2007 AMC 8 Problems

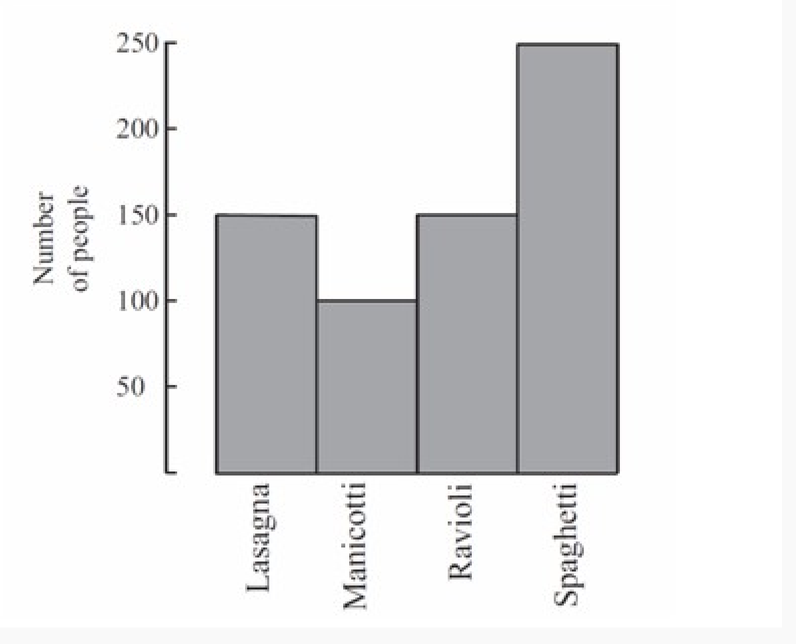
Problem 1

Theresa's parents have agreed to buy her tickets to see her favorite band if she spends an average of 10 hours per week helping around the house for 6 weeks. For the first 5 weeks she helps around the house for 8, 11, 7, 12 and 10 hours. How many hours must she work for the final week to earn the tickets?

$\mathrm{(A)}\ 9 \qquad\mathrm{(B)}\ 10 \qquad\mathrm{(C)}\ 11 \qquad\mathrm{(D)}\ 12 \qquad\mathrm{(E)}\ 13$

Problem 2

650 students were surveyed about their pasta preferences. The choices were lasagna, manicotti, ravioli and spaghetti. The results of the survey are displayed in the bar graph. What is the ratio of the number of students who preferred spaghetti to the number of students who preferred manicotti?



$\mathrm{(A)} \frac{2}{5} \qquad \mathrm{(B)} \frac{1}{2} \qquad \mathrm{(C)} \frac{5}{4} \qquad \mathrm{(D)} \frac{5}{3} \qquad \mathrm{(E)} \frac{5}{2}$

Problem 3

What is the sum of the two smallest prime factors of 250?

$\mathrm{(A)}\ 2 \qquad\mathrm{(B)}\ 5 \qquad\mathrm{(C)}\ 7 \qquad\mathrm{(D)}\ 10 \qquad\mathrm{(E)}\ 12$

Problem 4

A haunted house has six windows. In how many ways can Georgie the Ghost enter the house by one window and leave by a different window?

$\textbf{(A)}\ 12 \qquad \textbf{(B)}\ 15 \qquad \textbf{(C)}\ 18 \qquad \textbf{(D)}\ 30 \qquad \textbf{(E)}\ 36$

Problem 5

Chandler wants to buy a $\$500$ mountain bike. For his birthday, his grandparents send him $\$50$, his aunt sends him $\$35$ and his cousin gives him $\$15$. He earns $\$16$ per week for his paper route. He will use all of his birthday money and all of the money he earns from his paper route. In how many weeks will he be able to buy the mountain bike?

$\textbf{(A)}\ 24 \qquad \textbf{(B)}\ 25 \qquad \textbf{(C)}\ 26 \qquad \textbf{(D)}\ 27 \qquad \textbf{(E)}\ 28$

Problem 6

The average cost of a long-distance call in the USA in 1985 was 41 cents per minute, and the average cost of a long-distance call in the USA in 2005 was 7 cents per minute. Find the approximate percent decrease in the cost per minute of a long- distance call.

