1951 AHSME Problems

Problem 1

The percent that M is greater than N is:

$\textbf{(A)}\ \frac{100(M-N)}{M} \qquad \textbf{(B)}\ \frac{100(M-N)}{N} \qquad \textbf{(C)}\ \frac{M-N}{N} \qquad \textbf{(D)}\ \frac{M-N}{M} \qquad \textbf{(E)}\ \frac{100(M+N)}{N}$

Problem 2

A rectangular field is half as wide as it is long and is completely enclosed by x yards of fencing. The area in terms of x is:

$(\mathrm{A})\ \frac{x^2}{2} \qquad (\mathrm{B})\ 2x^2 \qquad (\mathrm{C})\ \frac{2x^2}{9} \qquad (\mathrm{D})\ \frac{x^2}{18} \qquad (\mathrm{E})\ \frac{x^2}{72}$

Problem 3

If the length of a diagonal of a square is $a + b$, then the area of the square is:

$\textbf{(A)}\ (a+b)^2 \qquad \textbf{(B)}\ \frac{1}{2}(a+b)^2 \qquad \textbf{(C)}\ a^2+b^2 \textbf{(D)}\ \frac{1}{2}(a^2+b^2) \qquad \textbf{(E)}\ \text{none of these}$

Problem 4

A barn with a flat roof is rectangular in shape, 10 yd. wide, 13 yd. long and 5 yd. high. It is to be painted inside and outside, and on the ceiling, but not on the roof or floor. The total number of sq. yd. to be painted is:

$\textbf{(A)}\ 360 \qquad \textbf{(B)}\ 460 \qquad \textbf{(C)}\ 490 \qquad \textbf{(D)}\ 590 \qquad \textbf{(E)}\ 720$

Problem 5

Mr. A owns a home worth $10,000$ dollars. He sells it to Mr. B at a 10% profit based on the worth of the house. Mr. B sells the house back to Mr. A at a 10% loss. Then:

$\textbf{(A)}\ \text{A comes out even} \qquad \textbf{(B)}\ \text{A makes 1100 on the deal} \qquad \textbf{(C)}\ \text{A makes 1000 on the deal}$

$\textbf{(D)}\ \text{A loses 900 on the deal} \qquad \textbf{(E)}\ \text{A loses 1000 on the deal}$

Problem 6

The bottom, side, and front areas of a rectangular box are known. The product of these areas is equal to:

$\textbf{(A)}\ \text{the volume of the box} \qquad \textbf{(B)}\ \text{the square root of the volume} \qquad \textbf{(C)}\ \text{twice the volume}$

$\textbf{(D)}\ \text{the square of the volume} \qquad \textbf{(E)}\ \text{the cube of the volume}$

Problem 7

An error of $.02$" is made in the measurement of a line 10" long, while an error of only $.2$" is made in a measurement of a line 100" long. In comparison with the relative error of the first measurement, the relative error of the second measurement is:

$\textbf{(A)}\ \text{greater by .18} \qquad \textbf{(B)}\ \text{the same} \qquad \textbf{(C)}\ \text{less} \qquad \textbf{(D)}\ \text{10 times as great} \qquad \textbf{(E)}\ \text{correctly described by both}$

Problem 8

The price of an article is cut 10%. To restore it to its former value, the new price must be increased by:

$\textbf{(A)}\ 10\% \qquad \textbf{(B)}\ 9\% \qquad \textbf{(C)}\ 11\frac{1}{9}\% \qquad \textbf{(D)}\ 11\% \qquad \textbf{(E)}\ \text{none of these answers}$

Problem 9

An equilateral triangle is drawn with a side length of a. A new equilateral triangle is formed by joining the midpoints of the sides of the first one. then a third equilateral triangle is formed by joining the midpoints of the sides of the second; and so on forever. the limit of the sum of the perimeters of all the triangles thus drawn is:

$\textbf{(A)}\ \text{Infinite} \qquad \textbf{(B)}\ 5\frac{1}{4}a \qquad \textbf{(C)}\ 2a \qquad \textbf{(D)}\ 6a \qquad \textbf{(E)}\ 4\frac{1}{2}a$

Problem 10

Of the following statements, the one that is incorrect is:

$\textbf{(A)}$ Doubling the base of a given rectangle doubles the area. $\textbf{(B)}$ Doubling the altitude of a triangle doubles the area.

