1985 AHSME Problems

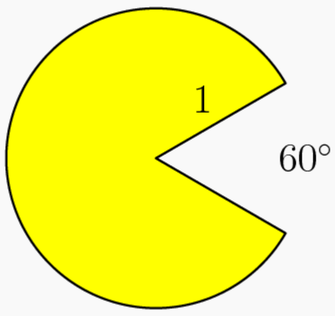
Problem 1

If $2x+1=8$, then $4x+1=$

$\text{(A)}\ 15 \qquad \text{(B)}\ 16 \qquad \text{(C)}\ 17 \qquad \text{(D)}\ 18 \qquad \text{(E)}\ 19$

Problem 2

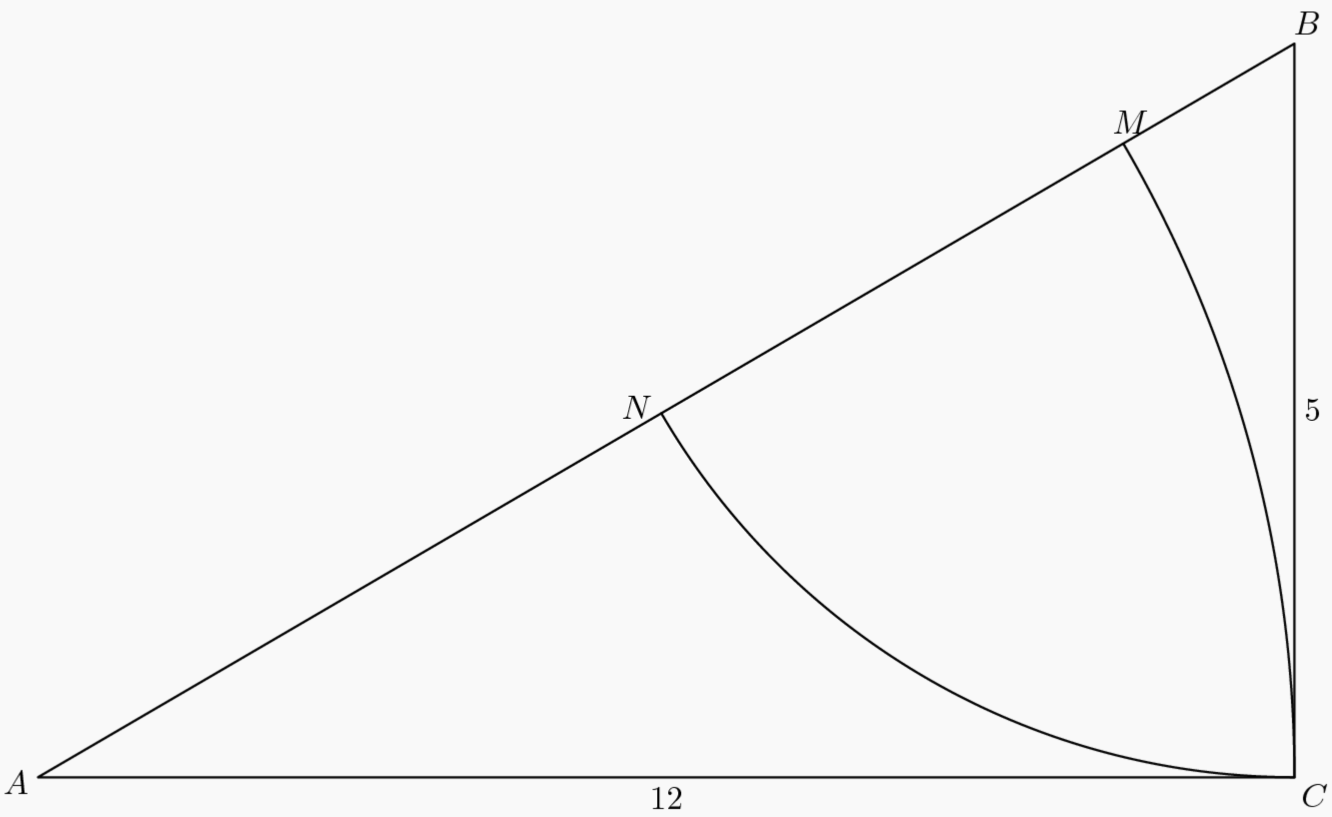
In an arcade game, the "monster" is the shaded sector of a circle of radius 1 cm, as shown in the figure. The missing piece (the mouth) has central angle 60°. What is the perimeter of the monster in cm?



