1989 AHSME Problems

Problem 1

$(-1)^{5^2}+1^{2^5}=$

$\text{(A)}\ -7 \qquad \text{(B)}\ -2 \qquad \text{(C)}\ 0 \qquad \text{(D)}\ 1 \qquad \text{(E)}\ 57$

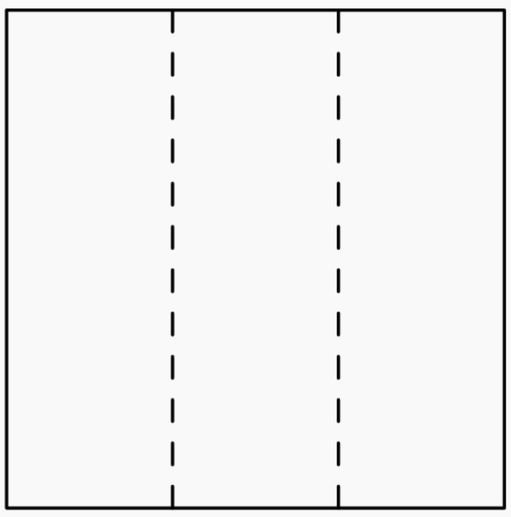
Problem 2

$\sqrt{\frac{1}{9}+\frac{1}{16}}=$

$\text{(A)}\ \frac{1}{5} \qquad \text{(B)}\ \frac{1}{4} \qquad \text{(C)}\ \frac{2}{7} \qquad \text{(D)}\ \frac{5}{12} \qquad \text{(E)}\ \frac{7}{12}$

Problem 3

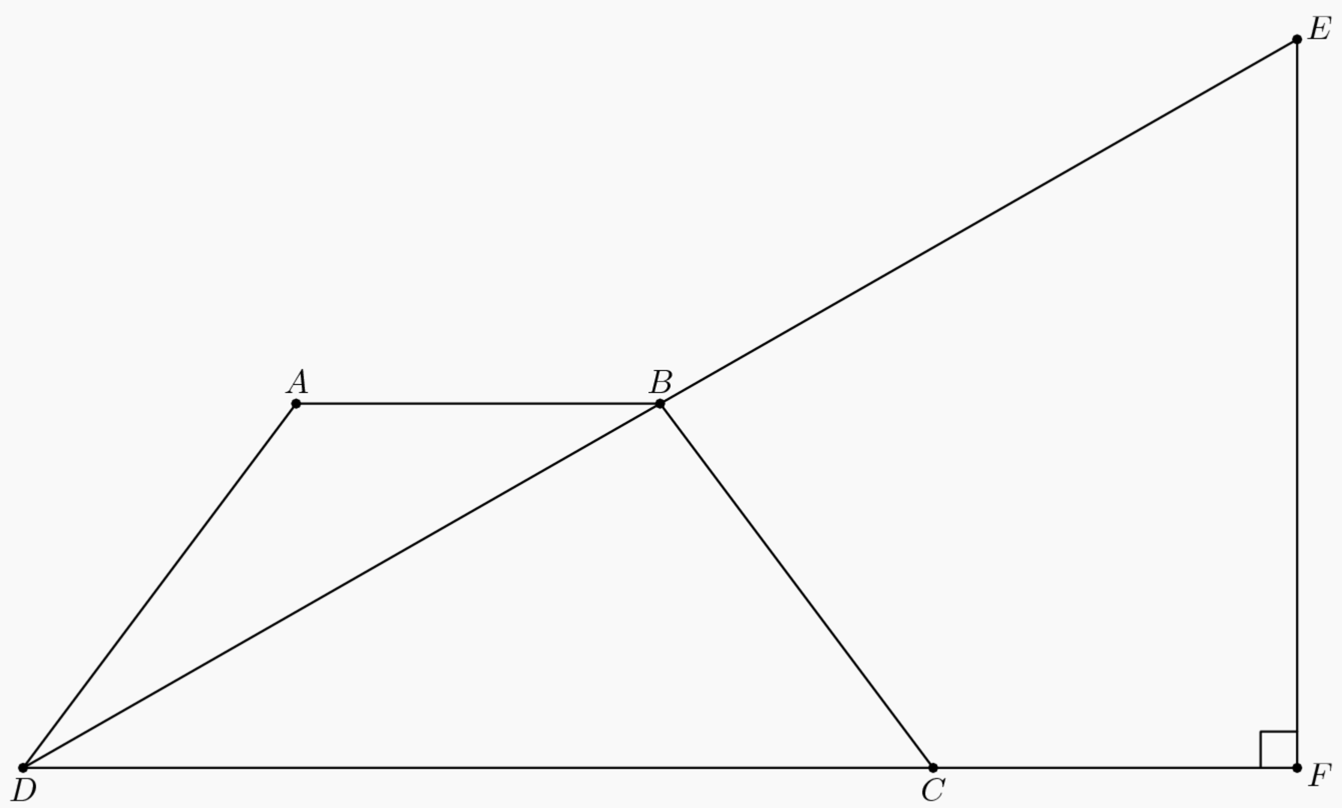
A square is cut into three rectangles along two lines parallel to a side, as shown. If the perimeter of each of the three rectangles is 24, then the area of the original square is



