# 1954 AHSME Problems

## Problem 1

The square of $5-\sqrt{y^2-25}$ is:

$\textbf{(A)}\ y^2-5\sqrt{y^2-25} \qquad \textbf{(B)}\ -y^2 \qquad \textbf{(C)}\ y^2 \\ \textbf{(D)}\ (5-y)^2 \qquad \textbf{(E)}\ y^2-10\sqrt{y^2-25}$

## Problem 2

The equation $\frac{2x^2}{x-1}-\frac{2x+7}{3}+\frac{4-6x}{x-1}+1=0$ can be transformed by eliminating fractions to the equation $x^2-5x+4=0$. The roots of the latter equation are $4$ and $1$. Then the roots of the first equation are:

$\textbf{(A)}\ 4\text{ and }1 \qquad \textbf{(B)}\ \text{only }1 \qquad \textbf{(C)}\ \text{only }4 \qquad \textbf{(D)}\ \text{neither }4\text{ nor }1 \qquad \textbf{(E)}\ 4\text{ and some other root}$

## Problem 3

If $x$ varies as the cube of $y$, and $y$ varies as the fifth root of $z$, then $x$ varies as the nth power of $z$, where n is:

$\textbf{(A)}\ \frac{1}{15} \qquad \textbf{(B)}\ \frac{5}{3} \qquad \textbf{(C)}\ \frac{3}{5} \qquad \textbf{(D)}\ 15 \qquad \textbf{(E)}\ 8$

## Problem 4

If the Highest Common Divisor of $6432$ and $132$ is diminished by $8$, it will equal:

$\textbf{(A)}\ -6 \qquad \textbf{(B)}\ 6 \qquad \textbf{(C)}\ -2 \qquad \textbf{(D)}\ 3 \qquad \textbf{(E)}\ 4$

## Problem 5

A regular hexagon is inscribed in a circle of radius $10$ inches. Its area is:

$\textbf{(A)}\ 150\sqrt{3} \text{ sq. in.} \qquad \textbf{(B)}\ \text{150 sq. in.} \qquad \textbf{(C)}\ 25\sqrt{3}\text{ sq. in.}\qquad \textbf{(D)}\ \text{600 sq. in.}\qquad \textbf{(E)}\ 300\sqrt{3}\text{ sq. in.}$

## Problem 6

The value of $\frac{1}{16}a^0+\left(\frac{1}{16a}\right)^0-\left(64^{-\frac{1}{2}}\right)-(-32)^{-\frac{4}{5}}$ is:

$\textbf{(A)}\ 1\frac{13}{16} \qquad \textbf{(B)}\ 1\frac{3}{16} \qquad \textbf{(C)}\ 1 \qquad \textbf{(D)}\ \frac{7}{8}\qquad \textbf{(E)}\ \frac{1}{16}$

## Problem 7

A housewife saved $\$2.50$ in buying a dress on sale. If she spent $\$25$ for the dress, she saved about:

$\textbf{(A)}\ 8\% \qquad \textbf{(B)}\ 9\% \qquad \textbf{(C)}\ 10\% \qquad \textbf{(D)}\ 11\% \qquad \textbf{(E)}\ 12\%$

## Problem 8

The base of a triangle is twice as long as a side of a square and their areas are the same. Then the ratio of the altitude of the triangle to the side of the square is:

**(A)** $\frac{1}{4}$ **(B)** $\frac{1}{2}$ **(C)** 1 **(D)** 2 **(E)** 4

## Problem 9

A point $P$ is outside a circle and is $13$ inches from the center. A secant from $P$ cuts the circle at $Q$ and $R$ so that the external segment of the secant $PQ$ is $9$ inches and $QR$ is $7$ inches. The radius of the circle is:

**(A)** 3" **(B)** 4" **(C)** 5" **(D)** 6" **(E)** 7"

## Problem 10

The sum of the numerical coefficients in the expansion of the binomial $(a+b)^6$ is:

**(A)** 32 **(B)** 16 **(C)** 64 **(D)** 48 **(E)** 7

## Problem 11

A merchant placed on display some dresses, each with a marked price. He then posted a sign "$\frac{1}{3}$ off on these dresses." The cost of the dresses was $\frac{3}{4}$ of the price at which he actually sold them. Then the ratio of the cost to the marked price was:

**(A)** $\frac{1}{2}$ **(B)** $\frac{1}{3}$ **(C)** $\frac{1}{4}$ **(D)** $\frac{2}{3}$ **(E)** $\frac{3}{4}$

## Problem 12

The solution of the equations

$2x - 3y = 7$
$4x - 6y = 20$

is:

**(A)** $x = 18, y = 12$ **(B)** $x = 0, y = 0$ **(C)** There is no solution
**(D)** There are an unlimited number of solutions **(E)** $x = 8, y = 5$

## Problem 13

A quadrilateral is inscribed in a circle. If angles are inscribed in the four arcs cut off by the sides of the quadrilateral, their sum will be:

**(A)** $180^\circ$ **(B)** $540^\circ$ **(C)** $360^\circ$ **(D)** $450^\circ$ **(E)** $1080^\circ$

## Problem 14

When simplified $\sqrt{1+\left(\frac{x^4-1}{2x^2}\right)^2}$ equals:

**(A)** $\frac{x^4+2x^2-1}{2x^2}$ **(B)** $\frac{x^4-1}{2x^2}$ **(C)** $\frac{\sqrt{x^2+1}}{2}$
**(D)** $\frac{x^2}{\sqrt{2}}$ **(E)** $\frac{x^2}{2}+\frac{1}{2x^2}$

## Problem 15

$\log 125$ equals:

$\textbf{(A)}\ 100\log 1.25 \qquad \textbf{(B)}\ 5\log 3 \qquad \textbf{(C)}\ 3\log 25$
$\textbf{(D)}\ 3 - 3\log 2 \qquad \textbf{(E)}\ (\log 25)(\log 5)$

## Problem 16

If $f(x)=5x^2-2x-1$, then $f(x+h)-f(x)$ equals:

$\textbf{(A)}\ 5h^2-2h \qquad \textbf{(B)}\ 10xh-4x+2 \qquad \textbf{(C)}\ 10xh-2x-2$
$\textbf{(D)}\ h(10x+5h-2) \qquad \textbf{(E)}\ 3h$

## Problem 17

The graph of the function $f(x)=2x^3-7$ goes:

$\textbf{(A)}$ up to the right and down to the left
$\textbf{(B)}$ down to the right and up to the left
$\textbf{(C)}$ up to the right and up to the left
$\textbf{(D)}$ down to the right and down to the left
$\textbf{(E)}$ none of these ways.

## Problem 18

Of the following sets, the one that includes all values of $x$ which will satisfy $2x-3>7-x$ is:

$\textbf{(A)}\ x>4 \qquad \textbf{(B)}\ x<\frac{10}{3} \qquad \textbf{(C)}\ x=\frac{10}{3} \qquad \textbf{(D)}\ x>\frac{10}{3} \qquad \textbf{(E)}\ x<0$

## Problem 19

If the three points of contact of a circle inscribed in a triangle are joined, the angles of the resulting triangle:

$\textbf{(A)}$ are always equal to $60^\circ$
$\textbf{(B)}$ are always one obtuse angle and two unequal acute angles
$\textbf{(C)}$ are always one obtuse angle and two equal acute angles
$\textbf{(D)}$ are always acute angles
$\textbf{(E)}$ are always unequal to each other

## Problem 20

The equation $x^3+6x^2+11x+6=0$ has:

$\textbf{(A)}$ no negative real roots $\qquad \textbf{(B)}$ no positive real roots $\qquad \textbf{(C)}$ no real roots
$\textbf{(D)}$ 1 positive and 2 negative roots $\qquad \textbf{(E)}$ 2 positive and 1 negative root