$\text{(A)}\ \pi+2 \qquad \text{(B)}\ 2\pi \qquad \text{(C)}\ \frac{5}{3}\pi \qquad \text{(D)}\ \frac{5}{6}\pi+2 \qquad \text{(E)}\ \frac{5}{3}\pi+2$

Problem 3

In right $\triangle ABC$ with legs 5 and 12, arcs of circles are drawn, one with center A and radius 12, the other with center B and radius 5. They intersect the hypotenuse at M and N. Then, MN has length:



$\text{(A)}\ 2 \qquad \text{(B)}\ \frac{13}{5} \qquad \text{(C)}\ 3 \qquad \text{(D)}\ 4 \qquad \text{(E)}\ \frac{24}{5}$

Problem 4

A large bag of coins contains pennies, dimes, and quarters. There are twice as many dimes as pennies and three times as many quarters as dimes. An amount of money which could be in the bag is

$\text{(A)}\ \$306 \qquad \text{(B)}\ \$333 \qquad \text{(C)}\ \$342 \qquad \text{(D)}\ \$348 \qquad \text{(E)}\ \$360$

Problem 5

Which terms must be removed from the sum

$\frac{1}{2}+\frac{1}{4}+\frac{1}{6}+\frac{1}{8}+\frac{1}{10}+\frac{1}{12}$

if the sum of the remaining terms is equal to 1?

$\textbf{(A)}\ \frac{1}{4}\text{ and }\frac{1}{8}\qquad \textbf{(B)}\ \frac{1}{4}\text{ and }\frac{1}{12}\qquad \textbf{(C)}\ \frac{1}{8}\text{ and }\frac{1}{12}\qquad \textbf{(D)}\ \frac{1}{6}\text{ and }\frac{1}{10}\qquad \textbf{(E)}\ \frac{1}{8}\text{ and }\frac{1}{10}$

Problem 6

One student in a class of boys and girls is chosen to represent the class. Each student is equally likely to be chosen and the probability that a boy is chosen is $\frac{2}{3}$ of the probability that a girl is chosen. The ratio of the number of boys to the total number of boys and girls is

$\textbf{(A)}\ \frac{1}{3}\qquad \textbf{(B)}\ \frac{2}{5}\qquad \textbf{(C)}\ \frac{1}{2}\qquad \textbf{(D)}\ \frac{3}{5}\qquad \textbf{(E)}\ \frac{2}{3}$

Problem 7

In some computer languages (such as APL), when there are no parentheses in an algebraic expression, the operations are grouped from left to right. Thus, $a\times b-c$ in such languages means the same as $a(b-c)$ in ordinary algebraic notation. If $a\div b-c+d$ is evaluated in such a language, the result in ordinary algebraic notation would be

$\textbf{(A)}\ \frac{a}{b}-c+d\qquad \textbf{(B)}\ \frac{a}{b}-c-d\qquad \textbf{(C)}\ \frac{d+c-b}{a}\qquad \textbf{(D)}\ \frac{a}{b-c+d}\qquad \textbf{(E)}\ \frac{a}{b-c-d}$

Problem 8

Let $a, a', b,$ and b' be real numbers with a and a' nonzero. The solution to $ax+b=0$ is less than the solution to $a'x+b'=0$ if and only if

$\textbf{(A)}\ a'b<ab'\qquad \textbf{(B)}\ ab'<a'b\qquad \textbf{(C)}\ ab<a'b'\qquad \textbf{(D)}\ \frac{b}{a}<\frac{b'}{a'}$

$\textbf{(E)}\ \frac{b'}{a'}<\frac{b}{a}$

Problem 9

The odd positive integers $1, 3, 5, 7, \cdots$, are arranged into five columns continuing with the pattern shown on the right. Counting from the left, the column in which 1985 appears in is the