$\textbf{(C)}$ Doubling the radius of a given circle doubles the area.

$\textbf{(D)}$ Doubling the divisor of a fraction and dividing its numerator by 2 changes the quotient.

$\textbf{(E)}$ Doubling a given quantity may make it less than it originally was.

Problem 11

The limit of the sum of an infinite number of terms in a geometric progression is $\frac{a}{1-r}$ where a denotes the first term and $-1 < r < 1$denotes the common ratio. The limit of the sum of their squares is:

$\textbf{(A)}\ \frac{a^2}{(1-r)^2} \qquad \textbf{(B)}\ \frac{a^2}{1+r^2} \qquad \textbf{(C)}\ \frac{a^2}{1-r^2} \qquad \textbf{(D)}\ \frac{4a^2}{1+r^2} \qquad \textbf{(E)}\ \text{none of these}$

Problem 12

At $2:15$ o'clock, the hour and minute hands of a clock form an angle of:

$\textbf{(A)}\ 30^{\circ} \qquad \textbf{(B)}\ 5^{\circ} \qquad \textbf{(C)}\ 22\frac{1}{2}^{\circ} \qquad \textbf{(D)}\ 7\frac{1}{2}^{\circ} \qquad \textbf{(E)}\ 28^{\circ}$

Problem 13

A can do a piece of work in 9 days. B is 50% more efficient than A. The number of days it takes B to do the same piece of work is:

$\textbf{(A)}\ 13\frac{1}{2} \qquad \textbf{(B)}\ 4\frac{1}{2} \qquad \textbf{(C)}\ 6 \qquad \textbf{(D)}\ 3 \qquad \textbf{(E)}\ \text{none of these answers}$

Problem 14

In connection with proof in geometry, indicate which one of the following statements is *incorrect*:

$\textbf{(A)}$ Some statements are accepted without being proved.

$\textbf{(B)}$ In some cases there is more than one correct order in proving certain propositions.

$\textbf{(C)}$ Every term used in a proof must have been defined previously.

$\textbf{(D)}$ It is not possible to arrive by correct reasoning at a true conclusion if, in the given, there is an untrue proposition.

$\textbf{(E)}$ Indirect proof can be used whenever there are two or more contrary propositions.

Problem 15

The largest number by which the expression $n^3 - n$ is divisible for all possible integral values of n, is:

$\textbf{(A)}\ 2 \qquad \textbf{(B)}\ 3 \qquad \textbf{(C)}\ 4 \qquad \textbf{(D)}\ 5 \qquad \textbf{(E)}\ 6$

Problem 16

If in applying the quadratic formula to a quadratic equation

$f(x) \equiv ax^2 + bx + c = 0,$

it happens that $c = \frac{b^2}{4a}$, then the graph of $y = f(x)$ will certainly:

$\textbf{(A)}\ \text{have a maximum} \qquad \textbf{(B)}\ \text{have a minimum} \qquad \textbf{(C)}\ \text{be tangent to the x-axis}$
$\textbf{(D)}\ \text{be tangent to the y-axis} \qquad \textbf{(E)}\ \text{lie in one quadrant only}$

Problem 17

Indicate in which one of the following equations y is neither directly nor inversely proportional to x:

$\textbf{(A)}\ x + y = 0 \qquad \textbf{(B)}\ 3xy = 10 \qquad \textbf{(C)}\ x = 5y \qquad \textbf{(D)}\ 3x + y = 10$

Problem 18

The expression $21x^2 + ax + 21$ is to be factored into two linear prime binomial factors with integer coefficients. This can be one if a is:

$\textbf{(A)}\ \text{any odd number} \qquad \textbf{(B)}\ \text{some odd number} \qquad \textbf{(C)}\ \text{any even number} \qquad \textbf{(D)}\ \text{some even number} \qquad \textbf{(E)}\ \text{zero}$

Problem 19

A six place number is formed by repeating a three place number; for example, 256256 or 678678, etc. Any number of this form is always exactly divisible by:

$\textbf{(A)}\ 7 \text{ only} \qquad \textbf{(B)}\ 11 \text{ only} \qquad \textbf{(C)}\ 13 \text{ only} \qquad \textbf{(D)}\ 101 \qquad \textbf{(E)}\ 1001$

Problem 20

When simplified and expressed with negative exponents, the expression $(x + y)^{-1}(x^{-1} + y^{-1})$ is equal to:

$\textbf{(A)}\ x^{-2} + 2x^{-1}y^{-1} + y^{-2} \qquad \textbf{(B)}\ x^{-2} + 2^{-1}x^{-1}y^{-1} + y^{-2} \qquad \textbf{(C)}\ x^{-1}y^{-1}$ $\textbf{(D)}\ \text{some even number} \qquad \textbf{(E)}\ \text{zero}$

Problem 21

Given: $x > 0, y > 0, x > y$ and $z \neq 0$. The inequality which is not always correct is:

$\textbf{(A)}\ x + z > y + z \qquad \textbf{(B)}\ x - z > y - z \qquad \textbf{(C)}\ xz > yz$ $\textbf{(D)}\ \frac{x}{z^2} > \frac{y}{z^2} \qquad \textbf{(E)}\ xz^2 > yz^2$

Problem 22

The values of a in the equation: $\log_{10}(a^2 - 15a) = 2$ are:

$\textbf{(A)}\ \frac{15 \pm \sqrt{233}}{2} \qquad \textbf{(B)}\ 20, -5 \qquad \textbf{(C)}\ \frac{15 \pm \sqrt{305}}{2} \qquad \textbf{(D)}\ \pm 20$ $\textbf{(E)}\ \text{none of these}$

Problem 23

The radius of a cylindrical box is 8 inches and the height is 3 inches. The number of inches that may be added to either the radius or the height to give the same nonzero increase in volume is:

$\textbf{(A)}\ 1 \qquad \textbf{(B)}\ 5\frac{1}{3} \qquad \textbf{(C)}\ \text{any number} \qquad \textbf{(D)}\ \text{non-existent} \qquad \textbf{(E)}\ \text{none of these}$

Problem 24

$\frac{2^{n+4}-2(2^n)}{2(2^{n+3})}$ when simplified is:

$\textbf{(A)}\ 2^{n+1}-\frac{1}{8} \qquad \textbf{(B)}\ -2^{n+1} \qquad \textbf{(C)}\ 1-2^n \qquad \textbf{(D)}\ \frac{7}{8} \qquad \textbf{(E)}\ \frac{7}{4}$

Problem 25

The apothem of a square having its area numerically equal to its perimeter is compared with the apothem of an equilateral triangle having its area numerically equal to its perimeter. The first apothem will be:

$\textbf{(A)}\ \text{equal to the second} \qquad \textbf{(B)}\ \frac{4}{3}\text{ times the second} \qquad \textbf{(C)}\ \frac{2}{\sqrt{3}}\text{ times the second}$
$\textbf{(D)}\ \frac{\sqrt{2}}{\sqrt{3}}\text{ times the second} \qquad \textbf{(E)}\ \text{indeterminately related to the second}$

Problem 26

In the equation $\frac{x(x-1)-(m+1)}{(x-1)(m-1)}=\frac{x}{m}$ the roots are equal when:

$\textbf{(A)}\ m=1 \qquad \textbf{(B)}\ m=\frac{1}{2} \qquad \textbf{(C)}\ m=0 \qquad \textbf{(D)}\ m=-1 \qquad \textbf{(E)}\ m=-\frac{1}{2}$

Problem 27

Through a point inside a triangle, three lines are drawn from the vertices to the opposite sides forming six triangular sections. Then:

$\textbf{(A)}\ \text{the triangles are similar in opposite pairs} \qquad \textbf{(B)}\ \text{the triangles are congruent in opposite pairs}$

$\textbf{(C)}\ \text{the triangles are equal in area in opposite pairs} \qquad \textbf{(D)}\ \text{three similar quadrilaterals are formed}$