## Problem 21

The roots of the equation $2\sqrt{x}+2x^{-\frac{1}{2}}=5$ can be found by solving:

$\textbf{(A)}\ 16x^2-92x+1=0 \qquad \textbf{(B)}\ 4x^2-25x+4=0 \qquad \textbf{(C)}\ 4x^2-17x+4=0$
$\textbf{(D)}\ 2x^2-21x+2=0 \qquad \textbf{(E)}\ 4x^2-25x-4=0$

## Problem 22

The expression $\frac{2x^2-x}{(x+1)(x-2)}-\frac{4+x}{(x+1)(x-2)}$ cannot be evaluated for $x=-1$ or $x=2$, since division by zero is not allowed. For other values of $x$:

$\textbf{(A)}$ The expression takes on many different values.
$\textbf{(B)}$ The expression has only the value 2.
$\textbf{(C)}$ The expression has only the value 1.
$\textbf{(D)}$ The expression always has a value between $-1$ and $+2$.
$\textbf{(E)}$ The expression has a value greater than 2 or less than $-1$.

## Problem 23

If the margin made on an article costing $C$ dollars and selling for $S$ dollars is $M=\frac{1}{n}C$, then the margin is given by:

$\textbf{(A)}\ M=\frac{1}{n-1}S \qquad \textbf{(B)}\ M=\frac{1}{n}S \qquad \textbf{(C)}\ M=\frac{n}{n+1}S$
$\textbf{(D)}\ M=\frac{1}{n+1}S \qquad \textbf{(E)}\ M=\frac{n}{n-1}S$

## Problem 24

The values of $k$ for which the equation $2x^2-kx+x+8=0$ will have real and equal roots are:

$\textbf{(A)}\ 9\text{ and }-7 \qquad \textbf{(B)}\ \text{only }-7 \qquad \textbf{(C)}\ 9\text{ and }7$
$\textbf{(D)}\ -9\text{ and }-7 \qquad \textbf{(E)}\ \text{only }9$

## Problem 25

The two roots of the equation $a(b-c)x^2+b(c-a)x+c(a-b)=0$ are 1 and:

$\textbf{(A)}\ \frac{b(c-a)}{a(b-c)} \qquad \textbf{(B)}\ \frac{a(b-c)}{c(a-b)} \qquad \textbf{(C)}\ \frac{a(b-c)}{b(c-a)} \qquad \textbf{(D)}\ \frac{c(a-b)}{a(b-c)} \qquad \textbf{(E)}\ \frac{c(a-b)}{b(c-a)}$

## Problem 26

The straight line $\overline{AB}$ is divided at $C$ so that $AC=3CB$. Circles are described on $\overline{AC}$ and $\overline{CB}$ as diameters and a common tangent meets $AB$ produced at $D$. Then $BD$ equals:

$\textbf{(A)}\ \text{diameter of the smaller circle}$
$\textbf{(B)}\ \text{radius of the smaller circle}$
$\textbf{(C)}\ \text{radius of the larger circle}$
$\textbf{(D)}\ CB\sqrt{3}$
$\textbf{(E)}\ \text{the difference of the two radii}$

## Problem 27

A right circular cone has for its base a circle having the same radius as a given sphere. The volume of the cone is one-half that of the sphere. The ratio of the altitude of the cone to the radius of its base is:

$\textbf{(A)}\ \frac{1}{1} \qquad \textbf{(B)}\ \frac{1}{2} \qquad \textbf{(C)}\ \frac{2}{3} \qquad \textbf{(D)}\ \frac{2}{1} \qquad \textbf{(E)}\ \sqrt{\frac{5}{4}}$

## Problem 28

If $\frac{m}{n}=\frac{4}{3}$ and $\frac{r}{t}=\frac{9}{14}$, the value of $\frac{3mr-nt}{4nt-7mr}$ is:

$\textbf{(A)}\ -5\frac{1}{2} \qquad \textbf{(B)}\ -\frac{11}{14} \qquad \textbf{(C)}\ -1\frac{1}{4} \qquad \textbf{(D)}\ \frac{11}{14} \qquad \textbf{(E)}\ -\frac{2}{3}$

## Problem 29

If the ratio of the legs of a right triangle is $1:2$, then the ratio of the corresponding segments of the hypotenuse made by a perpendicular upon it from the vertex is:

$\textbf{(A)}\ 1:4 \qquad \textbf{(B)}\ 1:\sqrt{2} \qquad \textbf{(C)}\ 1:2 \qquad \textbf{(D)}\ 1:\sqrt{5} \qquad \textbf{(E)}\ 1:5$

## Problem 30

$A$ and $B$ together can do a job in $2$ days; $B$ and $C$ can do it in four days; and $A$ and $C$ in $2\frac{2}{5}$ days. The number of days required for A to do the job alone is:

$\textbf{(A)}\ 1 \qquad \textbf{(B)}\ 3 \qquad \textbf{(C)}\ 6 \qquad \textbf{(D)}\ 12 \qquad \textbf{(E)}\ 2.8$

## Problem 31

In $\triangle ABC$, $AB = AC$, $\angle A = 40^\circ$. Point $O$ is within the triangle with $\angle OBC \cong \angle OCA$. The number of degrees in $\angle BOC$ is:

$\textbf{(A)}\ 110^{\circ} \qquad \textbf{(B)}\ 35^{\circ} \qquad \textbf{(C)}\ 140^{\circ} \qquad \textbf{(D)}\ 55^{\circ} \qquad \textbf{(E)}\ 70^{\circ}$

## Problem 32

The factors of $x^4+64$ are:

$\textbf{(A)}\ (x^2+8)^2 \qquad \textbf{(B)}\ (x^2+8)(x^2-8) \qquad \textbf{(C)}\ (x^2+2x+4)(x^2-8x+16)$
$\textbf{(D)}\ (x^2-4x+8)(x^2-4x-8) \qquad \textbf{(E)}\ (x^2-4x+8)(x^2+4x+8)$

## Problem 33

A bank charges $\$6$ for a loan of $\$120$. The borrower receives $\$114$ and repays the loan in $12$ easy installments of $\$10$ a month. The interest rate is approximately:

$\textbf{(A)}\ 5\% \qquad \textbf{(B)}\ 6\% \qquad \textbf{(C)}\ 7\% \qquad \textbf{(D)}\ 9\% \qquad \textbf{(E)}\ 15\%$

