prove the following statement:

$$\binom{n+1}{k} = \binom{n}{k} + \binom{n}{k-1}.$$

Ans: Suppose we are choosing k objects from a set of *n*+1 objects. There are $\binom{n+1}{k}$ ways to do so. The alternate explanation is:

Let X be one of the $n + 1$ objects. In order to count how many ways there are to choose k objects from $n + 1$ objects, break it into 2 cases:

Case 1: The selection does not contain X.

If we do not choose X, there are only n selectable objects. There are $\binom{n}{k}$ ways to choose k objects such that X is not one of the selected objects.

Case 2: The selection contains X.

After choosing X, we are choosing $k - 1$ objects from the remaining n objects. There are $\binom{n}{k-1}$ ways to choose k objects such that X is one of the selected objects.

Therefore, we see that $\binom{n+1}{k} = \binom{n}{k} + \binom{n}{k-1}$. □

<u>Question 20: (20 pts, note: paper with exceptional quality can get up to 40pts.)</u>

Probability Theory in Games

Games of chance, be they for religious purposes or for profit, can be traced all the way back to some of the earliest civilizations—the Babylonians, Assyrians, and ancient Egyptians were all known to play primitive dice games as well as board games of various kinds.

In this project you are to write a paper discussing the development of probability theory as it relates to the games that involve chance. One possible way to start this project is to select a game, describe what it is, and how the concept of probability is involved in the game.

Note: For this question, please write your answer on file "middle-school-answersheet.doc", downloadable together with this document at www.mathleague.cn, and submit file "middle-school-answersheet.doc" at www.mathleague.cn after you are done.

以下是一些同学提交的试卷, 供大家参考。这些试卷并不代表是写得最好、最正确、最全面、得分最高的，只是给大家参考，因为这道题目本身就是 open question, 没有标准答案或者最佳答案。

Sample Essay 1

我们每天要做的且必不可少的就是看电视了，看电视就大约看一些新闻联播，天气预报之类的，当然其中比较有实际与数学意义的就属天气预报了。

现在一般老年人在家里看电视不是看新闻就是天气预报，天气预报这个节目所谓永垂不朽，无论播放了多长时间还是一样受人关注，每天都会有很多人看，世界各地都是一样。这主要是因为天气的不确定性，我们不知道明天天气会是什么，有可能是令人感到格外舒适而欣喜的晴天，有可能是比较昏暗的阴天，还有可能是令人无聊烦躁的下雨天，当然除此之外还有其它不同天气或极端天气影响，那么天气预报就可以提前几天通知我们后几天天气会是什么，虽然准确度不太高但访看量十分大。

那又为什么说它有数学意义呢？主要是因为它富含概率这一大数学知识，因天气的总量非常大，而其下部分又分为每个省，城市等，而且播放时间不一样，每一个电视台都播放各自主省城市的天气，所以我们就可以用概率来计算一个天气平均每天出现的量，既准确有比较富含哲理。有一天出于好奇就耐不住性子看了看天气预报，那是时刚好是在浙江省，发现在中国有大太阳标志也就是晴天的城市有 5 个，而一共城市有 32 个，那晴天在浙江省四周出现的概率也就是 5/32 。这想法不禁激起了我继续看这“节目”浓厚的兴趣，于是我带着对数学概率的深刻思考继续看下去，过了几分钟后看视播放首都与他四周的天气状况，这不禁出乎了我的意料，首都北京当时雾霾是比较严重的，但晴天标志远远比浙江省周围晴天的标志多多了，经过仔细计算后算出晴天在北京周围出现的概率是 37.1 也就是 13/37。算到这里后一个更深的问题不禁跳到了我的脑海里“北京市周围晴天量与浙江省周围晴天量总和概率是多少”，想到这我不仅冷笑一下，“这不就是 5/32 加上 13/37 吗？”最后得出结论是 0.508 也就是 127/250，算好后转念又一想这值也太大了，以前好像也做过类似的题，难道是算错了，经过不断查作业本，猜测等方式后才发现这应该是各城市相加再除于除于晴天状态的城市之和最后等于 0.26 也就是 18/69，以这种方式也可以立刻算出不是晴天城市总和的概率为 0.74 也就是 51/69，当然也别看这个式子小没什么用途。通过这个式子可以发现两省周围大部分都处于阴天，下雨天或暴雪天，很少部分是晴天，一半都不到，从中也可以得出这两地区天气都不太好。

