

2015-2016 年度美国“数学大联盟杯赛”(中国赛区)初赛

(十、十一、十二年级)

(初赛时间: 2015年11月14日, 考试时间90分钟, 总分300分)

学生诚信协议: 考试期间, 我确定没有就所涉及的问题或结论, 与任何人、用任何方式交流或讨论, 我确定以下的答案均为我个人独立完成的成果, 否则愿接受本次成绩无效的处罚。

如果您同意遵守以上协议请在装订线内签名

一、选择题(每小题10分, 答对加10分, 答错不扣分, 共100分, 请将正确答案A、B、C或者D写在每题后面的圆括号内。)

正确答案填写示例如下:

$$20 - 5 \times 2 = 2 \times \underline{\quad ? \quad} \quad (\text{ A })$$

A) 5 B) 15 C) 25 D) 30

1. If a square has the same area as a circle whose radius is 10, then the side-length of the square is ()
A) $10\sqrt{\pi}$ B) 10π C) $100\sqrt{\pi}$ D) 100π
2. $x^2 - y^2 + x + y =$ ()
A) $(x + y - 1)(x - y)$ B) $(x + y)(x - y - 1)$
C) $(x + y + 1)(x - y)$ D) $(x + y)(x - y + 1)$
3. If $x + y = 25$ and $x^2 - y^2 = 50$. What is the value of xy ? ()
A) 150.25 B) 155.25 C) 175 D) 1250
4. Janet picked a number from 1 to 10 and rolled a die. What is the probability that the sum of the number she picked and the outcome on the die is an even number? ()
A) 1/5 B) 1/4 C) 1/3 D) 1/2
5. Let r be a solution of $x^2 - 7x + 11 = 0$. What is the value of $(r - 3)(r - 4) + (r - 12)(r + 5)$? ()
A) -71 B) -70 C) -69 D) 70
6. Last month the ratio of males to females in Miss Fox's company was 3:4. When 9 new males and 52 new females were employed this month, the new ratio of males to females is now 1/2. How many employees are there now in the company total? ()
A) 68 B) 120 C) 180 D) 240
7. On the quest, a knight rode his horse 30 mph from his house to the castle. After finishing his task, he returned 40 mph from the castle to home. What is his average speed, in mph, of his quest? ()
A) $120/7$ B) $240/7$ C) 35 D) 70
8. Today I want to practice shooting apples with darts. If each dart is used to shoot 3 apples, then when I use up the darts, I will be left with 35 apples; if each dart is used to shoot 4 apples, then when I use up the apples, I will be left with 5 darts.



I have ? apples at the beginning. ()

- A) 51 B) 55 C) 200 D) 240

9. $x/2 = y/3 = z/4$, what is the value of $x:y:z$? ()

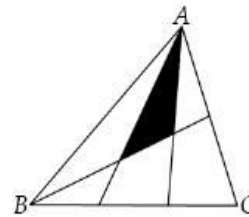
- A) 6:4:3 B) 3:4:6 C) 2:3:4 D) 4:3:2

10. Super Jack and Almighty Jill were doing the 100-mile walk at the same time and same starting point, at constant speeds. Jack took a 5-minute break at the end of every 10 miles; Jill took a 10-minute break at the end of each 20 miles. Jill's speed was $5/8$ of that of Jack. They finished at the same time. How long, in minutes, does the trip take? ()

- A) 53.333 B) 56.667 C) 60.333 D) 60.667

二、填空题(每小题 10 分, 答对加 10 分, 答错不扣分, 共 200 分。)

11. As shown at the right, $\triangle ABC$ has area 180. Two line segments are drawn from A to the trisection points of \overline{BC} , and a line segment is drawn from B to the midpoint of \overline{AC} . What is the area of the shaded triangular region?



Answer: _____.

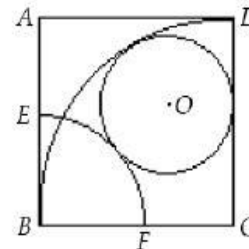
12. In trapezoid $ABCD$, $\overline{AB} \parallel \overline{CD}$, and both $\angle A$ and $\angle B$ are obtuse. What is the area of the trapezoid if $AB = 10$, $BC = 15$, $CD = 24$, and $DA = 13$?