$\textbf{(A)}\ 7 \qquad \textbf{(B)}\ 17 \qquad \textbf{(C)}\ 34 \qquad \textbf{(D)}\ 41 \qquad \textbf{(E)}\ 80$

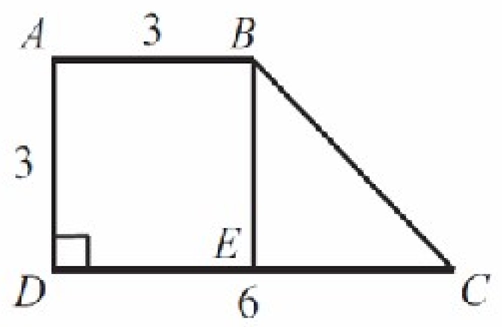
Problem 7

The average age of 5 people in a room is 30 years. An 18-year-old person leaves the room. What is the average age of the four remaining people?

$\textbf{(A)}\ 25 \qquad \textbf{(B)}\ 26 \qquad \textbf{(C)}\ 29 \qquad \textbf{(D)}\ 33 \qquad \textbf{(E)}\ 36$

Problem 8

In trapezoid $ABCD$, AD is perpendicular to DC, AD = AB = 3, and DC = 6. In addition, E is on DC, and BE is parallel to AD. Find the area of $\triangle BEC$.



(A) 3 (B) 4.5 (C) 6 (D) 9 (E) 18

Problem 9

To complete the grid below, each of the digits 1 through 4 must occur once in each row and once in each column. What number will occupy the lower right-hand square?

1		2	
2	3		
			4

(A) 1 (B) 2 (C) 3 (D) 4 (E) cannot be determined

Problem 10

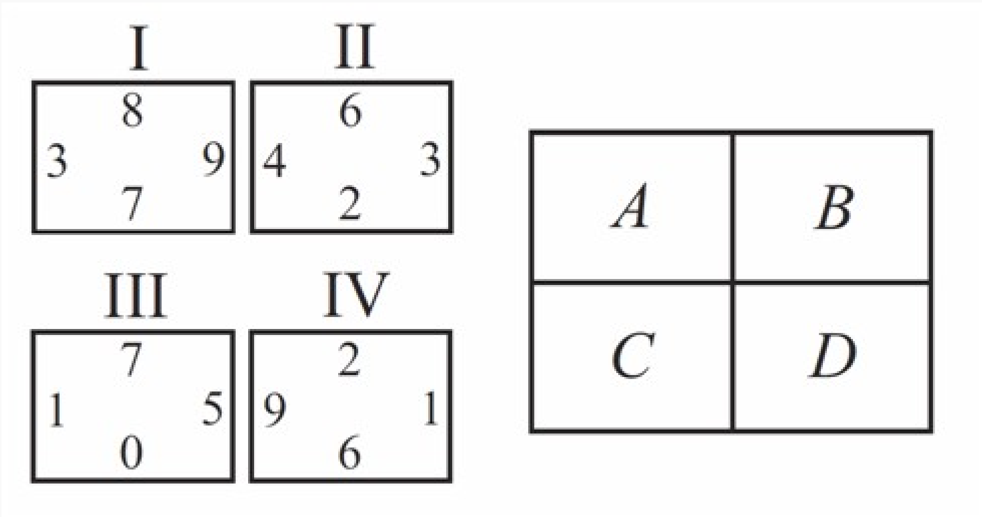
For any positive integer n, define $\boxed{n}$ to be the sum of the positive factors of n.

For example, $\boxed{6} = 1 + 2 + 3 + 6 = 12$. Find $\boxed{\boxed{11}}$.

$\textbf{(A)}\ 13 \qquad \textbf{(B)}\ 20 \qquad \textbf{(C)}\ 24 \qquad \textbf{(D)}\ 28 \qquad \textbf{(E)}\ 30$

Problem 11

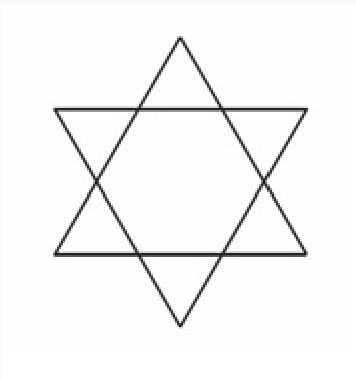
Tiles I, II, III and IV are translated so one tile coincides with each of the rectangles A, B, C and D. In the final arrangement, the two numbers on any side common to two adjacent tiles must be the same. Which of the tiles is translated to Rectangle C?



$\textbf{(A)}\ I \qquad \textbf{(B)}\ II \qquad \textbf{(C)}\ III \qquad \textbf{(D)}\ IV \qquad \textbf{(E)}$ cannot be determined

Problem 12

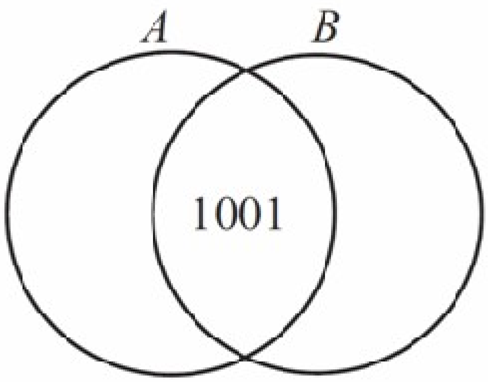
A unit hexagram is composed of a regular hexagon of side length 1 and its 6 equilateral triangular extensions, as shown in the diagram. What is the ratio of the area of the extensions to the area of the original hexagon?



