1996 AHSME Problems

Problem 1

The addition below is incorrect. What is the largest digit that can be changed to make the addition correct?

$$\begin{array}{rrrrr} & & 6 & 4 & 1 \\ & & 8 & 5 & 2 \\ + & & 9 & 7 & 3 \\ \hline & 2 & 4 & 5 & 6 \end{array}$$

$\text{(A)}\ 4 \qquad \text{(B)}\ 5 \qquad \text{(C)}\ 6 \qquad \text{(D)}\ 7 \qquad \text{(E)}\ 8$

Problem 2

Each day Walter gets 3 dollars for doing his chores or 5 dollars for doing them exceptionally well. After 10 days of doing his chores daily, Walter has received a total of 36 dollars. On how many days did Walter do them exceptionally well?

$\text{(A)}\ 3 \qquad \text{(B)}\ 4 \qquad \text{(C)}\ 5 \qquad \text{(D)}\ 6 \qquad \text{(E)}\ 7$

Problem 3

$\frac{(3!)!}{3!} =$

$\text{(A)}\ 1 \qquad \text{(B)}\ 2 \qquad \text{(C)}\ 6 \qquad \text{(D)}\ 40 \qquad \text{(E)}\ 120$

Problem 4

Six numbers from a list of nine integers are $7, 8, 3, 5, 9$ and 5. The largest possible value of the median of all nine numbers in this list is

$\text{(A)}\ 5 \qquad \text{(B)}\ 6 \qquad \text{(C)}\ 7 \qquad \text{(D)}\ 8 \qquad \text{(E)}\ 9$

Problem 5

Given that $0 < a < b < c < d$, which of the following is the largest?

$\text{(A)}\ \frac{a+b}{c+d} \qquad \text{(B)}\ \frac{a+d}{b+c} \qquad \text{(C)}\ \frac{b+c}{a+d} \qquad \text{(D)}\ \frac{b+d}{a+c} \qquad \text{(E)}\ \frac{c+d}{a+b}$

Problem 6

If $f(x) = x^{(x+1)}(x+2)^{(x+3)}$, then $f(0) + f(-1) + f(-2) + f(-3) =$

$\text{(A)}\ -\frac{8}{9} \qquad \text{(B)}\ 0 \qquad \text{(C)}\ \frac{8}{9} \qquad \text{(D)}\ 1 \qquad \text{(E)}\ \frac{10}{9}$

Problem 7

A father takes his twins and a younger child out to dinner on the twins' birthday. The restaurant charges 4.95 for the father and 0.45 for each year of a child's age, where age is defined as the age at the most recent birthday. If the bill is 9.45, which of the following could be the age of the youngest child?

$\text{(A)}\ 1 \qquad \text{(B)}\ 2 \qquad \text{(C)}\ 3 \qquad \text{(D)}\ 4 \qquad \text{(E)}\ 5$

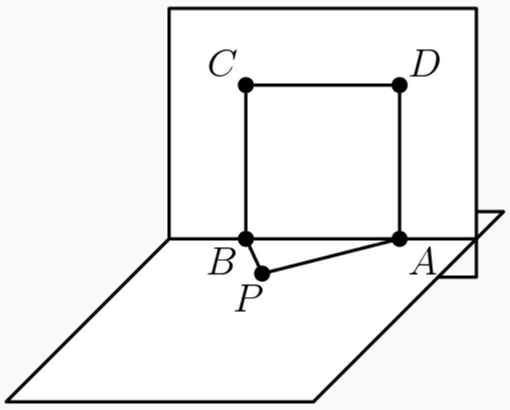
Problem 8

If $3 = k \cdot 2^r$ and $15 = k \cdot 4^r$, then $r =$

(A) $-\log_2 5$ (B) $\log_5 2$ (C) $\log_{10} 5$ (D) $\log_2 5$ (E) $\frac{5}{2}$

Problem 9

Triangle PAB and square $ABCD$ are in perpendicular planes. Given that $PA = 3, PB = 4$ and $AB = 5$, what is PD?



(A) 5 (B) $\sqrt{34}$ (C) $\sqrt{41}$ (D) $2\sqrt{13}$ (E) 8

Problem 10

How many line segments have both their endpoints located at the vertices of a given cube?

(A) 12 (B) 15 (C) 24 (D) 28 (E) 56

Problem 11

Given a circle of radius 2, there are many line segments of length 2 that are tangent to the circle at their midpoints. Find the area of the region consisting of all such line segments.

(A) $\frac{\pi}{4}$ (B) $4-\pi$ (C) $\frac{\pi}{2}$ (D) π (E) 2π

Problem 12

A function f from the integers to the integers is defined as follows:

$$f(x) = \begin{cases} n+3 & \text{if n is odd} \\ n/2 & \text{if n is even} \end{cases}$$

Suppose k is odd and $f(f(f(k))) = 27$. What is the sum of the digits of k?