	1	3	5	7
15	13	11	9	
	17	19	21	23
31	29	27	25	
	33	35	37	39
47	45	43	41	
	49	51	53	55
•	•	•	•	
	•	•	•	•
•	•	•	•	
	•	•	•	•

$\textbf{(A)}\ \text{first}\qquad \textbf{(B)}\ \text{second}\qquad \textbf{(C)}\ \text{third}\qquad \textbf{(D)}\ \text{fourth}\qquad \textbf{(E)}\ \text{fifth}$

Problem 10

An arbitrary circle can intersect the graph $y = \sin x$ in

$\text{(A)at most 2 points} \qquad \text{(B)at most 4 points} \qquad \text{(C)at most 6 points} \qquad \text{(D)at most 8 points} \qquad \text{(E)more than 16 points}$

Problem 11

How many **distinguishable** rearrangements of the letters in CONTEST have both the vowels first? (For instance, OETCNST is one such arrangement but OTETSNC is not.)

$\text{(A) } 60 \qquad \text{(B) } 120 \qquad \text{(C) } 240 \qquad \text{(D) } 720 \qquad \text{(E) } 2520$

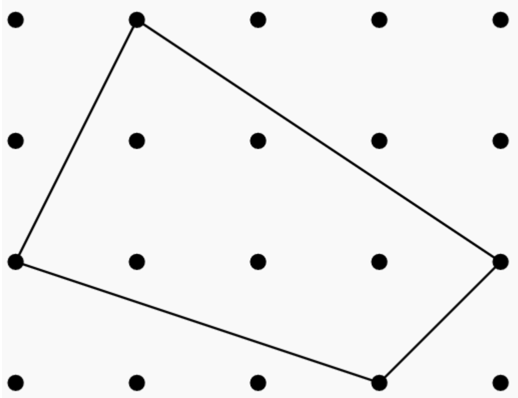
Problem 12

Let's write p, q, and r as three distinct prime numbers, where 1 is not a prime. Which of the following is the smallest positive perfect cube having $n = pq^2r^4$ as a divisor?

$\text{(A) } p^8q^8r^8 \qquad \text{(B) } (pq^2r^2)^3 \qquad \text{(C) } (p^2q^2r^2)^3 \qquad \text{(D) } (pqr^2)^3 \qquad \text{(E) } 4p^3q^3r^3$

Problem 13

Pegs are put in a board 1 unit apart both horizontally and vertically. A rubber band is stretched over 4 pegs as shown in the figure, forming a quadrilateral. Its area in square units is



$\text{(A) } 4 \qquad \text{(B) } 4.5 \qquad \text{(C) } 5 \qquad \text{(D) } 5.5 \qquad \text{(E) } 6$

Problem 14

Exactly three of the interior angles of a convex polygon are obtuse. What is the maximum number of sides of such a polygon?

$\text{(A) } 4 \qquad \text{(B) } 5 \qquad \text{(C) } 6 \qquad \text{(D) } 7 \qquad \text{(E) } 8$

Problem 15

If a and b are positive numbers such that $a^b = b^a$ and $b = 9a$, then the value of a is:

$\text{(A) } 9 \qquad \text{(B) } \frac{1}{9} \qquad \text{(C) } \sqrt[9]{9} \qquad \text{(D) } \sqrt[3]{9} \qquad \text{(E) } \sqrt[4]{3}$