不仅是这个天气预报的例子，在很多日常生活中也能遇到概率比如说抽彩票，抛硬币或扔筛子，这不禁让我更加深刻了解到了数学的广泛与范围面宽大，这有时也真挺有用的。

Sample Essay 2

Probability Theory in Games

Probability is always closely involved in our lives, rolling dices, playing round plates, weather reporting, even buying lotteries. Probability seems to be showing fair and justice, we always think we have the same chance on something.

Here is a game that you might be familiar with, now I have got more than 5 iPhone 233, 4 pieces of white paper and a piece of red paper. My five friends—A, B, C, D, E will choose one of these pieces of paper with replacement. Whoever gets the red piece of paper wins the iPhone233. What's the probability that all of them win the iPhone 233?

We all know that P(all my friends win it)=$\frac{1}{5^5}$, because everyone has five choices ,so there are totally 5^5 possibilities. However, this task won't be so hard for you, what I really want to talk about is the next situation.

We still keep the things I had in the first situation, but right now things are a little different. My friends will take the papers in the order of "A, B, C, D, E" without replacement. So the first question is, is this game fair? Then, what's the probability for everyone to win the iPhone 233?

"In order? It can't be fair!" You may think like that, I guess. Yes, it's sure that if A takes the red piece of paper, there will be no chance for others. The more rear you are, the smaller possibility you have, so E is definitely the most unfortunate person. But the fact is that, everyone has the same chance--$\frac{1}{5}$.

Let's get the answer together. A has a chance of-$\frac{1}{5}$ to get the red paper. B has a probability of $\frac{4}{5}$ to make sure that A hasn't got the red paper and still has a chance of $\frac{1}{4}$ to get it. You can get C, D, E's probability in the same way. Both of them are just $\frac{1}{5}$.

Why does that happen? Because this probability is an average amount, actually everybody has the chance to win it. Likewise, we have a lot of games which mentioned "order", but the game is still fair.

Sample Essay 3

Probability theory in traditional games

Sports games or folk games are required to have a certain rule and the formulation of these rules should be based on the principle of fairness reasonable. In fact most of the games and the rules can be achieved. But some games seem to be fair. In fact it has some small problems. I'll talk about that in the following.

[Grab 30]

A B two people take turns off, form 1 to 30, each people can count off one or two consecutive number. The winner is who can count 30 first if the winner. This is "Grab 30" game.

Just need a little analysis can find a method to victory. We want to grab 30. First we must grab 27. If A count 27.then B can just count 28 or 29. So A is the winner. Similarly, we have to grab 24.The multiples of three. So it seems the person behind can win.

It seems the rules have a tricky opportunity. But time passed, more and more people know this trick. It will lose its significance. But on the other hand, thinking about a problem, analysis it and research it are always good.

[Ballot question]

Many competitions need to draw before the game to decide the arena. The order and so on. In general, most of the way is tossing the coin. Because it's fair to everyone. But some important tournaments generally take a lottery which is to prevent cheating and foil the atmosphere of the game. Such as in the 1999 FA cup final. The decision to determine the sequence of the host and guest of the draw .we put it in abstract of the rules of math problems. Choose two numbers at random from 1-9, what is the probability of the sum of two numbers can be odd or even?

For this question, we make A="the sum of the two numbers". A1="the two numbers are all even"

A2=" the two numbers are all odd".

Then A=A1+A2

P(A1)=2C4/2C9=1/6 P(A2)=2C5/2CP=5/18

So P(A)=P(A1p+P(A2)=4/9

Then the sum of two numbers is odd's probability is 5/9, even is 4/9.

It seems choose odd is easier to win.

[Poker]

I think everyone has played cards before. There is a variety of placer game, everyone realizes that fight against and fords, fried golden flower, race, and so on. Sometimes we can use poker to decide something. But some rules seem to be fair. In fact, it doesn't. Let's look at this game rule is fair or not.

