## DUKE MATH MEET 2007: RELAY ROUND

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The first and second students may pass as many answers as they wish; however, the third student can only submit answers after three minutes or six minutes, and only the third students's answer will be graded. If a group submits an answer after six minutes then their answer after three minutes, if they gave one, will be discarded. If a group obtains the correct answer after three minutes, they will earn 4 points for their team. If a group obtains the correct answer after six minutes, they will earn 2 points for their team. Therefore, a group should not, for example, submit the same answer after six minutes that they did after three minutes. The moderator will give 15-second warnings before the three minute deadline and the six minute deadline.

## FIRST RELAY ROUND

1A. How many of the following eight integers are divisible by 36?

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0, 3636, 33636, 33663, 64278, 72702, 90000, 91980.

1B. Let k be TNYWR. The following polynomial equation in x has exactly one real solution. What is it?

$$(x-k)^{10} = x^{10}.$$

1C. Let k be TNYWR. How many positive prime numbers are less than or equal to 18k?

## SECOND RELAY ROUND

2A. The following equation has exactly one solution (x, y) where x and y are both positive integers. What is x + y?

$$\frac{x^3}{y} + \frac{y^3}{x} = 288.$$

 $f(x) = x^2 + \frac{k}{x^2 + 1}$ 

2B. Let k be TNYWR. A lattice point is an ordered pair (x, y) such that x and y are both integers. How many lattice points are either inside of or on the boundary of the triangle whose vertices have Withit the start the start is within the the 's coordinates (0, 0), (k, 0), and (0, k)?

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2C. Let k be TNYWR. Find the minimum of the function

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for  $x \in \mathbb{R}$ .

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