



The CENTRE for EDUCATION  
in MATHEMATICS and COMPUTING  
*cemc.uwaterloo.ca*

# *Fermat Contest*

(Grade 11)

**Tuesday, February 26, 2019**  
(in North America and South America)

**Wednesday, February 27, 2019**  
(outside of North America and South America)



UNIVERSITY OF  
**WATERLOO**

**Time:** 60 minutes

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Calculating devices are allowed, provided that they do not have any of the following features: (i) internet access, (ii) the ability to communicate with other devices, (iii) information previously stored by students (such as formulas, programs, notes, etc.), (iv) a computer algebra system, (v) dynamic geometry software.

## **Instructions**

1. Do not open the Contest booklet until you are told to do so.
2. You may use rulers, compasses and paper for rough work.
3. Be sure that you understand the coding system for your response form. If you are not sure, ask your teacher to clarify it. All coding must be done with a pencil, preferably HB. Fill in circles completely.
4. On your response form, print your school name and city/town in the box in the upper right corner.
5. **Be certain that you code your name, age, grade, and the Contest you are writing in the response form. Only those who do so can be counted as eligible students.**
6. This is a multiple-choice test. Each question is followed by five possible answers marked **A, B, C, D, and E**. Only one of these is correct. After making your choice, fill in the appropriate circle on the response form.
7. Scoring: Each correct answer is worth 5 in Part A, 6 in Part B, and 8 in Part C.  
There is *no penalty* for an incorrect answer.  
Each unanswered question is worth 2, to a maximum of 10 unanswered questions.
8. Diagrams are *not* drawn to scale. They are intended as aids only.
9. When your supervisor tells you to begin, you will have *sixty* minutes of working time.
10. You may not write more than one of the Pascal, Cayley and Fermat Contests in any given year.

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*Do not discuss the problems or solutions from this contest online for the next 48 hours.*

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*The name, grade, school and location, and score range of some top-scoring students will be published on our website, [cemc.uwaterloo.ca](http://cemc.uwaterloo.ca). In addition, the name, grade, school and location, and score of some top-scoring students may be shared with other mathematical organizations for other recognition opportunities.*

Scoring: There is *no penalty* for an incorrect answer.

Each unanswered question is worth 2, to a maximum of 10 unanswered questions.

**Part A: Each correct answer is worth 5.**

1. What is the remainder when 14 is divided by 5?

(A) 0                      (B) 1                      (C) 2                      (D) 3                      (E) 4

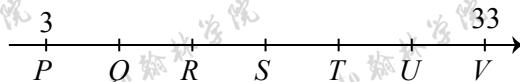
2. Which of the following is equal to  $20(x + y) - 19(y + x)$  for all values of  $x$  and  $y$ ?

(A)  $39x + 39y$     (B)  $x + y$                       (C)  $39x + y$                       (D)  $x + 39y$                       (E)  $19x - 18y$

3. The value of  $8 - \frac{6}{4 - 2}$  is

(A) 5                      (B) 1                      (C)  $\frac{7}{2}$                       (D)  $\frac{17}{2}$                       (E) 7

4. In the diagram, point  $P$  is on the number line at 3 and  $V$  is at 33. The number line between 3 and 33 is divided into six equal parts by the points  $Q, R, S, T, U$ .



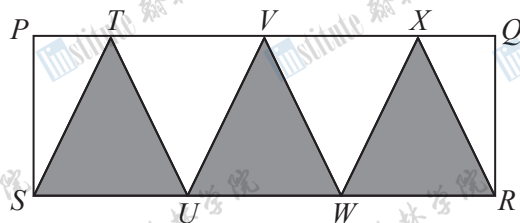
What is the sum of the lengths of  $PS$  and  $TV$ ?

(A) 25                      (B) 23                      (C) 24                      (D) 21                      (E) 27

5. Mike rides his bicycle at a constant speed of 30 km/h. How many kilometres does Mike travel in 20 minutes?

(A) 5                      (B) 6                      (C) 1.5                      (D) 15                      (E) 10

6. In the diagram,  $PQRS$  is a rectangle. Also,  $\triangle STU$ ,  $\triangle UVW$  and  $\triangle WXR$  are congruent.