Answer: _____.

13. What ordered pair of positive integers (a, b) satisfies $\sqrt{163 - 56\sqrt{3}} = a\sqrt{3} - b$?

Answer: _____.

14. The area of square $ABCD$ is 144. The respective midpoints of \overline{AB} and \overline{BC} are E and F , as shown. If B is the center of quarter-circle \overline{EF} and C is the center of quarter-circle \overline{BD} , then how long is a radius of the circle centered at O that is tangent to \overline{CD} , \overline{EF} , and \overline{BD} , as shown?



Answer: _____.

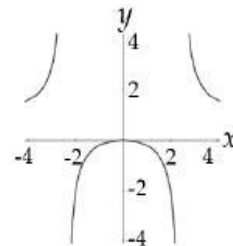
15. From a point interior to an equilateral triangle T , perpendiculars drawn to the sides of T have lengths of 1, 4, and 7. How long is each side of T ?

Answer: _____.

16. When organizing a parade, Professor Brainiac noticed that if the participants tried to arrange themselves into a square, there would be 15 people left over. He determined that the participants could arrange themselves into a rectangle whose width and length differed by 7, with no one left over. How many participants are in the parade?

Answer: _____.

17. The graph of $f(x) = \frac{x^2}{x^2 - 6}$ is shown at the right. How many solutions does $f(f(x)) = 3$ have?



Answer: _____.

18. In a takeout restaurant, there are 10 different food choices. A guest is given 4 boxes to take out the food he/she likes. Each box can only contain one type of food. The types of food in the boxes that the guest take out are not necessarily different. How many ways are there to put the food into 4 boxes?
 Answer: _____.
19. In rectangle $ABCD$, $AB = 8$, and $BC = 5$. When $ABCD$ is revolved around \overline{AB} in a 3-dimensional space, the result is right circular cylinder R . What is the volume of R ?
 Answer: _____.
20. Each student in a certain college dormitory takes at least one of three courses: Art 101, English 101, and Math 101. If 19 students study English, 22 students study math, 22 students study art, 7 students study English and art, 8 students study English and math, and 9 students study art and math, and 3 students study all 3 subjects, then how many students are there in the dormitory?
 Answer: _____.
21. If i represents the imaginary unit, what value of n satisfies $(1 - i)^n = 256$?
 Answer: _____.
22. The 2016 binomial factors $(x - 1)(x - 2)(x - 3) \dots (x - 2015)(x - 2016)$ are multiplied and the product is written as a polynomial P in standard form. What is the sum of all of P 's coefficients, including the constant term?
 Answer: _____.
23. What are both positive integer values of n which satisfy

$$\sin^2\left(\frac{\pi}{6}\right) + \sin^2\left(\frac{2\pi}{6}\right) + \sin^2\left(\frac{3\pi}{6}\right) + \dots + \sin^2\left(\frac{(n-1)\pi}{6}\right) + \sin^2\left(\frac{n\pi}{6}\right) = 2016?$$
 Answer: _____.
24. The first few terms of sequence S are 20, 4, 16, 37, If the sum of the squares of the digits of the n th term of S is the $(n + 1)$ st term of S , what is the 2015th term of S ?
 Answer: _____.
25. For how many of the integers from 100 to 999 inclusive is the product of its tens digit and its hundreds digit equal to its units digit?
 Answer: _____.
26. Four positive numbers, a , b , c , and d , have the property that $\frac{a}{b} = \frac{b}{c} = \frac{c}{d}$ and $a > d$. If $\frac{a-d}{b-c} > x$ for all such a , b , c , and d , what is the maximum value of x ?
 Answer: _____.
27. In a 10000-meter race between the hare and the tortoise, the hare's speed was 5 times that of the tortoise. They started at the same place and at the same time and ran at constant speeds. The hare ran for a while, then went to sleep. The tortoise ran all the time. When the hare awakened, he tried to catch the tortoise but was 100 meters behind when the tortoise crossed the finish line. How far, in meters, did the tortoise run while the hare was sleeping?
 Answer: _____.
28. Two different circles can have n common tangents. What are all possible values of n ?
 Answer: _____.