

# DUKE MATH MEET 2016

## TIEBREAKER ROUND

1. How many ordered triples of integers  $(a, b, c)$  where  $1 \leq a, b, c \leq 10$  are such that for every natural number, the equation  $(a + n)x^2 + (b + 2n)x + c + n = 0$  has at least one real root?
2. Find the smallest integer  $n$  such that we can cut a  $n \times n$  grid into 5 rectangles with distinct side lengths in  $\{1, 2, 3, \dots, 10\}$ . Every value is used exactly once.
3. A plane is flying at constant altitude along a circle of radius 12 miles with center at a point  $A$ . The speed of the aircraft is  $v$ . At some moment in time, a missile is fired at the aircraft from the point  $A$ , which has speed  $v$  and is guided so that its velocity vector always points towards the aircraft. How far does the missile travel before colliding with the aircraft?