$\text{(A)}\ 24 \qquad \text{(B)}\ 36 \qquad \text{(C)}\ 64 \qquad \text{(D)}\ 81 \qquad \text{(E)}\ 96$

Problem 4

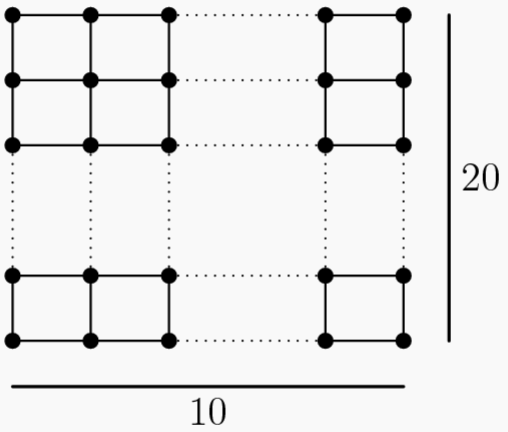
In the figure, $ABCD$ is an isosceles trapezoid with side lengths $AD=BC=5$, $AB=4$, and $DC=10$. The point C is on $\overline{DF}$ and $B$is the midpoint of hypotenuse $\overline{DE}$ in right triangle DEF. Then $CF=$



(A) 3.25 (B) 3.5 (C) 3.75 (D) 4.0 (E) 4.25

Problem 5

Toothpicks of equal length are used to build a rectangular grid as shown. If the grid is 20 toothpicks high and 10 toothpicks wide, then the number of toothpicks used is



(A) 30 (B) 200 (C) 410 (D) 420 (E) 430

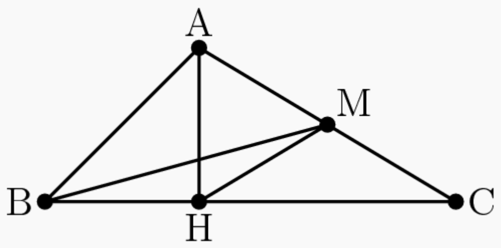
Problem 6

If $a, b > 0$ and the triangle in the first quadrant bounded by the coordinate axes and the graph of $ax + by = 6$ has area 6, then $ab =$

(A) 3 (B) 6 (C) 12 (D) 108 (E) 432

Problem 7

In $\triangle ABC$, $\angle A = 100^\circ$, $\angle B = 50^\circ$, $\angle C = 30^\circ$, $\overline{AH}$ is an altitude, and $\overline{BM}$ is a median. Then $\angle MHC =$



(A) 15° (B) 22.5° (C) 30° (D) 40° (E) 45°

Problem 8

For how many integers n between 1 and 100 does $x^2 + x - n$ factor into the product of two linear factors with integer coefficients?

(A) 0 (B) 1 (C) 2 (D) 9 (E) 10

Problem 9

Mr. and Mrs. Zeta want to name their baby Zeta so that its monogram (first, middle, and last initials) will be in alphabetical order with no letter repeated. How many such monograms are possible?

(A) 276 (B) 300 (C) 552 (D) 600 (E) 15600

Problem 10

Consider the sequence defined recursively by $u_1 = a$ (any positive integer), and $u_{n+1} = \dfrac{-1}{u_n + 1}$, $n = 1, 2, 3, \cdots$. For which of the following values of n must $u_n = a$?

