1990 AHSME Problems

Problem 1

If $\frac{\frac{x}{4}}{2}=\frac{4}{\frac{x}{2}}$, then $x=$

$\text{(A)} \ \pm\frac{1}{2} \qquad \text{(B)} \ \pm 1 \qquad \text{(C)} \ \pm 2 \qquad \text{(D)} \ \pm 4 \qquad \text{(E)} \ \pm 8$

Problem 2

$\left(\frac{1}{4}\right)^{-\frac{1}{4}}=$

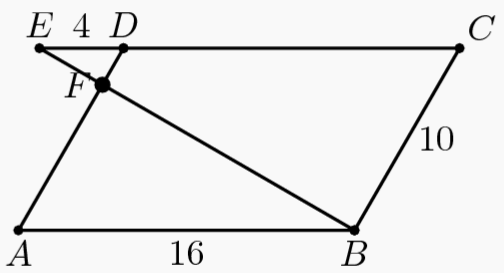
$\text{(A)} \ -16 \quad \text{(B)} \ -\sqrt{2} \quad \text{(C)} \ -\frac{1}{16} \quad \text{(D)} \ \frac{1}{256} \quad \text{(E)} \ \sqrt{2}$

Problem 3

The consecutive angles of a trapezoid form an arithmetic sequence. If the smallest angle is 75°, then the largest angle is

$\text{(A) } 95^\circ \quad \text{(B) } 100^\circ \quad \text{(C) } 105^\circ \quad \text{(D) } 110^\circ \quad \text{(E) } 115^\circ$

Problem 4



Let $ABCD$ be a parallelogram with $\angle ABC=120^\circ, AB=6$ and $BC=10.$ Extend $\overline{CD}$ through D to E so that $DE=4.$ If $\overline{BE}$ intersects $\overline{AD}$ at F, then FD is closest to

$\text{(A) } 1 \quad \text{(B) } 2 \quad \text{(C) } 3 \quad \text{(D) } 4 \quad \text{(E) } 5$

Problem 5

Which of these numbers is largest?

$\text{(A) } \sqrt{\sqrt[3]{5\cdot 6}} \quad \text{(B) } \sqrt{6\sqrt[3]{5}} \quad \text{(C) } \sqrt{5\sqrt[3]{6}} \quad \text{(D) } \sqrt[3]{5\sqrt{6}} \quad \text{(E) } \sqrt[3]{6\sqrt{5}}$

Problem 6

Points A and B are 5 units apart. How many lines in a given plane containing A and B are 2 units from A and 3 units from B?

$\text{(A) } 0 \quad \text{(B) } 1 \quad \text{(C) } 2 \quad \text{(D) } 3 \quad \text{(E) more than } 3$

Problem 7

A triangle with integral sides has perimeter 8. The area of the triangle is

$\text{(A) } 2\sqrt{2} \quad \text{(B) } \frac{16}{9}\sqrt{3} \quad \text{(C) } 2\sqrt{3} \quad \text{(D) } 4 \quad \text{(E) } 4\sqrt{2}$

Problem 8

The number of real solutions of the equation $|x-2|+|x-3|=1$ is

$\text{(A) } 0 \quad \text{(B) } 1 \quad \text{(C) } 2 \quad \text{(D) } 3 \quad \text{(E) more than } 3$

Problem 9

Each edge of a cube is colored either red or black. Every face of the cube has at least one black edge. The smallest number possible of black edges is

$\text{(A) } 2 \quad \text{(B) } 3 \quad \text{(C) } 4 \quad \text{(D) } 5 \quad \text{(E) } 6$

Problem 10

An $11\times 11\times 11$ wooden cube is formed by gluing together 11^3 unit cubes. What is the greatest number of unit cubes that can be seen from a single point?

$\text{(A) } 328 \quad \text{(B) } 329 \quad \text{(C) } 330 \quad \text{(D) } 331 \quad \text{(E) } 332$

Problem 11

How many positive integers less than 50 have an odd number of positive integer divisors?

$\text{(A) } 3 \quad \text{(B) } 5 \quad \text{(C) } 7 \quad \text{(D) } 9 \quad \text{(E) } 11$

Problem 12

Let f be the function defined by $f(x)=ax^2-\sqrt{2}$ for some positive a. If $f(f(\sqrt{2}))=-\sqrt{2}$ then $a=$

$\text{(A) } \frac{2-\sqrt{2}}{2} \quad \text{(B) } \frac{1}{2} \quad \text{(C) } 2-\sqrt{2} \quad \text{(D) } \frac{\sqrt{2}}{2} \quad \text{(E) } \frac{2+\sqrt{2}}{2}$

Problem 13

If the following instructions are carried out by a computer, which value of X will be printed because of instruction 5?