What fraction of the area of rectangle  $PQRS$  is shaded?

(A)  $\frac{3}{7}$                       (B)  $\frac{3}{4}$                       (C)  $\frac{1}{2}$                       (D)  $\frac{3}{5}$                       (E)  $\frac{2}{3}$

7. The town of Cans is north of the town of Ernie. The town of Dundee is south of Cans but north of Ernie. The town of Arva is south of the town of Blythe and is north of both Dundee and Cans. The town that is the most north is

(A) Arva                      (B) Blythe                      (C) Cans                      (D) Dundee                      (E) Ernie

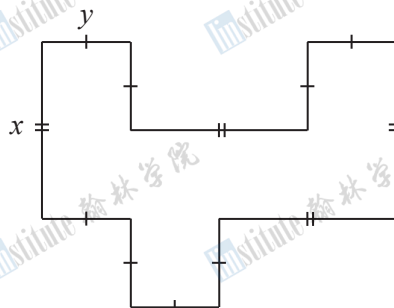
8. The product  $8 \times 48 \times 81$  is divisible by  $6^k$ . The largest possible integer value of  $k$  is  
 (A) 7 (B) 6 (C) 5 (D) 4 (E) 3
9. The average of  $\frac{1}{8}$  and  $\frac{1}{6}$  is  
 (A)  $\frac{1}{9}$  (B)  $\frac{7}{24}$  (C)  $\frac{1}{5}$  (D)  $\frac{1}{48}$  (E)  $\frac{7}{48}$
10. The digits 2, 3, 5, 7, and 8 can be used, each exactly once, to form many five-digit integers. Of these integers,  $N$  is the one that is as close as possible to 30 000. What is the tens digit of  $N$ ?  
 (A) 2 (B) 5 (C) 3 (D) 8 (E) 7

**Part B: Each correct answer is worth 6.**

11. Line  $\ell$  is perpendicular to the line with equation  $y = x - 3$ . Line  $\ell$  has the same  $x$ -intercept as the line with equation  $y = x - 3$ . The  $y$ -intercept of line  $\ell$  is  
 (A)  $-3$  (B)  $\frac{1}{3}$  (C) 3 (D)  $-1$  (E) 0
12. The first part of the Genius Quiz has 30 questions and the second part has 50 questions. Alberto answered exactly 70% of the 30 questions in the first part correctly. He answered exactly 40% of the 50 questions in the second part correctly. The percentage of all of the questions on the quiz that Alberto answered correctly is closest to  
 (A) 59 (B) 57 (C) 48 (D) 51 (E) 41
13. Tanis looked at her watch and noticed that, at that moment, it was  $8x$  minutes after 7:00 a.m. and  $7x$  minutes before 8:00 a.m. for some value of  $x$ . What time was it at that moment?  
 (A) 7:08 a.m. (B) 7:40 a.m. (C) 7:32 a.m. (D) 7:36 a.m. (E) 7:31 a.m.
14. The letters A, B, C, D, and E are to be placed in the grid so that each of these letters appears exactly once in each row and exactly once in each column. Which letter will go in the square marked with \*?  
 (A) A (B) B (C) C  
 (D) D (E) E

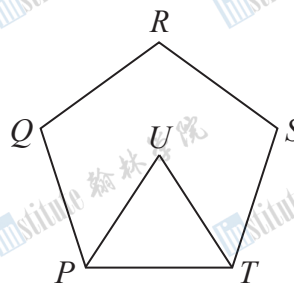
A				E
		C	A	
E		B	C	
	*			
B			D	

16. In the diagram, each line segment has length  $x$  or  $y$ . Also, each pair of adjacent sides is perpendicular.



If the area of the figure is 252 and  $x = 2y$ , the perimeter of the figure is

- (A) 96 (B) 192 (C) 288 (D) 72 (E) 168
17. The five sides of a regular pentagon are all equal in length. Also, all interior angles of a regular pentagon have the same measure. In the diagram,  $PQRST$  is a regular pentagon and  $\triangle PUT$  is equilateral. The measure of obtuse  $\angle QUS$  is
- (A)  $172^\circ$  (B)  $168^\circ$  (C)  $170^\circ$   
(D)  $176^\circ$  (E)  $174^\circ$