(A) 14 (B) 15 (C) 16 (D) 17 (E) 18

Problem 11

Let a, b, c, and d be positive integers with $a < 2b$, $b < 3c$, and $c < 4d$. If $d < 100$, the largest possible value for a is

$\text{(A)}\ 2367 \qquad \text{(B)}\ 2375 \qquad \text{(C)}\ 2391 \qquad \text{(D)}\ 2399 \qquad \text{(E)}\ 2400$

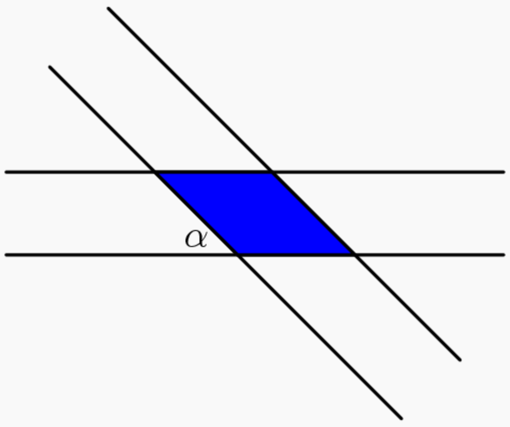
Problem 12

The traffic on a certain east-west highway moves at a constant speed of 60 miles per hour in both directions. An eastbound driver passes 20 west-bound vehicles in a five-minute interval. Assume vehicles in the westbound lane are equally spaced. Which of the following is closest to the number of westbound vehicles present in a 100-mile section of highway?

$\text{(A)}\ 100 \qquad \text{(B)}\ 120 \qquad \text{(C)}\ 200 \qquad \text{(D)}\ 240 \qquad \text{(E)}\ 400$

Problem 13

Two strips of width 1 overlap at an angle of α as shown. The area of the overlap (shown shaded) is



$\text{(A)}\ \sin\alpha \qquad \text{(B)}\ \frac{1}{\sin\alpha} \qquad \text{(C)}\ \frac{1}{1-\cos\alpha} \qquad \text{(D)}\ \frac{1}{\sin^{2}\alpha} \qquad \text{(E)}\ \frac{1}{(1-\cos\alpha)^2}$

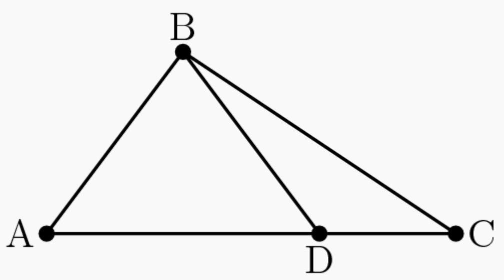
Problem 14

$\cot 10 + \tan 5 =$

$\text{(A)}\ \csc 5 \qquad \text{(B)}\ \csc 10 \qquad \text{(C)}\ \sec 5 \qquad \text{(D)}\ \sec 10 \qquad \text{(E)}\ \sin 15$

Problem 15

In $\triangle ABC$, $AB = 5$, $BC = 7$, $AC = 9$, and D is on $\overline{AC}$ with $BD = 5$. Find the ratio of $AD : DC$.



$\text{(A)}\ 4:3 \qquad \text{(B)}\ 7:5 \qquad \text{(C)}\ 11:6 \qquad \text{(D)}\ 13:5 \qquad \text{(E)}\ 19:8$

Problem 16

A lattice point is a point in the plane with integer coordinates. How many lattice points are on the line segment whose endpoints are (3,17) and (48,281)? (Include both endpoints of the segment in your count.)

$\text{(A)}\ 2 \qquad \text{(B)}\ 4 \qquad \text{(C)}\ 6 \qquad \text{(D)}\ 16 \qquad \text{(E)}\ 46$

Problem 17

The perimeter of an equilateral triangle exceeds the perimeter of a square by 1989 cm. The length of each side of the triangle exceeds the length of each side of the square by d cm. The square has perimeter greater than 0. How many positive integers are NOT a possible value for d?

(A) 0 (B) 9 (C) 221 (D) 663 (E) infinitely many

Problem 18

The set of all numbers x for which $x+\sqrt{x^2+1}-\frac{1}{x+\sqrt{x^2+1}}$ is a rational number is the set of all:

(A) integers x (B) rational x (C) real x (D) x for which $\sqrt{x^2+1}$ is rational (E) x for which $x+\sqrt{x^2+1}$ is rational

Problem 19

A triangle is inscribed in a circle. The vertices of the triangle divide the circle into three arcs of lengths 3, 4, and 5. What is the area of the triangle?