Now there are two cards. One is big joker, and the other because of the printing error so both sides are all printed the same of the opposite side. Now we put the two cards into the pocket for a shuffle. The rule is: after the shuffle you take a card without looking. Then put the card on the table. If the card is already on the side of Big joker, then put it back and draw again. If the up-word side is the opposite side, then we can continue play the game. Now there are two possibilities the card is the Big joker of the printed error card. If the card is Big joker, then you win, other wise you lose, so is this game fair?

Let's have an analysis about this problem. It seems the probability of your winning is 1/2. But in fact, you have only 1/3 of winning probability, because if you get the big joker is positive up word, you must put it back and take again, that reduces possibility of winning. In other words there are 4 possible: the big joker upward or downward, the error card upward or downward. But the rule get rid of one possible (the big joker upward). So the probability of winning is only one possible.

We can see, the gambling games are often in the hands of the magician or mathematics fans. So the game becomes an unfair game. But in the other hand, if we have some knowledge about math, then we may become the master of the games. So it's very important to learn mathematics.

Sample Essay 4

概率与骰子

自古以来，人们就十分喜爱赌博游戏，赌博游戏的一大常见形式就是投骰子，中国古代的六博、樗蒲、双陆，五木等都属于骰子博戏，因为投掷骰子带有很大的偶然性，所以也吸引了古今中外各个阶层的人们。为了让这个游戏更加刺激，人们还设计了各种各样变换的规则，同时，为了增加获胜率，数学的概率论在此基础上得以发展。

如樗蒲，它有黑、白、雉、犊四种花色，每个骰子上有两种花色，共能产生十二种五个花色的组合，彩名有十种，为卢、塞、秃、雉、枭、撅、犊、塔、开、白。以其中概率最低的卢彩、雉彩、犊彩、白彩为贵彩，因为几率只有 1/32。这里的 1/32 表示了共有 32 种情况，而中卢彩是其中的 1 种情况。因为有五个骰子，每个骰子有两面，所以共有 32 种情况。这体现了概率在骰子游戏中的应用。

这只是简单的概率计算，随着人们研究游戏中的概率的兴趣越来越多，骰子的情况也越来越复杂。

比如以下的一个游戏：掷硬币，正面朝上，A 给 B 一个筹码；反面朝上，B 给 A 两个筹码，但 B 输了后可以再掷一次。这个游戏听起来很公平，因为共有三种情况，1 种 A 赢，2 种 B 赢，而 A 赢了得两个硬币，B 赢了得一个硬币。但是，这个游戏其实是暗藏玄机的，B 很快就会赢走 A 所有的筹码。

让我们来分析一下，这个游戏有 3 种情况：①第一次为正面，A 给 B 一个筹码，概率为 1/2；②第一次为反面，第二次为正面，A 给 B 一个筹码，概率为 1/4；③第一次和第二次都为反面，B 给 A 两个筹码，概率为 1/4。由此可见，B 有 3/4 的几率赢，而 A 只有 1/4 的几率赢，所以 B 输时给 A 三个硬币才公平。

还有一个让骰子游戏更有意思的原因是骰子是没有记忆的，你不能保证投硬币十次一定是五次正面和五次反面，就像你不能保证掷一个骰子六次一定是 1、2、3、4、5、6 各出现一次一样，所以你有可能掷一千次硬币，却总是为反面。尽管可能性不大，但这也让赌博游戏更加刺激，因为人们往往会认为掷骰子连续出现 5 次 2 点后，下一次不可能还是 2 点，但不管之前你掷了多少次，下一次每一种的几率都还是 1/6。

情况的变幻多样和骰子的“无记忆”，让骰子游戏激动人心，从古至今都深受各年龄段的人们的喜爱。

Part 2 Essay (75 pts)

Let's define the word "discipline" as follows:

A way of behaving that shows a willingness to obey rules or orders (*Webster's*).

Without discipline, there's no life at all.

--Katharine Hepburn

Do you agree or disagree with this bolded quotation? Plan and write an essayin which you develop your point of view on this issue. Support your position with reasoning and examples taken from your reading, studies, experience, or observations.

