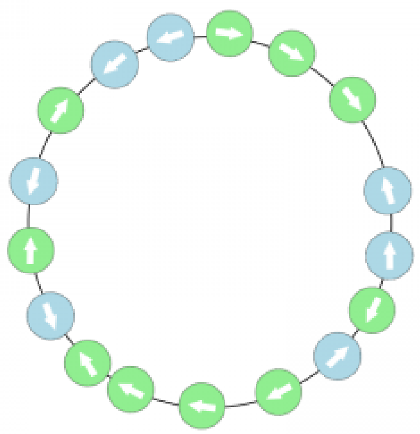
Problems which at first seem to lack sufficient information:

1. Each of three teams, A, B, and C, enters one contestant in each event of a track meet. In each event, p points are awarded to first place, q points for second place and r points for third. p, q, and r are positive integers, and $p > q > r$. Team A won the first event and finished with a total of 9 points; B also finished with a total of 9 points; and C won the meet with 22 points. How many events were there and which team won the second event?
2. The circle inscribed in a triangle with perimeter 17cm. has a radius of 2cm. What is the area of the circle? What is the area of the triangle?
3. Two-thirds of the way across a narrow railroad bridge, Willy Gope hears a train coming towards him. He knows that the train's speed is 45 miles per hour and that he has just enough time to reach safety at either end of the bridge. How fast can Willy run?
4. From the third term on, each term of the sequence of real numbers $a_1, a_2, \ldots, a_{10}$ is the sum of the preceding two terms; that is, $a_n = a_{n-1} + a_{n-2}$, for $n = 3, 4, 5, \ldots 10$. If $a_7 = 17$, what is the sum of all the ten terms?

5. Lewis Carroll posed the following problem: Two travelers spent from 2 o'clock until 9 walking along a level road up a hill and home again; their pace on the level being x miles per hour, uphill y mph, and downhill 2y mph. Find the distance walked. In Carroll's formulation x and y were given integers. Making use of the additional assumption that the original problem was solvable, find the distance walked.

Bouncing Electrons:

At time t=0, seventeen electrons are situated at various points on a circular track, seventeen miles in circumference. Some of the electrons are moving along the track in a clockwise direction and others are moving in a counterclockwise direction, but all of the electrons are moving at the same speed: 17 miles per second. Whenever two electrons collide, each instantly reverses direction, losing no time or velocity.



- Prove that, at time t=1 second, each of the positions initially occupied by an electron will again be occupied by an electron traveling in the same direction as the electron initially at that location.
- Prove that there will be a time when all 17 electrons will have simultaneously returned to their initial positions and directions.

Paul Halmos' Wife Shakes Hands: *(this problem was posed by Professor Paul R. Halmos of Indiana University)*

My wife and I were invited to a party recently, a party attended by four other couples. Some of the 10 knew some of the others and some did not, and some were polite and some were not. As a result, a certain amount of handshaking took place in an unpredictable way, subject only to two obvious conditions: No one shook his or her own hand and no husband shook his wife's hand. When it was all over, I became curious, and I went around the party asking each person: "How many hands did you shake? …And you? …And you?" What answers could I have received?

Conceivably, some people could have said None, and others could have given me any number between 1 and 8 inclusive. That's right isn't it? Since self-handshakes and spouse-handshakes were ruled out, eight is the maximum number of hands that any one of the party of 10 could have shaken.

I asked nine people (everybody, including my own wife), and each answer could have been any one of the nine numbers 0 to 8 inclusive. I was interested to note, and I hereby report, that the nine different people gave me nine different answers: someone said “none,” someone said “one,” and so on and, finally, someone said “eight.” Next morning, I told the story to my colleagues, and I challenged them, on the basis of the information just given, to tell me how many hands my wife shook.

Inefficiently Emptying the Warehouse:

A large warehouse contains a number of boxes, each of which is labeled with a natural number (positive integer); there may be boxes with the same number. Every minute a box is removed from the warehouse. If the box has a label greater than 1, it may be replaced by arbitrarily many boxes with smaller labels. For instance, when a box labeled 17 is removed, it might be replaced by 153 boxes labeled 3, 1017 boxes labeled 8, 289000 boxes labeled 9, 3 boxes labeled 14, and $17!$ boxes labeled 16. Show that the warehouse will be emptied within a finite period of time. You may want to begin with the special case in which all the boxes initially the warehouse are labeled with 1’s and 2’s.

Sums of Two or More Consecutive Positive Integers:

Some numbers can be expressed as the sum of 2 or more consecutive positive integers (sometimes in more than 1 way):

$17=8+9$,

$51=25+\_\_\_=\_\_\_+\_\_\_+\_\_\_$,

$34=\_\_\_\_\_\_\_\_\_\_\_\_\_$.

Other numbers, including 1, 2, and 8, can not be expressed as the sum of 2 or more consecutive positive integers. Determine, with proof, the set of numbers which can not be expressed as the sum of two or more consecutive positive integers.

Sums of Three or More Consecutive Positive Integers:

Some numbers can be expressed as the sum of three or more consecutive positive integers (sometimes in more than one way):

$30=\_\_\_+\_\_\_+\_\_\_=\_\_\_+\_\_\_+\_\_\_+\_\_\_=\_\_\_+\_\_\_+\_\_\_+\_\_\_+\_\_\_$,

$33=10+11+12=3+4+5+6+7+8$,

$34=\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_$,

$35=5+6+7+8+9=\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_$.

Other numbers, including 1, 2, 3, 31, and 32, can not be expressed as the sum of three or more consecutive positive integers. Determine, with proof, the set of numbers which can not be expressed as the sum of three or more consecutive positive integers.