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MIDDLE SCHOOL - PROBLEMS

Copyright ©Titu Andreescu and Jonathan Kane itute # ***

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Problem 1

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Caden, Zoe, Noah, and Sophia shared a pizza. Caden ate 20 percent of the pizza. Zoe ate 50 percent more of the pizza than Caden ate. Noah ate 50 percent more of the pizza than Zoe ate, and Sophia ate the rest itute mark 's PR of the pizza. Find the percentage of the pizza that Sophia ate. itute ## 资本 奶秋

Problem 2

The figure below was made by gluing together 5 non-overlapping congruent squares. The figure has area

45. Find the perimeter of the figure. TUNITH AT # 3 1% 而时间的新林塔梯

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Problem 3

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The Stromquist Comet is visible every 61 years. If the comet is visible in 2017, what is the next leap year Asitute ## # 3 Institute the the '3 stitute the the "B nstitute ## when the comet will be visible?

nstitute Problem 4

The following diagram includes six circles with radius 4, one circle with radius 8, and one circle with radius 而如此他教祥後 而时间推新林塔像

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16. The area of the shaded region is $k\pi$. Find k. -ne mutine # # mstitute ###

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Problem 5

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Find the greatest odd divisor of 160^3

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Problem 6

On a typical morning Aiden gets out of bed, goes through his morning preparation, rides the bus, and walks from the bus stop to work arriving at work 120 minutes after getting out of bed. One morning Aiden got out of bed late, so he rushed through his morning preparation getting onto the bus in half the usual time, the bus ride took 25 percent longer than usual, and he ran from the bus stop to work in half the usual time it takes him to walk arriving at work 96 minutes after he got out of bed. The next morning Aiden got out of bed extra early, leisurely went through his morning preparation taking 25 percent longer than usual to get onto the bus, his bus ride took 25 percent less time than usual, and he walked slowly from the bus stop to work taking 25 percent longer than usual. How many minutes after Aiden got out of bed did he arrive at work that day?

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Problem 7

stitute \$15 Ht 'S 18 Find the number of positive integers less than 100 that are divisors of 300.

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Problem 8

The positive integer m is a multiple of 111, and the positive integer n is a multiple of 31. Their sum is 2017. Find n - m. 加斯林塔梯 · 13. 19% · /3. 1%

Problem 9

In $\triangle ADE$ points B and C are on side \overline{AD} and points F and G are on side \overline{AE} so that $\overline{BG} \parallel \overline{CF} \parallel \overline{DE}$, as shown. The area of $\triangle ABG$ is 36, the area of trapezoid CFED is 144, and AB = CD. Find the area of Milling H 3 12 mutule ## # '& K 面的机机称林塔像 mutule ## # '& P& trapezoid BGFC. astitute the the 'S PR 面动机能称林塔

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Problem 10

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titute # H 'S P stitute ## stitute ## Find the number of rearrangements of the letters in the word **MATHMEET** that begin and end with the same letter such as TAMEMHET.

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Problem 11

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Find the greatest prime divisor of 29! + 33!.

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Problem 12

Let x, y, and z be real numbers such that

 $\begin{array}{rcl}
12x - 9y^2 &=& 7\\
6y - 9z^2 &=& -2\\
12z - 9x^2 &=& 4.
\end{array}$

Find $6x^2 + 9y^2 + 12z^2$.

Find the number of positive integer divisors of 20^{17} that are either perfect squares or perfect cubes.

Problem 14

Let a and b be positive integers such that a + ab = 1443 and ab + b = 1444. Find 10a + b. minitute # # #

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Problem 15

Find the remainder when 7^{7^7} is divided by 1000.

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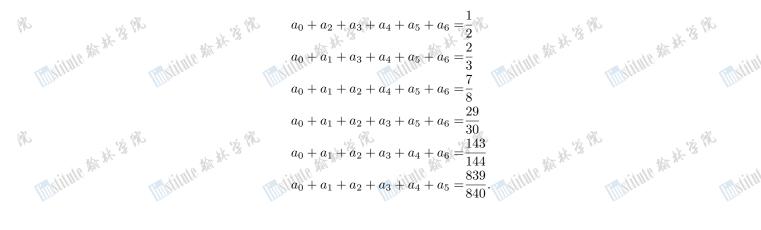
Problem 16

The set of positive real numbers x that satisfy $2|x^2 - 9| \le 9|x|$ is an interval [m, M]. Find 10m + M. **Problem 17**

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Let a_0, a_1, \dots, a_6 be real numbers such that $a_0 + a_1 + \dots + a_6 = 1$ and





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ente 新来送祭 加加斯林资料 Find the sum of all values of a + b, where (a, b) is an ordered pair of positive integers and $a^2 + \sqrt{2017 - b^2}$ is a perfect square.

Problem 20

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A right circular cone has a height equal to three times its base radius and has volume 1. The cone is inscribed inside a sphere as charge TV inscribed inside a sphere as shown. The volume of the sphere is $\frac{m}{n}$, where m and n are relatively prime positive integers. Find m + n.