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1. START X AT 3 AND S AT 0.
2. INCREASE THE VALUE OF X BY 2.
3. INCREASE THE VALUE OF S BY THE VALUE OF X.
4. IF S IS AT LEAST 10000,
       THEN GO TO INSTRUCTION 5;
       OTHERWISE, GO TO INSTRUCTION 2.
       AND PROCEED FROM THERE.
5. PRINT THE VALUE OF X.
6.STOP.
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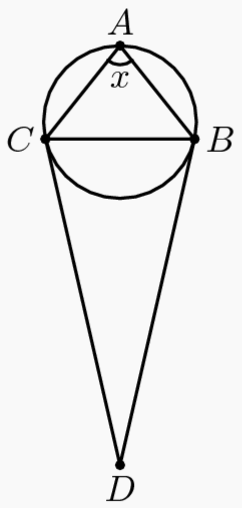
$\text{(A) } 19 \quad \text{(B) } 21 \quad \text{(C) } 23 \quad \text{(D) } 199 \quad \text{(E) } 201$

Problem 14



An acute isosceles triangle, ABC, is inscribed in a circle. Through B and C, tangents to the circle are drawn, meeting at point D. If $\angle ABC = \angle ACB = 2\angle D$ and x is the radian measure of $\angle A$, then $x =$

$\text{(A)}\ \frac{3\pi}{7} \qquad \text{(B)}\ \frac{4\pi}{9} \qquad \text{(C)}\ \frac{5\pi}{11} \qquad \text{(D)}\ \frac{6\pi}{13} \qquad \text{(E)}\ \frac{7\pi}{15}$

Problem 15

Four whole numbers, when added three at a time, give the sums $180, 197, 208$ and 222. What is the largest of the four numbers?

$\text{(A) } 77 \quad \text{(B) } 83 \quad \text{(C) } 89 \quad \text{(D) } 95 \quad \text{(E) cannot be determined from the given information}$

Problem 16

At one of George Washington's parties, each man shook hands with everyone except his spouse, and no handshakes took place between women. If 13 married couples attended, how many handshakes were there among these 26 people?

$\text{(A) } 78 \quad \text{(B) } 185 \quad \text{(C) } 234 \quad \text{(D) } 312 \quad \text{(E) } 325$

Problem 17

How many of the numbers, $100, 101, \cdots, 999$ have three different digits in increasing order or in decreasing order?

$\text{(A) } 120 \quad \text{(B) } 168 \quad \text{(C) } 204 \quad \text{(D) } 216 \quad \text{(E) } 240$

Problem 18

First a is chosen at random from the set $\{1,2,3,\cdots,99,100\}$, and then b is chosen at random from the same set. The probability that the integer 3^a+7^b has units digit 8 is

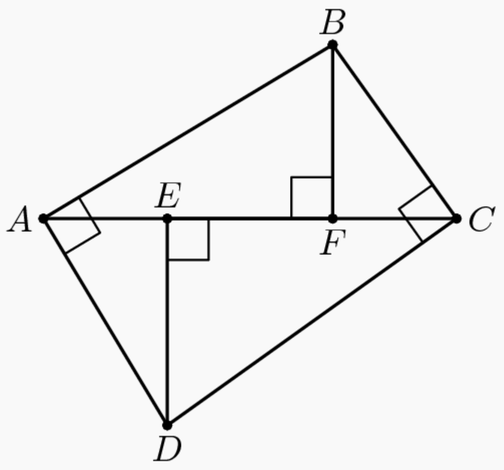
$\text{(A) } \frac{1}{16} \quad \text{(B) } \frac{1}{8} \quad \text{(C) } \frac{3}{16} \quad \text{(D) } \frac{1}{5} \quad \text{(E) } \frac{1}{4}$

Problem 19

For how many integers N between 1 and 1990 is the improper fraction $\frac{N^2+7}{N+4}$ $\underline{not}$ in lowest terms?