18. How many 7-digit positive integers are made up of the digits 0 and 1 only, and are divisible by 6?
- (A) 16 (B) 11 (C) 21 (D) 10 (E) 33
19. The function  $f$  has the properties that  $f(1) = 6$  and  $f(2x + 1) = 3f(x)$  for every integer  $x$ . What is the value of  $f(63)$ ?
- (A) 4374 (B) 1162 (C) 54 (D) 1458 (E) 486
20. The vertices of an equilateral triangle lie on a circle with radius 2. The area of the triangle is
- (A)  $3\sqrt{3}$  (B)  $4\sqrt{3}$  (C)  $6\sqrt{3}$  (D)  $5\sqrt{3}$  (E)  $2\sqrt{3}$

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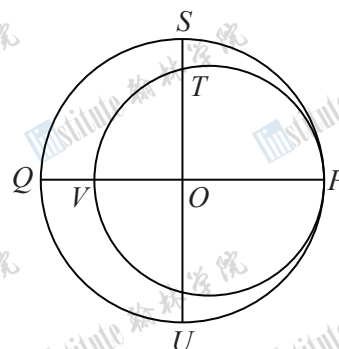
**Part C: Each correct answer is worth 8.**

21. In the multiplication shown, each of  $P$ ,  $Q$ ,  $R$ ,  $S$ , and  $T$  is a digit. The value of  $P + Q + R + S + T$  is
- (A) 14 (B) 20 (C) 16  
(D) 17 (E) 13

$$\begin{array}{r} P Q R S T 4 \\ \times \quad \quad \quad 4 \\ \hline 4 P Q R S T \end{array}$$



22. In the diagram, two circles touch at  $P$ . Also,  $QP$  and  $SU$  are perpendicular diameters of the larger circle that intersect at  $O$ . Point  $V$  is on  $QP$  and  $VP$  is a diameter of the smaller circle. The smaller circle intersects  $SU$  at  $T$ , as shown. If  $QV = 9$  and  $ST = 5$ , what is the sum of the lengths of the diameters of the two circles?



- (A) 50                      (B) 91                      (C) 41  
(D) 82                      (E) 100

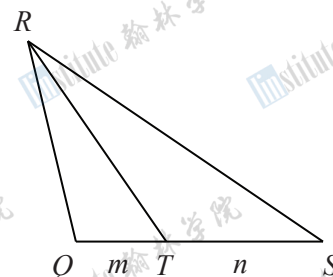
23. How many positive integers  $n$  with  $n \leq 100$  can be expressed as the sum of four or more consecutive positive integers?

- (A) 64                      (B) 63                      (C) 66                      (D) 65                      (E) 69

24. Consider the quadratic equation  $x^2 - (r + 7)x + r + 87 = 0$  where  $r$  is a real number. This equation has two distinct real solutions  $x$  which are both negative exactly when  $p < r < q$ , for some real numbers  $p$  and  $q$ . The value of  $p^2 + q^2$  is

- (A) 7618                      (B) 698                      (C) 1738                      (D) 7508                      (E) 8098

25. In  $\triangle QRS$ , point  $T$  is on  $QS$  with  $\angle QRT = \angle SRT$ . Suppose that  $QT = m$  and  $TS = n$  for some integers  $m$  and  $n$  with  $n > m$  and for which  $n + m$  is a multiple of  $n - m$ . Suppose also that the perimeter of  $\triangle QRS$  is  $p$  and that the number of possible integer values for  $p$  is  $m^2 + 2m - 1$ . The value of  $n - m$  is



- (A) 4                      (B) 1                      (C) 3  
(D) 2                      (E) 5



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**For students...**

Thank you for writing the 2019 Fermat Contest! Each year, more than 260 000 students from more than 80 countries register to write the CEMC's Contests.

Encourage your teacher to register you for the Hypatia Contest which will be written in April.

Visit our website [cemc.uwaterloo.ca](http://cemc.uwaterloo.ca) to find

- More information about the Hypatia Contest
- Free copies of past contests
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- Information about careers in and applications of mathematics and computer science

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Visit our website [cemc.uwaterloo.ca](http://cemc.uwaterloo.ca) to

- Register your students for the Fryer, Galois and Hypatia Contests which will be written in April
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