$\textbf{(A)}\ 1:1 \qquad \textbf{(B)}\ 6:5 \qquad \textbf{(C)}\ 3:2 \qquad \textbf{(D)}\ 2:1 \qquad \textbf{(E)}\ 3:1$

Problem 13

Sets A and B, shown in the Venn diagram, have the same number of elements. Their union has 2007 elements and their intersection has 1001 elements. Find the number of elements in A.



$\text{(A)}\ 503 \qquad \text{(B)}\ 1006 \qquad \text{(C)}\ 1504 \qquad \text{(D)}\ 1507 \qquad \text{(E)}\ 1510$

Problem 14

The base of isosceles $\triangle ABC$ is 24 and its area is 60. What is the length of one of the congruent sides?

$\text{(A)}\ 5 \qquad \text{(B)}\ 8 \qquad \text{(C)}\ 13 \qquad \text{(D)}\ 14 \qquad \text{(E)}\ 18$

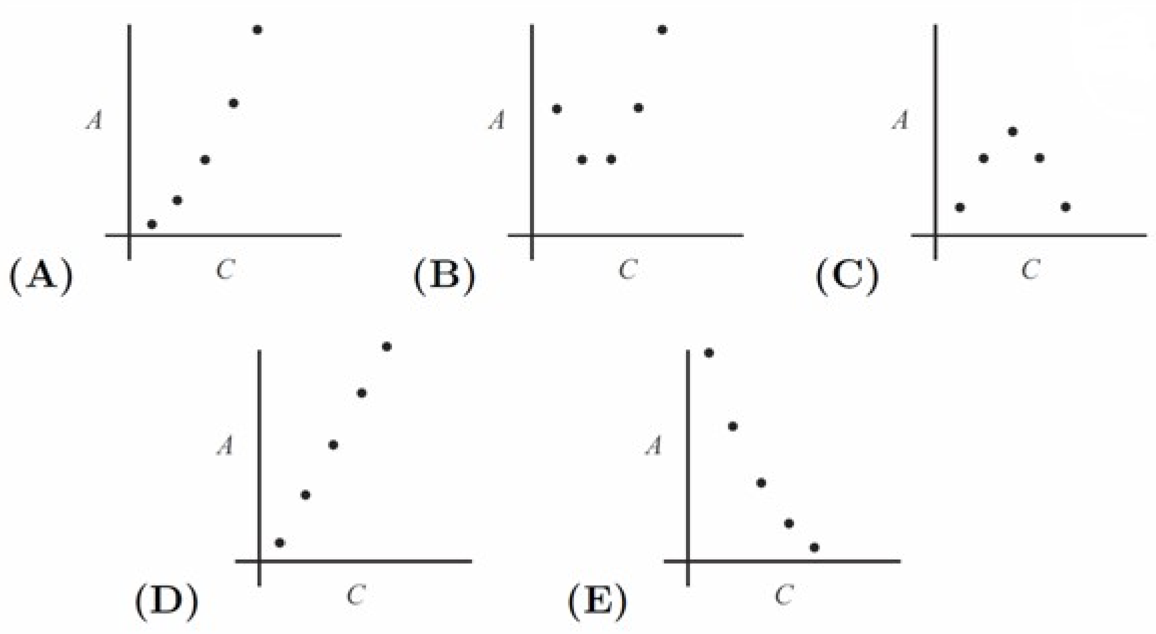
Problem 15

Let a, b and c be numbers with $0 < a < b < c$. Which of the following is impossible?

$\text{(A)}\ a + c < b \qquad \text{(B)}\ a \cdot b < c \qquad \text{(C)}\ a + b < c \qquad \text{(D)}\ a \cdot c < b \qquad \text{(E)} \frac{b}{c} = a$

Problem 16

Amanda draws five circles with radii $1, 2, 3, 4$ and 5. Then for each circle she plots the point (C, A), where C is its circumference and $A$is its area. Which of the following could be her graph?



Problem 17

A mixture of 30 liters of paint is 25% red tint, 30% yellow tint and 45% water. Five liters of yellow tint are added to the original mixture. What is the percent of yellow tint in the new mixture?