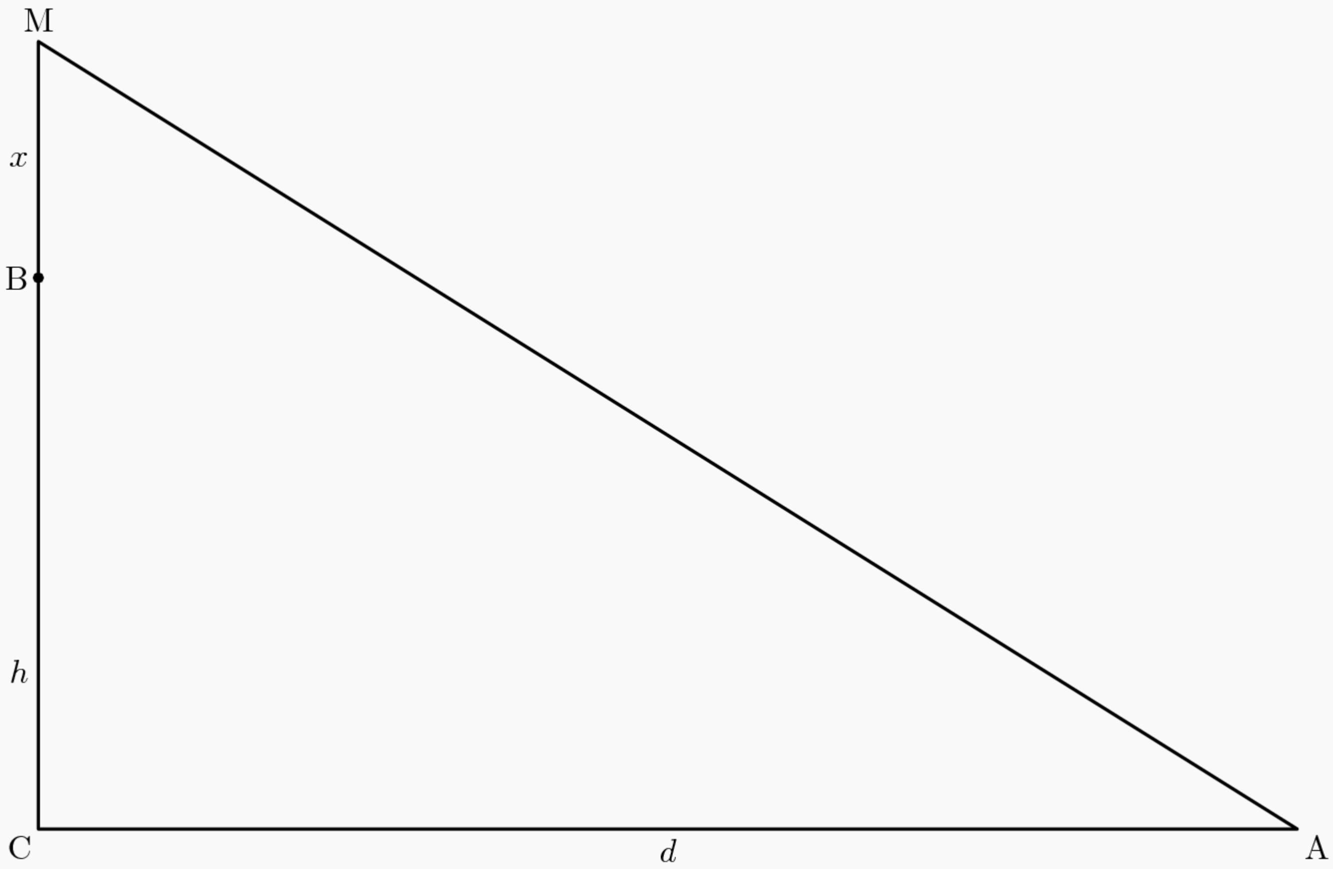
## Problem 34

The fraction $\frac{1}{3}$:

$\textbf{(A)}\ \text{equals } 0.33333333 \qquad \textbf{(B)}\ \text{is less than } 0.33333333 \text{ by } \frac{1}{3\cdot 10^8}$
$\textbf{(C)}\ \text{is less than } 0.33333333 \text{ by } \frac{1}{3\cdot 10^9}$
$\textbf{(D)}\ \text{is greater than } 0.33333333 \text{ by } \frac{1}{3\cdot 10^8}$
$\textbf{(E)}\ \text{is greater than } 0.33333333 \text{ by } \frac{1}{3\cdot 10^9}$

## Problem 35

In the right triangle shown the sum of the distances $BM$ and $MA$ is equal to the sum of the distances $BC$ and $CA$. If $MB = x, CB = h$, and $CA = d$, then $x$ equals:




**(A)** $\frac{hd}{2h+d}$ **(B)** $d-h$ **(C)** $\frac{1}{2}d$ **(D)** $h+d-\sqrt{2d}$ **(E)** $\sqrt{h^2+d^2}-h$

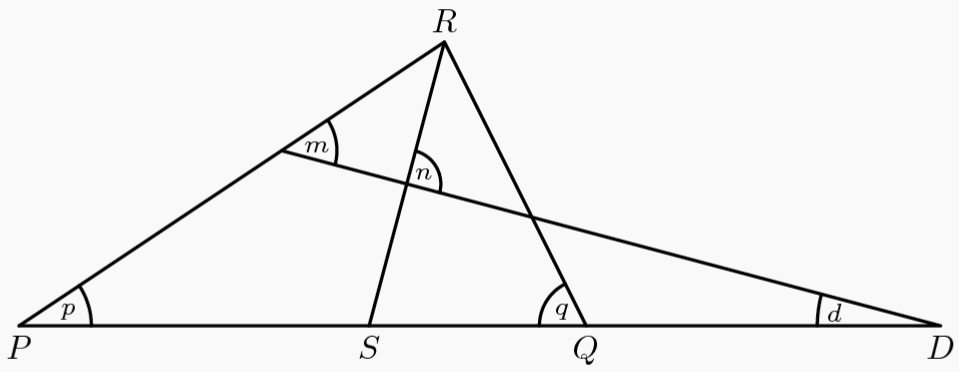
## Problem 36

A boat has a speed of $15$ mph in still water. In a stream that has a current of $5$ mph it travels a certain distance downstream and returns. The ratio of the average speed for the round trip to the speed in still water is:

**(A)** $\frac{5}{4}$ **(B)** $\frac{1}{1}$ **(C)** $\frac{8}{9}$ **(D)** $\frac{7}{8}$ **(E)** $\frac{9}{8}$

## Problem 37

Given $\triangle PQR$ with $\overline{RS}$ bisecting $\angle R$, $PQ$ extended to $D$ and $\angle n$ a right angle, then:




**(A)** $\angle m = \frac{1}{2}(\angle p - \angle q)$ **(B)** $\angle m = \frac{1}{2}(\angle p + \angle q)$

**(C)** $\angle d = \frac{1}{2}(\angle q + \angle p)$ **(D)** $\angle d = \frac{1}{2}\angle m$ **(E)** none of these is correct

## Problem 38

If $\log 2 = .3010$ and $\log 3 = .4771$, the value of $x$ when $3^{x+3} = 135$ is approximately:

**(A)** $5$ **(B)** $1.47$ **(C)** $1.67$ **(D)** $1.78$ **(E)** $1.63$

## Problem 39

The locus of the midpoint of a line segment that is drawn from a given external point $P$ to a given circle with center $O$ and radius $r$, is:

**(A)** a straight line perpendicular to $\overline{PO}$
**(B)** a straight line parallel to $\overline{PO}$
**(C)** a circle with center $P$ and radius $r$
**(D)** a circle with center at the midpoint of $\overline{PO}$ and radius $2r$
**(E)** a circle with center at the midpoint $\overline{PO}$ and radius $\frac{1}{2}r$

## Problem 40

If $\left(a+\frac{1}{a}\right)^2=3$, then $a^3+\frac{1}{a^3}$ equals:

**(A)** $\frac{10\sqrt{3}}{3}$ **(B)** $3\sqrt{3}$ **(C)** $0$ **(D)** $7\sqrt{7}$ **(E)** $6\sqrt{3}$

## Problem 41

The sum of all the roots of $4x^3-8x^2-63x-9=0$ is:

**(A)** $8$ **(B)** $2$ **(C)** $-8$ **(D)** $-2$ **(E)** $0$

## Problem 42

Consider the graphs of $(1)\qquad y=x^2-\frac{1}{2}x+2$ and $(2)\qquad y=x^2+\frac{1}{2}x+2$ on the same set of axis. These parabolas are exactly the same shape. Then:

**(A)** the graphs coincide.
**(B)** the graph of (1) is lower than the graph of (2).
**(C)** the graph of (1) is to the left of the graph of (2).
**(D)** the graph of (1) is to the right of the graph of (2).
**(E)** the graph of (1) is higher than the graph of (2).