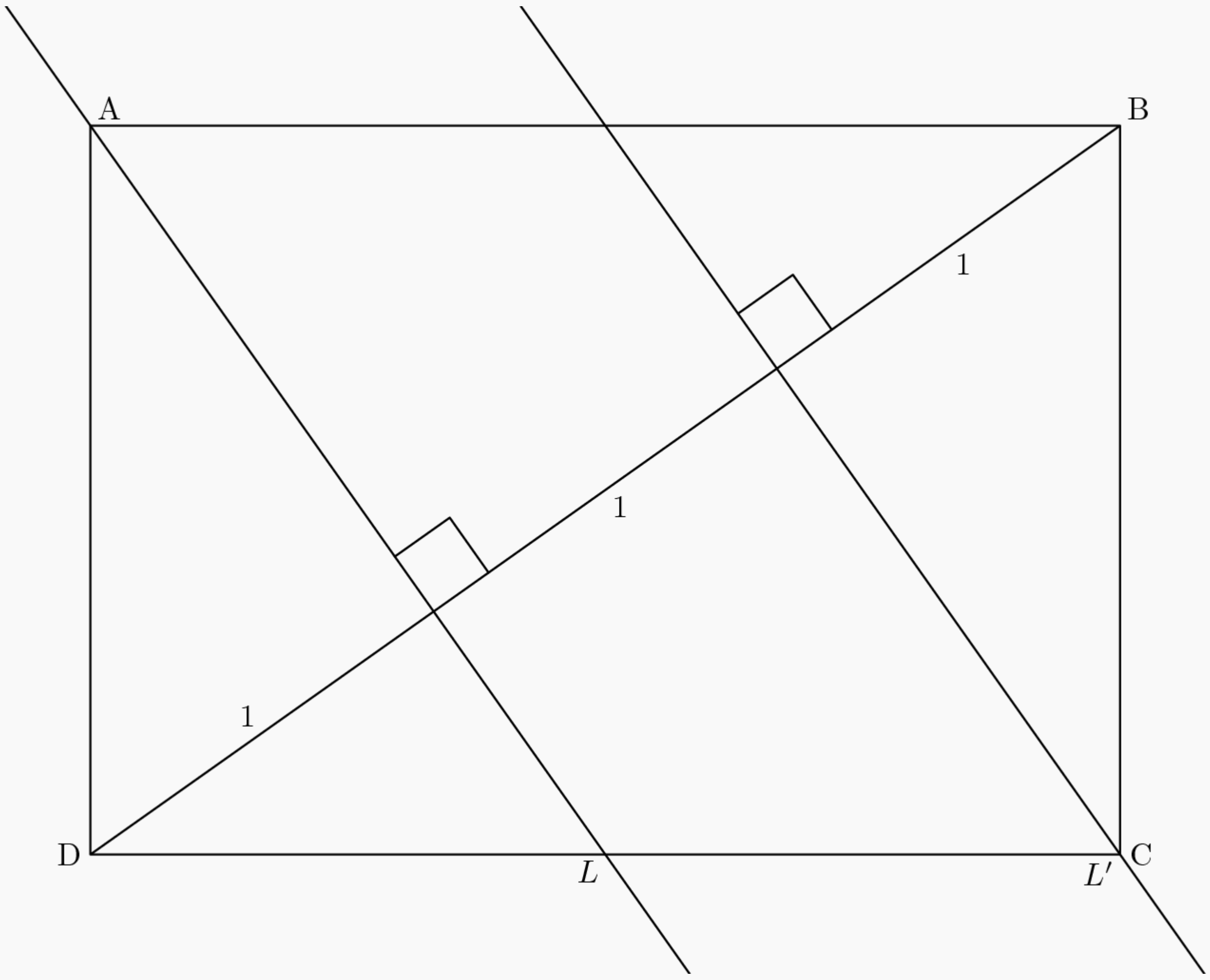
Problem 16

If $A = 20^\circ$ and $B = 25^\circ$, then the value of $(1 + \tan A)(1 + \tan B)$ is

$\text{(A) } \sqrt{3} \qquad \text{(B) } 2 \qquad \text{(C) } 1 + \sqrt{2} \qquad \text{(D) } 2(\tan A + \tan B) \qquad \text{(E) none of these}$

Problem 17

Diagonal DB of rectangle $ABCD$ is divided into 3 segments of length 1 by parallel lines L and L' that pass through A and C and are perpendicular to DB. The area of $ABCD$, rounded to the nearest tenth, is



$\text{(A) } 4.1 \qquad \text{(B) } 4.2 \qquad \text{(C) } 4.3 \qquad \text{(D) } 4.4 \qquad \text{(E) } 4.5$

Problem 18

Six bags of marbles contain $18, 19, 21, 23, 25,$ and 34 marbles, respectively. One bag contains chipped marbles only. The other 5 bags contain no chipped marbles. Jane takes three of the bags and George takes two of the others. Only the bag of chipped marbles remains. If Jane gets twice as many marbles as George, how many chipped marbles are there?