Hints: Here are some questions that help you to craft the essay. You do not have to answer them, but think of them as you write:

1. What are some examples of having discipline?
2. What will happen if one violates discipline in a particular circumstance? Give a specific example and explain your points.
3. Will too much discipline hurt freedom and innovation? When is discipline too much? How much discipline is necessary?
4. Will different people treat discipline differently? For example, will soldiers, students, scientists, politicians, and etc. treat discipline differently?
5. What is the role of discipline in civilized society?
6. In your school, teachers might emphasize as much discipline as possible, while students might want as little discipline as possible. Who are right? How to balance this?

Note: For this question, please write your answer on file "middle-school-answersheet.doc", downloadable together with this document at www.mathleague.cn, and submit file "middle-school-answersheet.doc" at www.mathleague.cn after you are done.

以下是一些同学提交的试卷, 供大家参考。这些试卷并不代表是写得最好、最正确、最全面、得分最高的，只是给大家参考，因为这道题目本身就是 open question, 没有标准答案或者最佳答案。

Sample Essay 1

Nothing can be accomplished without norms or standards

Rules and orders are everywhere in our daily life. It is even more complicated in ancient China. As years go by, it seems that we have less unreasonable rules and more freedom. While some people criticize bureaucracy with complex rules, others see too much freedom will harm freedom itself. The question is how much freedom is adequate.

Critics suggest that too many rules hinder people's freedom and innovation. As Richelle Mead said, throughout history people with "new ideas" who think differently and try to change things have always been called trouble-makers. If people just follow the path of their ancestors, the world will never advance forward. For example the ancient famous politician Shang Yang criticized the ministers for being inflexible on the policy basis and refused to made a reform. He did not follow the rules of his ancestors, but enhanced the administration through an emphasis on meritocracy. This propels the Qin kingdom to military and administrative superpower and the rest is history. A unified China was made possible. It shows how important it is to innovate, rather than trailing behind the beaten path. If people just walk the chalks, how can Einstein's theory of relativity move beyond Newton's classical mechanics, the Renaissance break away from the bondage of the medieval darkness, or the discovery of penicillin?

So if rule-breaking can promote social development, why do we have many rules to obey? While some people criticize that the large amount of legislation limits the creativity of people, others believe it is fundamental that rules keep the social order. Rules set and coordinate expectations. For example, students need to get to school on time. In conversations, when someone is speaking, the others should listen or politely interrupt. In public places, people have to line up to get on to the train. In fact, people are obliged to obey the rules or regulations to keep society operate smoothly. If law-breaking behaviors occur, such as murdering, drug abusing, black-mailing, there is both necessity and urgency to stop them and hold the perpetrators accountable, in part because they undermine the basic rights of others. If rules are flouted or unenforced, there is simply no significant innovation, since people do not feel secure. As an old Chinese idiom said, nothing can be accomplished without norms or standards.

Therefore, the proper social arrangement should not restrict freedom and innovation of the people, but defend people's rights and freedom through rule-based system. Rules are like a dam, and the freedom is the water inside it. Without the dam the water will flood the villages and towns and lose its potential to benefit agriculture nearby. We have to abide by the rule in the daily life, but don't lose our critical thinking to move beyond where we are today.

Sample Essay 2

We live all the time with discipline. Traffic intersection, the station ticket office, restaurant ordering department, workshop processing area, product development department, discipline is strict attitude. The vastness of the universe, each galaxy, each star, the planets follow the discipline of their operation, in order to build a harmonious and beautiful space. Small molecules, all the things you see in front of you, are built up by the molecules, or everything will collapse.

Discipline, is the largest wild society and resolution marked civilized people. Primitive human beings from the social start, slowly learned the behavior, the exchange of cooperation and the development of discipline and regulations. In ancient China, the Confucian thought may be the beginning of discipline. In the promotion of this kind of discipline progress thought, the ancient Chinese people's life is becoming better and better. In today's world, with the presence of discipline, our lives are a lot less trouble. You in the busy streets, the need to worry about the traffic jam; you live in the community, the environment clean and tidy; people from all walks of life in an orderly manner to do the things that should be done in an orderly manner. In a word, discipline makes the world a better and more free.

If there is no discipline, it is very terrible. Chinese the Spring Festival approaching, Spring Festival is too rash and too much in haste. Once, the government did not strengthen the discipline of this, often at this time the traffic system on the paralysis, disputes occur everywhere, the train was late, we cannot return home, the whole country's order will be chaos. Fortunately, China government strengthened its control; the Spring Festival is a year disciplined, orderly.