$\text{(A) } 0 \quad \text{(B) } 86 \quad \text{(C) } 90 \quad \text{(D) } 104 \quad \text{(E) } 105$

Problem 20



In the figure $ABCD$ is a quadrilateral with right angles at A and C. Points E and F are on $\overline{AC}$, and $\overline{DE}$ and $\overline{BF}$ are perpendicular to $\overline{AC}$. If $AE = 3, DE = 5,$ and $CE = 7$, then $BF =$

$\text{(A) } 3.6 \quad \text{(B) } 4 \quad \text{(C) } 4.2 \quad \text{(D) } 4.5 \quad \text{(E) } 5$

Problem 21

Consider a pyramid $P-ABCD$ whose base $ABCD$ is square and whose vertex P is equidistant from A, B, C and D. If $AB = 1$ and $\angle APB = 2\theta$, then the volume of the pyramid is

$$\text{(A) } \frac{\sin(\theta)}{6} \quad \text{(B) } \frac{\cot(\theta)}{6} \quad \text{(C) } \frac{1}{6\sin(\theta)} \quad \text{(D) } \frac{1-\sin(2\theta)}{6} \quad \text{(E) } \frac{\sqrt{\cos(2\theta)}}{6\sin(\theta)}$$

Problem 22

If the six solutions of $x^6 = -64$ are written in the form $a + bi$, where a and b are real, then the product of those solutions with $a > 0$ is

$\text{(A) } -2 \quad \text{(B) } 0 \quad \text{(C) } 2i \quad \text{(D) } 4 \quad \text{(E) } 16$

Problem 23

If $x, y > 0, \log_y(x) + \log_x(y) = \frac{10}{3} \text{ and } xy = 144, \text{ then } \frac{x+y}{2} =$

$\text{(A) } 12\sqrt{2} \quad \text{(B) } 13\sqrt{3} \quad \text{(C) } 24 \quad \text{(D) } 30 \quad \text{(E) } 36$

Problem 24

All students at Adams High School and at Baker High School take a certain exam. The average scores for boys, for girls, and for boys and girls combined, at Adams HS and Baker HS are shown in the table, as is the average for boys at the two schools combined. What is the average score for the girls at the two schools combined?

Average Scores

Category	Adams	Baker	Adams&Baker
Boys	71	81	79
Girls	76	90	?
Boys&Girls	74	84	

$\text{(A) } 81 \quad \text{(B) } 82 \quad \text{(C) } 83 \quad \text{(D) } 84 \quad \text{(E) } 85$

Problem 25

Nine congruent spheres are packed inside a unit cube in such a way that one of them has its center at the center of the cube and each of the others is tangent to the center sphere and to three faces of the cube. What is the radius of each sphere?

$\text{(A) } 1-\frac{\sqrt{3}}{2} \quad \text{(B) } \frac{2\sqrt{3}-3}{2} \quad \text{(C) } \frac{\sqrt{2}}{6} \quad \text{(D) } \frac{1}{4} \quad \text{(E) } \frac{\sqrt{3}(2-\sqrt(2))}{4}$

Problem 26

Ten people form a circle. Each picks a number and tells it to the two neighbors adjacent to him in the circle. Then each person computes and announces the average of the numbers of his two neighbors. The average announced by each person was (in order around the circle) 1, 2, 3, 4, 5, 6, 7, 8, 9, 10 (NOT the original number the person picked). The number picked by the person who announced the average 6 was

$\text{(A) } 1 \quad \text{(B) } 5 \quad \text{(C) } 6 \quad \text{(D) } 10 \quad \text{(E) not uniquely determined from the given information}$

Problem 27

Which of these triples could $\underline{not}$ be the lengths of the three altitudes of a triangle?

$\text{(A) } 1,\sqrt{3},2 \quad \text{(B) } 3,4,5 \quad \text{(C) } 5,12,13 \quad \text{(D) } 7,8,\sqrt{113} \quad \text{(E) } 8,15,17$

Problem 28

A quadrilateral that has consecutive sides of lengths $70,90,130$ and 110 is inscribed in a circle and also has a circle inscribed in it. The point of tangency of the inscribed circle to the side of length 130 divides that side into segments of length x and y.

Find $|x-y|$.

$\text{(A) } 12 \quad \text{(B) } 13 \quad \text{(C) } 14 \quad \text{(D) } 15 \quad \text{(E) } 16$

Problem 29

A subset of the integers $1,2,\cdots,100$ has the property that none of its members is 3 times another. What is the largest number of members such a subset can have?

$\text{(A) } 50 \quad \text{(B) } 66 \quad \text{(C) } 67 \quad \text{(D) } 76 \quad \text{(E) } 78$

Problem 30

If $R_n=\frac{1}{2}(a^n+b^n)$ where $a=3+2\sqrt{2}$ and $b=3-2\sqrt{2}$, and $n=0,1,2,\cdots,$ then R_{12345} is an integer. Its units digit is

$\text{(A) } 1 \quad \text{(B) } 3 \quad \text{(C) } 5 \quad \text{(D) } 7 \quad \text{(E) } 9$