Tiebreaker Round

Duke Math Meet

February 15, 2014

Instructions

- The tiebreaker round will consist of three questions. The contestants do not have a time limit.
- The contestants must face the audience. They may not talk, except to ask the judges for clarifications.
- Solutions for tiebreaker round questions should be simplified as much as possible. Rationalize denominators, reduce fractions, etc.
- Contestants will receive ten points for each correct answer and lose one point for each incorrect answer. Ties in total points will be broken first by number of problems solved, then by total time to solve the last problem both contestants solved.

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1. A light beam shines from the origin into the unit square at an angle of θ to one of the sides such that $\tan\theta = \frac{13}{17}$. The light beam is reflected by the sides of the square. How many times does the light beam hit a side of the square before hitting a vertex of the square?

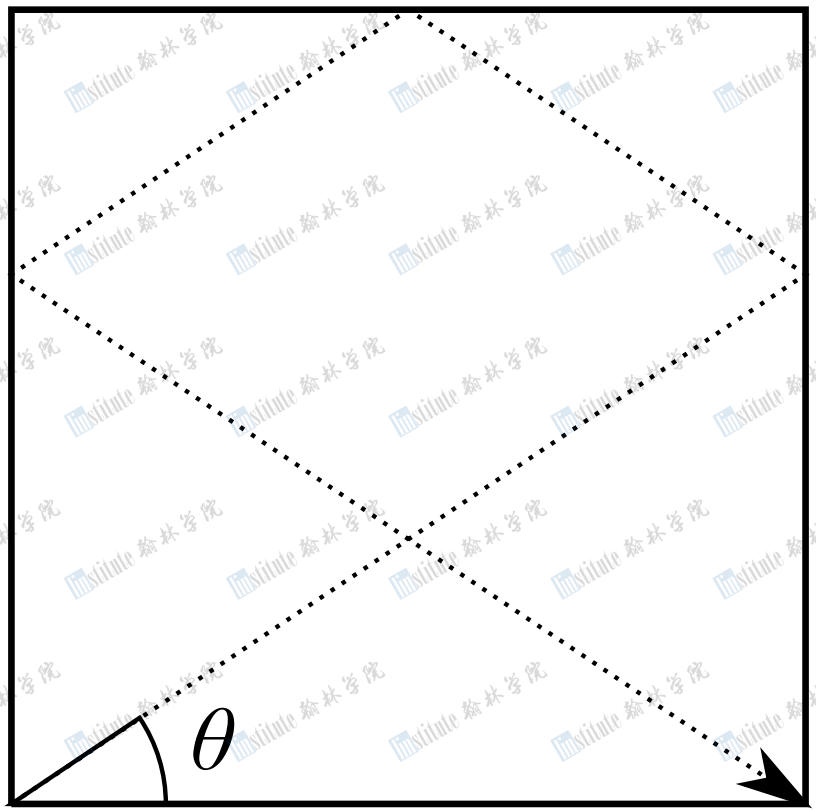

Figure: An example trajectory for $\tan\theta = 2/3$

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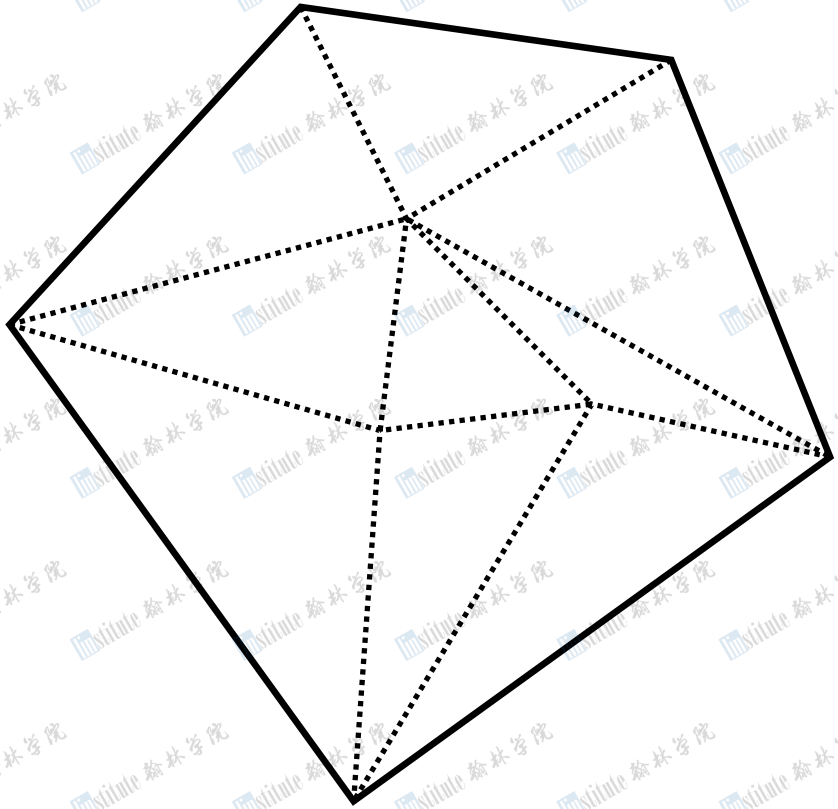
2. Alex is given points $A_1, A_2, \cdots, A_{150}$ in the plane such that no three are collinear and $A_1, A_2, \cdots, A_{100}$ are the vertices of a convex polygon $\mathcal{P}$ containing $A_{101}, A_{102}, \cdots, A_{150}$ in its interior. He proceeds to draw edges A_iA_j such that no two edges intersect (except possibly at their endpoints), eventually dividing $\mathcal{P}$ up into triangles. How many triangles are there?



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3. The polynomial $P(x)$ has the property that $P(1)$, $P(2)$, $P(3)$, $P(4)$, and $P(5)$ are equal to $1, 2, 3, 4, 5$ in some order. How many possibilities are there for the polynomial P, given that the degree of P is strictly less than 4?