Discipline is varied. As a scientist, to carry out R & D and innovation theory to prove rigorous; as soldiers, to obey the command, brave and unity; as a politician to strict observation of the political situation, in front of reporters, calm, calmly analyze the situation. People in different occupations, with different aspects of discipline. But the discipline is interlinked. It all has the same characteristics: make your work more rigorous, rigorous. It is said that discipline will hinder innovation and freedom. I think on the contrary, the appropriate discipline although had a constraint on certain things, but because it is bound by the live event that has a negative impact on your career, let your career detours, thus becoming a promoting agent, let you from success further. But discipline is moderate, and too much discipline does hinder your freedom.

Let us look back. Just talk about the learning life we have to experience every day. School, the teacher will teach us the school and outside the discipline. And students tend to feel very tired, always want to be less disciplined. So who's right? I think that this problem for the students of China and the United States have a different answer. Chinese students were taught etiquette thought and life in a prim box, some of the harsh discipline, such as a month no watching TV, the Internet and other, and the one who is to break down; and the student life in the United States than Chinese students and do something to control and need to gradually accept some reasonable discipline and not curb their freedom to grow. Only by reaching the realm of free growth under the restraint of restraint, is the most ideal. For example, when you are tired, you go outside to breathe fresh air. This will be more effective. More in detail, if the discipline is likened to a math reasoning problems, the need for discipline is: prescribed format, the calculation is correct and independent thinking. Do not need discipline is: each letter should be written in the specification, each symbol can not be left, must be in accordance with the teacher's standard format to write.

It is the duty of every one of us to obey the discipline. Let's build up a harmonious society with discipline!

Sample Essay 3

As it is defined in Webster, “discipline” is a way of behaving that shows a willingness to obey rules or orders. And Katharine Hepburn once said, “Without discipline, there’s no life at all.” I can’t agree more with this quotation. As far as I’m concerned, discipline is important and indispensable to every aspect of modern life.

Discipline permeates our life. In school, students have to wear school uniforms instead of casual clothes. In a supermarket, people have to stand in line waiting for payment. In a factory, workers can’t be absent for no reason...

Discipline gives our lives structure and prevents us from making costly mistakes. Examples to illustrate this point can be found everywhere. For instance, if someone crosses the road against the red light, he may be hit by a car and loses his life. Here the man violates the discipline-- the traffic laws. Since discipline is the practice of people to obey rules or orders, it means obedience to the established rules of conduct. Every society has certain rules to control and regulate the life and activity of its member in order that the society as a whole may progress in harmony and peace. If any of these rules are broken, there are troubles and the society suffers.

However, because discipline is associated with strict rules and strong self-control, some people argue that they sacrifice freedom to obey discipline. Is it true?Of course not. In a sense, discipline is freedom. According to Aristotle,“Through discipline comes freedom.” It is because of discipline that everything goes smoothly in our society.The kind of freedom that we get from discipline enables us to live healthily and happily. Without discipline, people are enslaved to their weaknesses and impulses, then the society will be in chaos and offences against law will soar. How can most people’s justified freedom be guaranteed?

Obviously, there is no real freedom without discipline. Likewise, there is no real discipline unless it creates freedom. People don’t need the kind of discipline where someone is put in a prison and told that he can’t do anything except follow the rules. If it is the case, discipline is meaningless to people. The criterion for necessary discipline, I suppose, is that it creates freedom for the majority of people.

Similarly, innovation benefits from discipline. Innovation includes the process by which new ideas are generated and converted into useful products, new technologies and methods. Certain discipline contributes to innovation because established discipline is mostly scientific. To some degree, innovation is the improvement and breakthrough of established rules and it is based on discipline. In other words, innovation is to make discipline more scientific and valid. Too much discipline, on the other hand, do hinder innovation, because new ideas cannot find an outlet among various stiff and rigorous rules. A certain amount of scientific discipline that provides the bases for innovation is necessary.