(A) 6 (B) $\frac{18}{\pi^2}$ (C) $\frac{9}{\pi^2}\left(\sqrt{3}-1\right)$ (D) $\frac{9}{\pi^2}\left(\sqrt{3}+1\right)$ (E) $\frac{9}{\pi^2}\left(\sqrt{3}+3\right)$

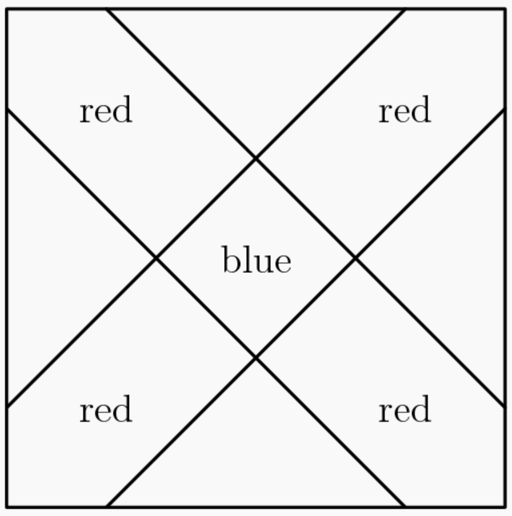
Problem 20

Let x be a real number selected uniformly at random between 100 and 200. If $\lfloor\sqrt{x}\rfloor=12$, find the probability that $\lfloor\sqrt{100x}\rfloor=120$. ($\lfloor v\rfloor$ means the greatest integer less than or equal to v.)

(A) $\frac{2}{25}$ (B) $\frac{241}{2500}$ (C) $\frac{1}{10}$ (D) $\frac{96}{625}$ (E) 1

Problem 21

A square flag has a red cross of uniform width with a blue square in the center on a white background as shown. (The cross is symmetric with respect to each of the diagonals of the square.) If the entire cross (both the red arms and the blue center) takes up 36% of the area of the flag, what percent of the area of the flag is blue?