$\textbf{(E)}\ \text{none of the above relations are true}$

Problem 28

The pressure (P) of wind on a sail varies jointly as the area (A) of the sail and the square of the velocity (V) of the wind. The pressure on a square foot is 1 pound when the velocity is 16 miles per hour. The velocity of the wind when the pressure on a square yard is 36pounds is:

$\textbf{(A)}\ 10\frac{2}{3}\text{ mph} \qquad \textbf{(B)}\ 96\text{ mph} \qquad \textbf{(C)}\ 32\text{ mph} \qquad \textbf{(D)}\ 1\frac{2}{3}\text{ mph} \qquad \textbf{(E)}\ 16\text{ mph}$

Problem 29

Of the following sets of data the only one that does not determine the shape of a triangle is:

(A) the ratio of two sides and the included angle
(B) the ratios of the three altitudes
(C) the ratios of the three medians
(D) the ratio of the altitude to the corresponding base
(E) two angles

Problem 30

If two poles $20''$ and $80''$ high are $100''$ apart, then the height of the intersection of the lines joining the top of each pole to the foot of the opposite pole is:

(A) $50''$ **(B)** $40''$ **(C)** $16''$ **(D)** $60''$ **(E)** none of these

Problem 31

A total of 28 handshakes were exchanged at the conclusion of a party. Assuming that each participant was equally polite toward all the others, the number of people present was:

(A) 14 **(B)** 28 **(C)** 56 **(D)** 8 **(E)** 7

Problem 32

If $\triangle ABC$ is inscribed in a semicircle whose diameter is AB, then $AC+BC$ must be

(A) equal to AB **(B)** equal to $AB\sqrt{2}$ **(C)** $\geq AB\sqrt{2}$ **(D)** $\leq AB\sqrt{2}$ **(E)** AB^2

Problem 33

The roots of the equation $x^2-2x=0$ can be obtained graphically by finding the abscissas of the points of intersection of each of the following pairs of equations except the pair:

(A) $y=x^2, y=2x$ **(B)** $y=x^2-2x, y=0$ **(C)** $y=x, y=x-2$ **(D)** $y=x^2-2x+1, y=1$
(E) $y=x^2-1, y=2x-1$

[Note: Abscissas means x-coordinate.]

Problem 34

The value of $10^{\log_{10}7}$ is:

(A) 7 **(B)** 1 **(C)** 10 **(D)** $\log_{10}7$ **(E)** $\log_{7}10$

Problem 35

If $a^x=c^q=b$ and $c^y=a^z=d$, then

(A) $xy=qz$ **(B)** $\frac{x}{y}=\frac{q}{z}$ **(C)** $x+y=q+z$ **(D)** $x-y=q-z$ **(E)** $x^y=q^z$

Problem 36

Which of the following methods of proving a geometric figure a locus is not correct?

(A) Every point of the locus satisfies the conditions and every point not on the locus does not satisfy the conditions.

(B) Every point not satisfying the conditions is not on the locus and every point on the locus does satisfy the conditions.

(C) Every point satisfying the conditions is on the locus and every point on the locus satisfies the conditions.

(D) Every point not on the locus does not satisfy the conditions and every point not satisfying the conditions is not on the locus.
(E) Every point satisfying the conditions is on the locus and every point not satisfying the conditions is not on the locus.

Problem 37

A number which when divided by 10 leaves a remainder of 9, when divided by 9 leaves a remainder of 8, by 8 leaves a remainder of 7, etc., down to where, when divided by 2, it leaves a remainder of 1, is:

$\textbf{(A)}\ 59 \qquad \textbf{(B)}\ 419 \qquad \textbf{(C)}\ 1259 \qquad \textbf{(D)}\ 2519 \qquad \textbf{(E)}\ \text{none of these answers}$

Problem 38

A rise of 600 feet is required to get a railroad line over a mountain. The grade can be kept down by lengthening the track and curving it around the mountain peak. The additional length of track required to reduce the grade from 3% to 2% is approximately:

$\textbf{(A)}\ 10000\text{ ft.}\qquad\textbf{(B)}\ 20000\text{ ft.}\qquad\textbf{(C)}\ 30000\text{ ft.}\qquad\textbf{(D)}\ 12000\text{ ft.}\qquad\textbf{(E)}\ \text{none of these}$