$\text{(A) } 18 \qquad \text{(B) } 19 \qquad \text{(C) } 21 \qquad \text{(D) } 23 \qquad \text{(E) } 25$

Problem 19

Consider the graphs $y = Ax^2$ and $y^2 + 3 = x^2 + 4y$, where A is a positive constant and x and y are real variables. In how many points do the two graphs intersect?

$\text{(A) exactly 4} \qquad \text{(B) exactly 2}$

$\text{(C) at least } 1, \text{ but the number varies for different positive values of } A$

$\text{(D) 0 for at least one positive value of } A \qquad \text{(E) none of these}$

Problem 20

A wooden cube with edge length n units (where n is an integer > 2) is painted black all over. By slices parallel to its faces, the cube is cut into n^3 smaller cubes each of unit length. If the number of smaller cubes with just one face painted black is equal to the number of smaller cubes completely free of paint, what is n?

$\text{(A) } 5 \qquad \text{(B) } 6 \qquad \text{(C) } 7 \qquad \text{(D) } 8 \qquad \text{(E) none of these}$

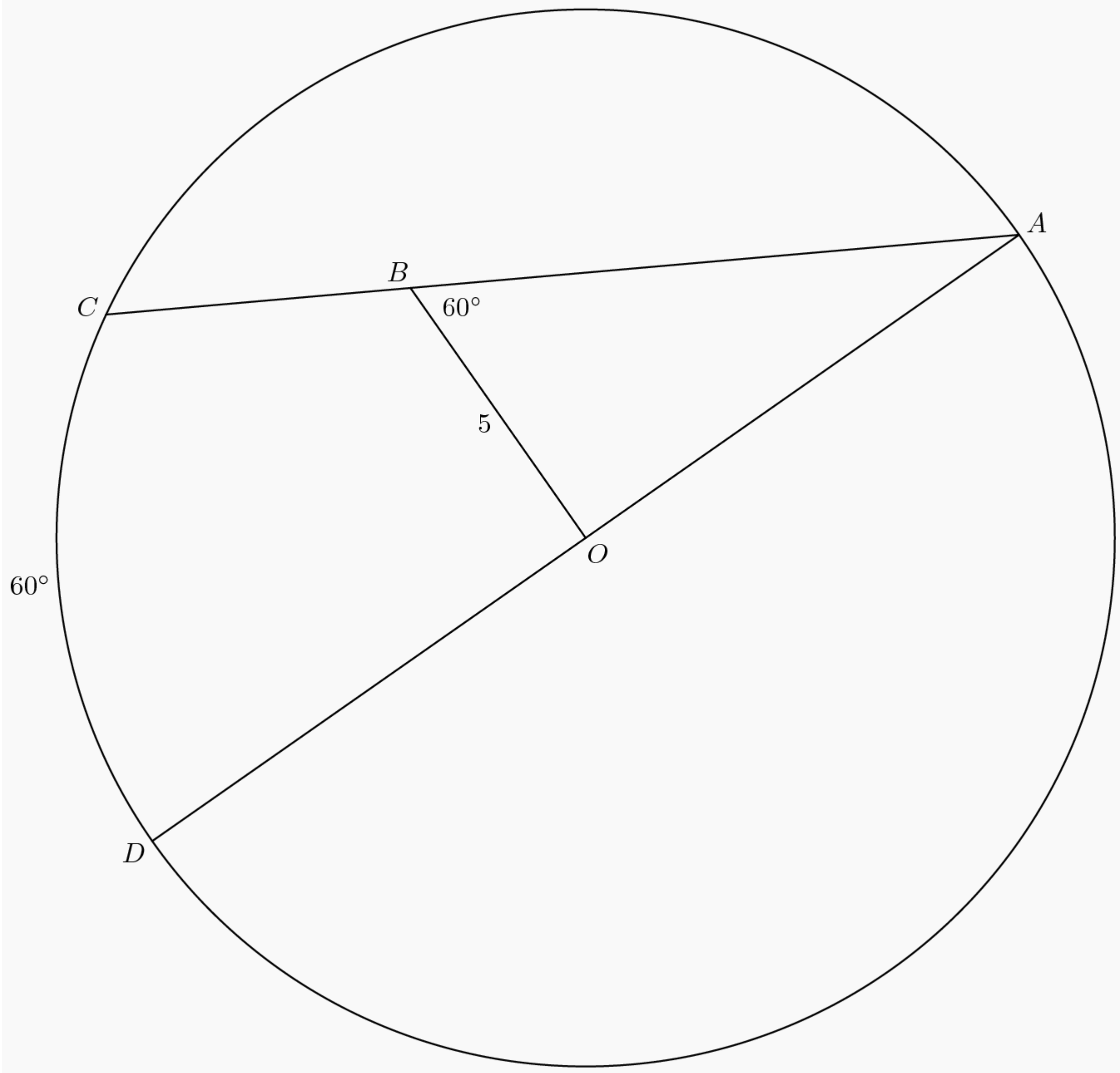
Problem 21

How many integers x satisfy the equation $(x^2 - x - 1)^{x+2} = 1$

$\text{(A) } 2 \qquad \text{(B) } 3 \qquad \text{(C) } 4 \qquad \text{(D) } 5 \qquad \text{(E) none of these}$

Problem 22

In a circle with center O, AD is a diameter, ABC is a chord, $BO = 5$, and $\angle ABO = \overset{\frown}{CD} = 60^\circ$. Then the length of BC is:



$(\text{A})\ 3 \qquad (\text{B})\ 3+\sqrt{3} \qquad (\text{C})\ 5-\frac{\sqrt{3}}{2} \qquad (\text{D})\ 5 \qquad (\text{E})\ \text{none of the above}$

Problem 23

If $x = \frac{-1+i\sqrt{3}}{2}$ and $y = \frac{-1-i\sqrt{3}}{2}$, where $i^2 = -1$, then which of the following is *not* correct?

$(\text{A})\ x^5+y^5=-1 \qquad (\text{B})\ x^7+y^7=-1 \qquad (\text{C})\ x^9+y^9=-1$
$(\text{D})\ x^{11}+y^{11}=-1 \qquad (\text{E})\ x^{13}+y^{13}=-1$

Problem 24

A non-zero digit is chosen in such a way that the probability of choosing digit d is $\log_{10}{(d+1)}-\log_{10}{d}$. The probability that the digit 2 is chosen is exactly $\frac{1}{2}$ the probability that the digit is chosen in the set

$(\text{A})\ \{2, 3\} \qquad (\text{B})\ \{3, 4\} \qquad (\text{C})\ \{4, 5, 6, 7, 8\} \qquad (\text{D})\ \{5, 6, 7, 8, 9\} \qquad (\text{E})\ \{4, 5, 6, 7, 8, 9\}$

Problem 25

The volume of a certain rectangular solid is 8cm^3, its total surface area is 32cm^2, and its three dimensions are in geometric progression. The sums of the lengths in cm of all the edges of this solid is

$(\text{A})\ 28 \qquad (\text{B})\ 32 \qquad (\text{C})\ 36 \qquad (\text{D})\ 40 \qquad (\text{E})\ 44$

Problem 26

Find the least positive integer n for which $\frac{n-13}{5n+6}$ is a non-zero reducible fraction.

