1 Problem 1

Problem 1.1. Let a p - coin be a coin that lands on heads with probability p. So, a fair coin is a $\frac{1}{2}$ - coin and a rigged coin that lands on heads 75 percent of the time is a $\frac{3}{4}$ - coin. Joe flips a $\frac{12}{23}$ - coin until it lands heads. What is the expected number of tails that he flips?

Problem 1.2. Let T = TNYWR. T should be of the form $\frac{a}{b}$. Let $c = a \cdot b$. Take the equation

$$4^x + c = y^2.$$

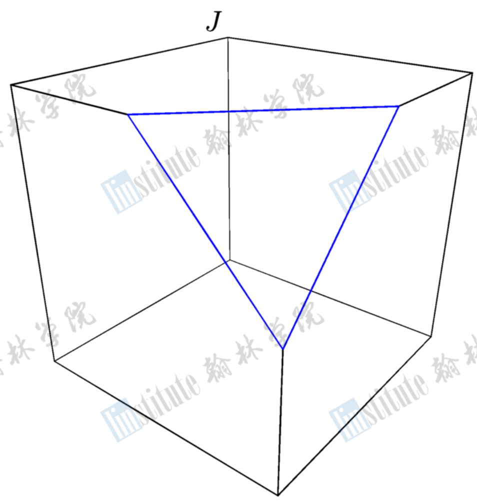
This equation has two integer solutions, (x_1, y_1) and (x_2, y_2). What is $(x_1 + x_2, y_1 + y_2)$?

Problem 1.3. Let T = TNYWR. T should be of the form (a, b). How many lattice points are on the interior of triangle formed by the points $(a, 0)$, $(0, b)$, and (a, b)?

2 Problem 2

Problem 2.1. Triangle ABC has side lengths $AB = 6$, $BC = 8$, and $CA = 10$. Let O be the center of the circumcircle of ABC. If circle Γ passes through points A, B, and O, and intersects segment BC at point D, find BD. Express your answer as a common fraction.

Problem 2.2. Let T = TNYWR, rounded to the nearest integer. Jung is an explorer, and one day finds himself on vertex J of the cube shown below. (One day, Bermuda, an evil demon, decides to mess with Jung and cuts off the vertex opposite the face he currently standing on. So, the cube now becomes a solid with 3 square faces, 3 pentagonal faces, and 1 triangular face. Bermuda haunts the triangle, so Jung now shudders with fear when he thinks about walking on the edges of the triangle. [can be deleted lol]) If Jung takes T steps, how many possible paths can he take without walking on the edges of Bermuda's blue triangle?



Problem 2.3. Let T = TYNWR. Let $f(x) = x^3 + ax^2 + Tx - 90$, where a is an integer. The roots of this polynomial are integers r_1, r_2, and r_3, with $r_1 < r_2 < r_3$. What is the area of the triangle with side lengths $r_1 + 13$, $r_2 + 12$, $r_3 + 11$?