## Problem 43

The hypotenuse of a right triangle is $10$ inches and the radius of the inscribed circle is $1$ inch. The perimeter of the triangle in inches is:

**(A)** $15$ **(B)** $22$ **(C)** $24$ **(D)** $26$ **(E)** $30$

## Problem 44

A man born in the first half of the nineteenth century was $x$ years old in the year $x^2$. He was born in:

**(A)** $1849$ **(B)** $1825$ **(C)** $1812$ **(D)** $1836$ **(E)** $1806$

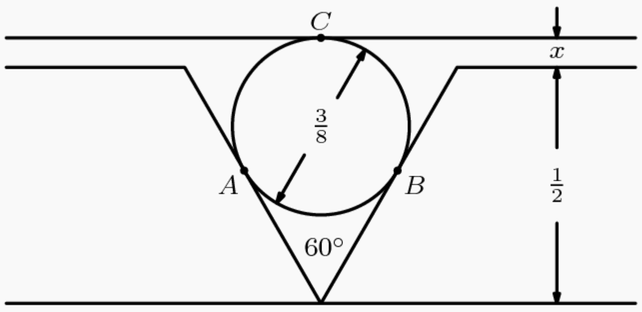
## Problem 45

In a rhombus, $ABCD$, line segments are drawn within the rhombus, parallel to diagonal $BD$, and terminated in the sides of the rhombus. A graph is drawn showing the length of a segment as a function of its distance from vertex $A$. The graph is:

**(A)** A straight line passing through the origin.
**(B)** A straight line cutting across the upper right quadrant.
**(C)** Two line segments forming an upright V.
**(D)** Two line segments forming an inverted V.
**(E)** None of these.

## Problem 46

In the diagram, if points $A, B$ and $C$ are points of tangency, then $x$ equals:




$\textbf{(A)}\ \frac{3}{16}"\qquad\textbf{(B)}\ \frac{1}{8}"\qquad\textbf{(C)}\ \frac{1}{32}"\qquad\textbf{(D)}\ \frac{3}{32}"\qquad\textbf{(E)}\ \frac{1}{16}"$

## Problem 47

At the midpoint of line segment $AB$ which is $p$ units long, a perpendicular $MR$ is erected with length $q$ units. An arc is described from $R$ with a radius equal to $\frac{1}{2}AB$, meeting $AB$ at $T$. Then $AT$ and $TB$ are the roots of:

$\textbf{(A)}\ x^2+px+q^2=0$
$\textbf{(B)}\ x^2-px+q^2=0$
$\textbf{(C)}\ x^2+px-q^2=0$
$\textbf{(D)}\ x^2-px-q^2=0$
$\textbf{(E)}\ x^2-px+q=0$

## Problem 48

A train, an hour after starting, meets with an accident which detains it a half hour, after which it proceeds at $\frac{3}{4}$ of its former rate and arrives $3\tfrac{1}{2}$ hours late. Had the accident happened $90$ miles farther along the line, it would have arrived only $3$ hours late. The length of the trip in miles was:

$\textbf{(A)}\ 400\qquad\textbf{(B)}\ 465\qquad\textbf{(C)}\ 600\qquad\textbf{(D)}\ 640\qquad\textbf{(E)}\ 550$

## Problem 49

The difference of the squares of two odd numbers is always divisible by $8$. If $a>b$, and $2a+1$ and $2b+1$ are the odd numbers, to prove the given statement we put the difference of the squares in the form:

$\textbf{(A)}\ (2a+1)^2-(2b+1)^2$
$\textbf{(B)}\ 4a^2-4b^2+4a-4b$
$\textbf{(C)}\ 4[a(a+1)-b(b+1)]$
$\textbf{(D)}\ 4(a-b)(a+b+1)$
$\textbf{(E)}\ 4(a^2+a-b^2-b)$

## Problem 50

The times between $7$ and $8$ o'clock, correct to the nearest minute, when the hands of a clock will form an angle of $84^{\circ}$ are:

$\textbf{(A)}\ \text{7: 23 and 7: 53}\qquad\textbf{(B)}\ \text{7: 20 and 7: 50}\qquad\textbf{(C)}\ \text{7: 22 and 7: 53}$
$\textbf{(D)}\ \text{7: 23 and 7: 52}\qquad\textbf{(E)}\ \text{7: 21 and 7: 49}$