$\text{(A)}\ 45 \qquad \text{(B)}\ 68 \qquad \text{(C)}\ 155 \qquad \text{(D)}\ 226 \qquad \text{(E)}\ \text{none of these}$

Problem 27

Consider a sequence $x_1, x_2, x_3, \cdots$ defined by

$x_1 = \sqrt[3]{3}$

$x_2 = \sqrt[3]{3}^{\sqrt[3]{3}}$

and in general

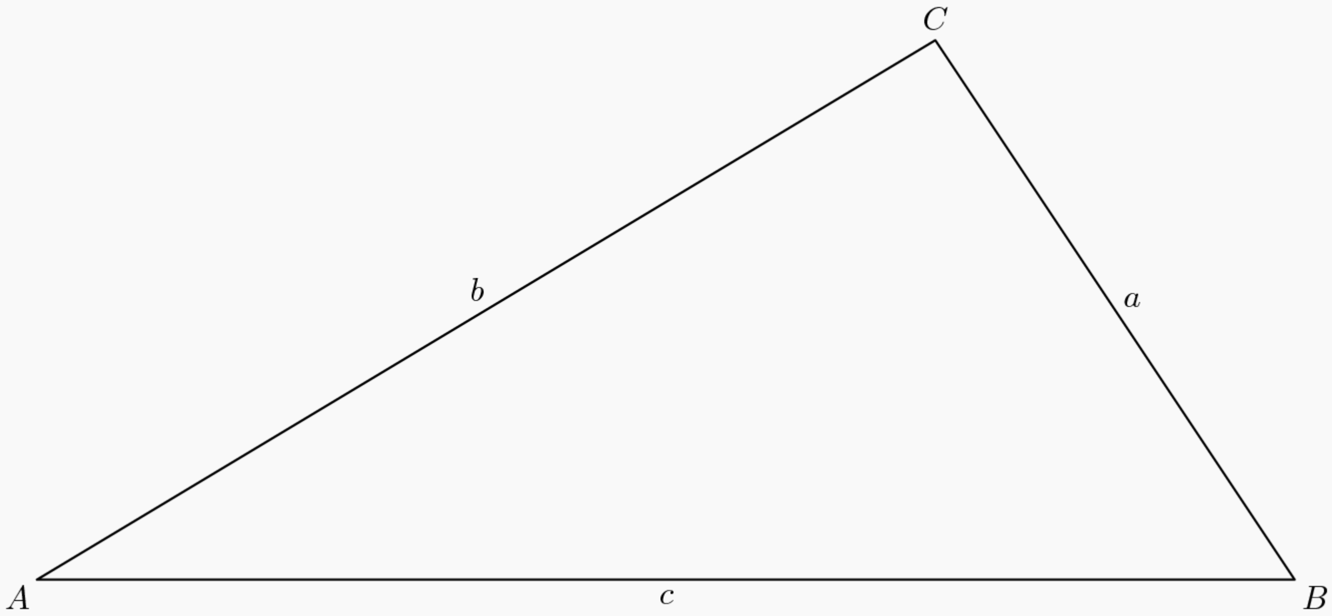
$x_n = (x_{n-1})^{\sqrt[3]{3}}$ for $n > 1$.

What is the smallest value of n for which x_n is an integer?

$\text{(A)}\ 2 \qquad \text{(B)}\ 3 \qquad \text{(C)}\ 4 \qquad \text{(D)}\ 9 \qquad \text{(E)}\ 27$

Problem 28

In $\triangle ABC$, we have $\angle C = 3\angle A$, $a = 27$, and $c = 48$. What is b?



$\text{(A)}\ 33 \qquad \text{(B)}\ 35 \qquad \text{(C)}\ 37 \qquad \text{(D)}\ 39 \qquad \text{(E)}\ \text{not uniquely determined}$

Problem 29

In their base 10 representation, the integer a consists of a sequence of 1985 eights and the integer b consists of a sequence of 1985fives. What is the sum of the digits of the base 10 representation of $9ab$?

$\text{(A)}\ 15880 \qquad \text{(B)}\ 17856 \qquad \text{(C)}\ 17865 \qquad \text{(D)}\ 17874 \qquad \text{(E)}\ 19851$

Problem 30

Let $\lfloor x \rfloor$ be the greatest integer less than or equal to x. Then the number of real solutions to $4x^2 - 40\lfloor x \rfloor - 51 = 0$ is

$\text{(A)}\ 0 \qquad \text{(B)}\ 1 \qquad \text{(C)}\ 2 \qquad \text{(D)}\ 3 \qquad \text{(E)}\ 4$