$\textbf{(A)}\ 25 \qquad \textbf{(B)}\ 35 \qquad \textbf{(C)}\ 40 \qquad \textbf{(D)}\ 45 \qquad \textbf{(E)}\ 50$

Problem 18

The product of the two 99-digit numbers

$303{,}030{,}303{,}\ldots{,}030{,}303$ and $505{,}050{,}505{,}\ldots{,}050{,}505$

has thousands digit A and units digit B. What is the sum of A and B?

$\textbf{(A)}\ 3 \qquad \textbf{(B)}\ 5 \qquad \textbf{(C)}\ 6 \qquad \textbf{(D)}\ 8 \qquad \textbf{(E)}\ 10$

Problem 19

Pick two consecutive positive integers whose sum is less than 100. Square both of those integers and then find the difference of the squares. Which of the following could be the difference?

$\text{(A)}\ 2 \qquad \text{(B)}\ 64 \qquad \text{(C)}\ 79 \qquad \text{(D)}\ 96 \qquad \text{(E)}\ 131$

Problem 20

Before district play, the Unicorns had won 45% of their basketball games. During district play, they won six more games and lost two, to finish the season having won half their games. How many games did the Unicorns play in all?

$\text{(A)}\ 48 \qquad \text{(B)}\ 50 \qquad \text{(C)}\ 52 \qquad \text{(D)}\ 54 \qquad \text{(E)}\ 60$

Problem 21

Two cards are dealt from a deck of four red cards labeled A, B, C, D and four green cards labeled A, B, C, D. A winning pair is two of the same color or two of the same letter. What is the probability of drawing a winning pair?

$\text{(A)}\frac{2}{7} \qquad \text{(B)}\frac{3}{8} \qquad \text{(C)}\frac{1}{2} \qquad \text{(D)}\frac{4}{7} \qquad \text{(E)}\frac{5}{8}$

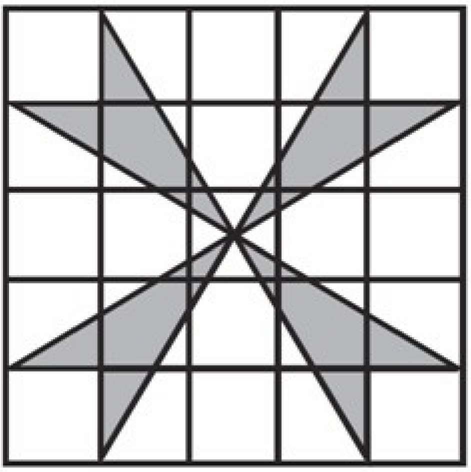
Problem 22

A lemming sits at a corner of a square with side length 10 meters. The lemming runs 6.2 meters along a diagonal toward the opposite corner. It stops, makes a 90 degree right turn and runs 2 more meters. A scientist measures the shortest distance between the lemming and each side of the square. What is the average of these four distances in meters?

$\text{(A)}\ 2 \qquad \text{(B)}\ 4.5 \qquad \text{(C)}\ 5 \qquad \text{(D)}\ 6.2 \qquad \text{(E)}\ 7$

Problem 23

What is the area of the shaded pinwheel shown in the 5 x 5 grid?



(A) 4 (B) 6 (C) 8 (D) 10 (E) 12

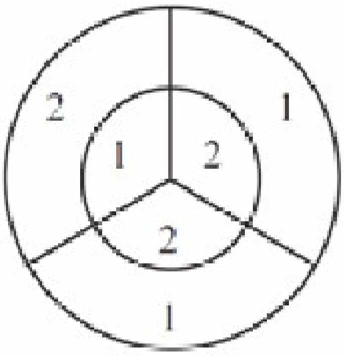
Problem 24

A bag contains four pieces of paper, each labeled with one of the digits "1, 2, 3" or "4", with no repeats. Three of these pieces are drawn, one at a time without replacement, to construct a three-digit number. What is the probability that the three-digit number is a multiple of 3?

(A) $\frac{1}{4}$ (B) $\frac{1}{3}$ (C) $\frac{1}{2}$ (D) $\frac{2}{3}$ (E) $\frac{3}{4}$

Problem 25

On the dart board shown in the Figure, the outer circle has radius 6 and the inner circle has radius 3. Three radii divide each circle into three congruent regions, with point values shown. The probability that a dart will hit a given region is proportional to the area of the region. When two darts hit this board, the score is the sum of the point values in the regions. What is the probability that the score is odd?



(A) $\frac{17}{36}$ (B) $\frac{35}{72}$ (C) $\frac{1}{2}$ (D) $\frac{37}{72}$ (E) $\frac{19}{36}$