(A) .5 (B) 1 (C) 2 (D) 3 (E) 6

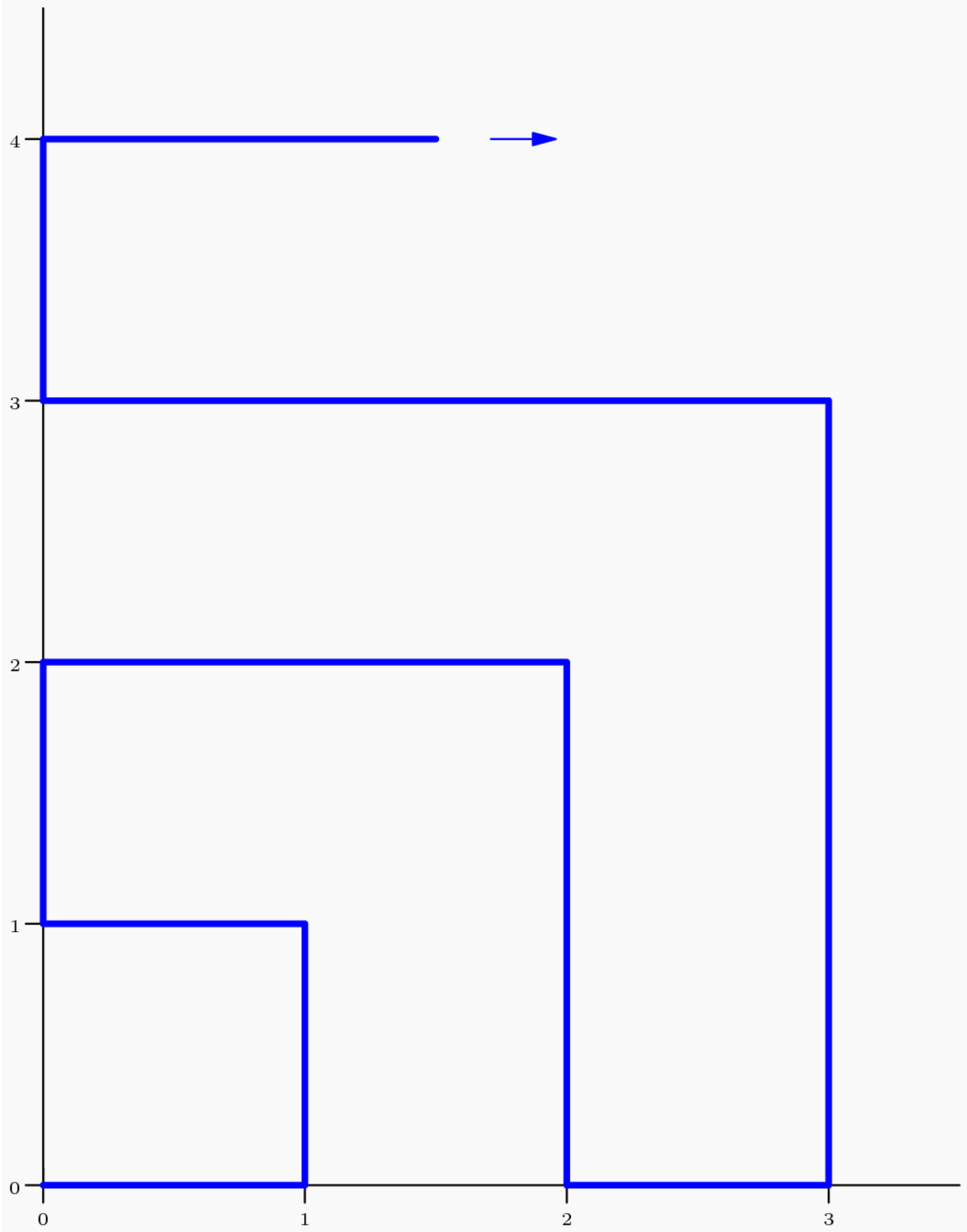
Problem 22

A child has a set of 96 distinct blocks. Each block is one of 2 materials (plastic,wood), 3 sizes (small, medium, large), 4 colors (blue, green, red, yellow), and 4 shapes (circle, hexagon, square, triangle). How many blocks in the set different from the 'plastic medium red circle' in exactly 2 ways? (The 'wood medium red square' is such a block)

(A) 29 (B) 39 (C) 48 (D) 56 (E) 62

Problem 23

A particle moves through the first quadrant as follows. During the first minute it moves from the origin to $(1,0)$. Thereafter, it continues to follow the directions indicated in the figure, going back and forth between the positive x and y axes, moving one unit of distance parallel to an axis in each minute. At which point will the particle be after exactly 1989 minutes?



$\text{(A)}\ (35,44) \qquad \text{(B)}\ (36,45) \qquad \text{(C)}\ (37,45) \qquad \text{(D)}\ (44,35) \qquad \text{(E)}\ (45,36)$

Problem 24

Five people are sitting at a round table. Let $f \geq 0$ be the number of people sitting next to at least 1 female and $m \geq 0$ be the number of people sitting next to at least one male. The number of possible ordered pairs (f,m) is

$\text{(A)}\ 7 \qquad \text{(B)}\ 8 \qquad \text{(C)}\ 9 \qquad \text{(D)}\ 10 \qquad \text{(E)}\ 11$

Problem 25

In a certain cross country meet between 2 teams of 5 runners each, a runner who finishes in the nth position contributes n to his teams score. The team with the lower score wins. If there are no ties among the runners, how many different winning scores are possible?

(A) 10 (B) 13 (C) 27 (D) 120 (E) 126

Problem 26

A regular octahedron is formed by joining the centers of adjoining faces of a cube. The ratio of the volume of the octahedron to the volume of the cube is

$(\mathrm{A})\frac{\sqrt{3}}{12} \qquad (\mathrm{B})\frac{\sqrt{6}}{16} \qquad (\mathrm{C})\frac{1}{6} \qquad (\mathrm{D})\frac{\sqrt{2}}{8} \qquad (\mathrm{E})\frac{1}{4}$

Problem 27

Let n be a positive integer. If the equation $2x+2y+z=n$ has 28 solutions in positive integers x, y, and z, then n must be either

$\mathrm{(A)}\ 14 \text{ or } 15\ \mathrm{(B)}\ 15 \text{ or } 16\ \mathrm{(C)}\ 16 \text{ or } 17\ \mathrm{(D)}\ 17 \text{ or } 18\ \mathrm{(E)}\ 18 \text{ or } 19$

Problem 28

Find the sum of the roots of $\tan^2 x - 9\tan x + 1 = 0$ that are between $x=0$ and $x=2\pi$ radians.

$(\mathrm{A})\frac{\pi}{2} \qquad (\mathrm{B})\pi \qquad (\mathrm{C})\frac{3\pi}{2} \qquad (\mathrm{D})3\pi \qquad (\mathrm{E})4\pi$

Problem 29

Find $\sum_{k=0}^{49}(-1)^k\binom{99}{2k}$, where $\binom{n}{j} = \frac{n!}{j!(n-j)!}$

(A) -2^{50} (B) -2^{49} (C) 0 (D) 2^{49} (E) 2^{50}

Problem 30

Suppose that 7 boys and 13 girls line up in a row. Let S be the number of places in the row where a boy and a girl are standing next to each other. For example, for the row $GBBGGGBGBGGGBGBGGBGG$ we have that $S=12$. The average value of S (if all possible orders of these 20 people are considered) is closest to

(A) 9 (B) 10 (C) 11 (D) 12 (E) 13