(A) 3 **(B)** 6 **(C)** 9 **(D)** 12 **(E)** 15

Problem 13

Sunny runs at a steady rate, and Moonbeam runs m times as fast, where m is a number greater than 1. If Moonbeam gives Sunny a head start of h meters, how many meters must Moonbeam run to overtake Sunny?

(A) hm (B) $\frac{h}{h+m}$ (C) $\frac{h}{m-1}$ (D) $\frac{hm}{m-1}$ (E) $\frac{h+m}{m-1}$

Problem 14

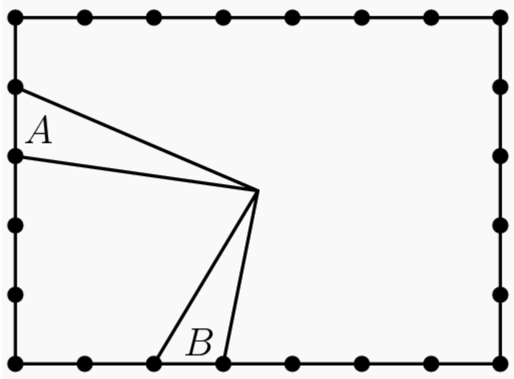
Let $E(n)$ denote the sum of the even digits of n. For example, $E(5681) = 6+8 = 14$.

Find $E(1)+E(2)+E(3)+\cdots+E(100)$

(A) 200 (B) 360 (C) 400 (D) 900 (E) 2250

Problem 15

Two opposite sides of a rectangle are each divided into n congruent segments, and the endpoints of one segment are joined to the center to form triangle A. The other sides are each divided into m congruent segments, and the endpoints of one of these segments are joined to the center to form triangle B. [See figure for $n = 5, m = 7$.] What is the ratio of the area of triangle A to the area of triangle B?



(A) 1 (B) m/n (C) n/m (D) $2m/n$ (E) $2n/m$

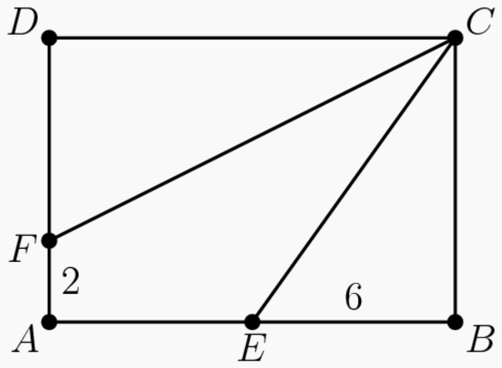
Problem 16

A fair standard six-sided dice is tossed three times. Given that the sum of the first two tosses equal the third, what is the probability that at least one "2" is tossed?

(A) $\frac{1}{6}$ (B) $\frac{91}{216}$ (C) $\frac{1}{2}$ (D) $\frac{8}{15}$ (E) $\frac{7}{12}$

Problem 17

In rectangle $ABCD$, angle C is trisected by $\overline{CF}$ and $\overline{CE}$, where E is on $\overline{AB}$, F is on $\overline{AD}$, $BE = 6$ and $AF = 2$. Which of the following is closest to the area of the rectangle $ABCD$?



(A) 110 (B) 120 (C) 130 (D) 140 (E) 150

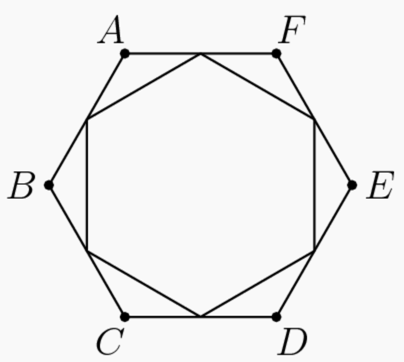
Problem 18

A circle of radius 2 has center at $(2,0)$. A circle of radius 1 has center at $(5,0)$. A line is tangent to the two circles at points in the first quadrant. Which of the following is closest to the y-intercept of the line?

(A) $\sqrt{2}/4$ (B) $8/3$ (C) $1+\sqrt{3}$ (D) $2\sqrt{2}$ (E) 3

Problem 19

The midpoints of the sides of a regular hexagon $ABCDEF$ are joined to form a smaller hexagon. What fraction of the area of $ABCDEF$ is enclosed by the smaller hexagon?



(A) $\frac{1}{2}$ (B) $\frac{\sqrt{3}}{3}$ (C) $\frac{2}{3}$ (D) $\frac{3}{4}$ (E) $\frac{\sqrt{3}}{2}$

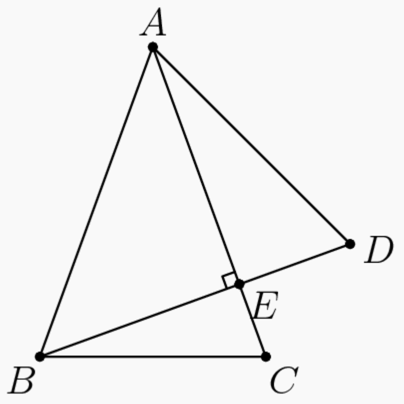
Problem 20

In the xy-plane, what is the length of the shortest path from $(0,0)$ to $(12,16)$ that does not go inside the circle $(x-6)^2+(y-8)^2=25$?