Different people treat discipline differently. Soldiers get the most strict discipline, to them, all actions are to listen to the command. In scientific research, scientists must follow scientific discipline to make inventions and innovations. Politicians need to obey certain social rules to make their policies accepted by people and the society. We students should also act according to school discipline. Every day teachers emphasize as much discipline as possible, while we students want as little discipline as possible. Indeed, too much discipline might kill students' inspiration and creativity, so it's necessary to achieve a balance between the two. On the one hand, certain discipline is needed to make the school run smoothly and create a good learning environment. On the other hand, students need to be given freedom to grow up and make minor mistakes.

From what's mentioned above, we may naturally draw a conclusion that discipline plays a significant role in civilized society. Without discipline, there won't be a harmonious life and everything will be in disorder. Positive behaviors and habits can only be implemented through discipline and discipline is to bring us real pleasure and freedom.

Sample Essay 4

Discipline seems to be an abstract concept, but actually it can be directly shown in people's lives. The combination of two famous quotes "the first and best victory is to conquer self" and "the best way to conquer self is using self discipline" just shows how vital discipline is to people.

Because of the importance, a famous American actress Katharine Hepburn once said: "Without discipline, there is no life at all". Although there might be some biases existing in this comment, it is true that three main parts of lives: study life, family life, and social life cannot continue without discipline.

First of all, discipline is the foundation of study life. Not only in class study, but also after class study requires students' discipline. Without this self-controlling action, students are likely to chat with others during class thus miss many knowledge. Moreover, when students are staying at home doing homework, it is easier for them to do other things because no one is supervising them, and they cannot finish the amount of work unless they have strong discipline. For instance, recently there is news online about a 15-year-old boy. This boy just entered a famous private high school in America with his fantastic previous grade. However, after he got in the school, he started to play a video game that was introduced by his friend, and he soon got obsessed with it. Afterwards, he began to not listen to the teachers in classes and always daydreamed about the game; he also played the game during every study time at home because of the lack of discipline. This directly caused horrible scores for the first term, but the boy did not care at all. In the end, he even played truant, so the school expelled him after several warnings. Undoubtedly, this boy's study life was totally ruined. In brief, as the example shows, study life cannot be success without discipline, thus students should know how to control themselves.

Secondly, discipline plays a great role in family life. From the aspect of children, as they grow older and older, sometimes they will lie to their parents for some reasons. This is not a positive phenomenon for normal families due to he fact that if the parents find out the truths, the harmonious relationships will break. On the contrary, if the children have self-discipline and they always force themselves to tell the veracities, this situation will only occur in little cases and the family life will not be destroyed. On the other hand, parents should also have discipline to keep the family life going. Adults are often willing to try new things that may be good or bad, thus they should have a sense of self-denial to help them get away from those negative things. Only with this discipline in their minds, they can care about their families and do the right action. Hence discipline is the tool to help maintain families; without it, there is no family life.

Thirdly, discipline cannot be moved away from social life. All the societies consist rules and a majority of people that obey rules, so social life is based on the rule followers. Nevertheless, to become real listeners, people need to sacrifice many personal benefits and spend their time on remembering the directions. Therefore people who are lack of self-controlling ability would not let themselves do such nonreciprocal things. Without the support of the huge number of people, the society will definitely collapse and social life will not exist. About this topic, there is a famous fairy tale related. Once there were two neighboring countries fighting against each other for several years, but nothing came out except the death of people. This was because these two countries had similar power and technologies, so the only way to triumph is by wisdom. After figuring out violence was not working, the king of one country collected thousands of ideas from people. Luckily, he was satisfied with the method to put magic powder everywhere in another country to make their people lose discipline. A week later, the king carried out this plan and it worked surprisingly well. Poor people in the other country started robbing rich people because they did not want to obey the rules any more, rich people began to protest the government for not letting them enjoy the best things, the government officials even stopped working due to the fact that their salaries were too low… This was undisputedly a huge catastrophe to the other country so the country broke down in expectation, and the wise king won the war in the end. Although this fairy tale may contain some unrealistic scenes, the main idea that discipline is necessary to the society is clear and factual. So without discipline, social life will be gone.

To summarize, Katharine Hepburn's statement that "Without discipline, there is no life at all" is appropriate because study life, family life, and social life are all based on discipline. However, this assertion could have been milder due to the fact that exceptions might occur. Next time when Katharine Hepburn gave out a claim, it would be smarter for her to not be so sharp.