(A) $10\sqrt{3}$ (B) $10\sqrt{5}$ (C) $10\sqrt{3}+\frac{5\pi}{3}$ (D) $40\frac{\sqrt{3}}{3}$ (E) $10+5\pi$

Problem 21

Triangles ABC and ABD are isosceles with $AB=AC=BD$, and BD intersects AC at E. If BD is perpendicular to AC, then $\angle C+\angle D$ is



(A) 115° (B) 120° (C) 130° (D) 135° (E) not uniquely determined

Problem 22

Four distinct points, A, B, C, and D, are to be selected from 1996 points evenly spaced around a circle. All quadruples are equally likely to be chosen. What is the probability that the chord AB intersects the chord CD?

(A) $\frac{1}{4}$ (B) $\frac{1}{3}$ (C) $\frac{1}{2}$ (D) $\frac{2}{3}$ (E) $\frac{3}{4}$

Problem 23

The sum of the lengths of the twelve edges of a rectangular box is 140, and the distance from one corner of the box to the farthest corner is 21. The total surface area of the box is

(A) 776 (B) 784 (C) 798 (D) 800 (E) 812

Problem 24

The sequence $1,2,1,2,2,1,2,2,2,1,2,2,2,2,1,2,2,2,2,2,1,2,\ldots$ consists of 1's separated by blocks of 2's with n 2's in the n^{th}block. The sum of the first 1234 terms of this sequence is

(A) 1996 (B) 2419 (C) 2429 (D) 2439 (E) 2449

Problem 25

Given that $x^2 + y^2 = 14x + 6y + 6$, what is the largest possible value that $3x + 4y$ can have?

$\text{(A) } 72 \qquad \text{(B) } 73 \qquad \text{(C) } 74 \qquad \text{(D) } 75 \qquad \text{(E) } 76$

Problem 26

An urn contains marbles of four colors: red, white, blue, and green. When four marbles are drawn without replacement, the following events are equally likely:

(a) the selection of four red marbles;

(b) the selection of one white and three red marbles;

(c) the selection of one white, one blue, and two red marbles; and

(d) the selection of one marble of each color.

What is the smallest number of marbles satisfying the given condition?

$\text{(A) } 19 \qquad \text{(B) } 21 \qquad \text{(C) } 46 \qquad \text{(D) } 69 \qquad \text{(E) more than } 69$

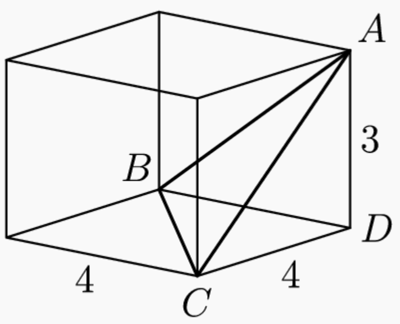
Problem 27

Consider two solid spherical balls, one centered at $(0, 0, \frac{21}{2})$ with radius 6, and the other centered at $(0, 0, 1)$ with radius $\frac{9}{2}$. How many points with only integer coordinates (lattice points) are there in the intersection of the balls?

$\text{(A) } 7 \qquad \text{(B) } 9 \qquad \text{(C) } 11 \qquad \text{(D) } 13 \qquad \text{(E) } 15$

Problem 28

On a $4 \times 4 \times 3$ rectangular parallelepiped, vertices A, B, and C are adjacent to vertex D. The perpendicular distance from D to the plane containing A, B, and C is closest to



$\text{(A) } 1.6 \qquad \text{(B) } 1.9 \qquad \text{(C) } 2.1 \qquad \text{(D) } 2.7 \qquad \text{(E) } 2.9$

Problem 29

If n is a positive integer such that $2n$ has 28 positive divisors and $3n$ has 30 positive divisors, then how many positive divisors does $6n$have?

$\text{(A) } 32 \qquad \text{(B) } 34 \qquad \text{(C) } 35 \qquad \text{(D) } 36 \qquad \text{(E) } 38$

Problem 30

A hexagon inscribed in a circle has three consecutive sides each of length 3 and three consecutive sides each of length 5. The chord of the circle that divides the hexagon into two trapezoids, one with three sides each of length 3 and the other with three sides each of length 5, has length equal to $\frac{m}{n}$, where m and n are relatively prime positive integers. Find $m + n$.

$\text{(A) } 309 \qquad \text{(B) } 349 \qquad \text{(C) } 369 \qquad \text{(D) } 389 \qquad \text{(E) } 409$