Problem 39

A stone is dropped into a well and the report of the stone striking the bottom is heard 7.7 seconds after it is dropped. Assume that the stone falls $16t^2$ feet in t seconds and that the velocity of sound is 1120 feet per second. The depth of the well is:

$\textbf{(A)}\ 784\text{ ft.}\qquad\textbf{(B)}\ 342\text{ ft.}\qquad\textbf{(C)}\ 1568\text{ ft.}\qquad\textbf{(D)}\ 156.8\text{ ft.}\qquad\textbf{(E)}\ \text{none of these}$

Problem 40

$\left(\frac{(x+1)^{2}(x^{2}-x+1)^{2}}{(x^{3}+1)^{2}}\right)^{2}\cdot\left(\frac{(x-1)^{2}(x^{2}+x+1)^{2}}{(x^{3}-1)^{2}}\right)^{2}$ equals:

$\textbf{(A)}\ (x+1)^{4}\qquad\textbf{(B)}\ (x^{3}+1)^{4}\qquad\textbf{(C)}\ 1\qquad\textbf{(D)}\ [(x^{3}+1)(x^{3}-1)]^{2}$ $\textbf{(E)}\ [(x^{3}-1)^{2}]^{2}$

Problem 41

The formula expressing the relationship between x and y in the table is:

x	2	3	4	5	6
y	0	2	6	12	20

$\textbf{(A)}\ y = 2x-4\qquad\textbf{(B)}\ y = x^{2}-3x+2\qquad\textbf{(C)}\ y = x^{3}-3x^{2}+2x$ $\textbf{(D)}\ y = x^{2}-4x\qquad\textbf{(E)}\ y = x^{2}-4$

Problem 42

If $x=\sqrt{1+\sqrt{1+\sqrt{1+\sqrt{1+\cdots}}}}$, then:

$\textbf{(A)}\ x=1\qquad\textbf{(B)}\ 0<x<1\qquad\textbf{(C)}\ 1<x<2\qquad\textbf{(D)}\ x\text{ is infinite}$ $\textbf{(E)}\ x>2\text{ but finite}$

Problem 43

Of the following statements, the only one that is incorrect is:

$\textbf{(A)}\ \text{An inequality will remain true after each side is increased,}$

$\text{ decreased, multiplied or divided zero excluded by the same positive quantity.}$

$\textbf{(B)}\ \text{The arithmetic mean of two unequal positive quantities is greater than their geometric mean.}$

$\textbf{(C)}\ \text{If the sum of two positive quantities is given, ther product is largest when they are equal.}$

$\textbf{(D)}\ \text{If }a\text{ and }b\text{ are positive and unequal, }\frac{1}{2}(a^{2}+b^{2})\text{ is greater than }[\frac{1}{2}(a+b)]^{2}.$

(E) If the product of two positive quantities is given, their sum is greatest when they are equal.

Problem 44

If $\frac{xy}{x+y}=a, \frac{xz}{x+z}=b, \frac{yz}{y+z}=c$, where a, b, c are other than zero, then x equals:

(A) $\frac{abc}{ab+ac+bc}$ **(B)** $\frac{2abc}{ab+bc+ac}$ **(C)** $\frac{2abc}{ab+ac-bc}$ **(D)** $\frac{2abc}{ab+bc-ac}$ **(E)** $\frac{2abc}{ac+bc-ab}$

Problem 45

If you are given $\log 8 \approx .9031$ and $\log 9 \approx .9542$, then the only logarithm that cannot be found without the use of tables is:

(A) $\log 17$ **(B)** $\log \frac{5}{4}$ **(C)** $\log 15$ **(D)** $\log 600$ **(E)** $\log .4$

Problem 46

AB is a fixed diameter of a circle whose center is O. From C, any point on the circle, a chord CD is drawn perpendicular to AB. Then, as C moves over a semicircle, the bisector of angle OCD cuts the circle in a point that always:

(A) bisects the arc AB **(B)** trisects the arc AB **(C)** varies

(D) is as far from AB as from D **(E)** is equidistant from B and C

Problem 47

If r and s are the roots of the equation $ax^2+bx+c=0$, the value of $\frac{1}{r^2}+\frac{1}{s^2}$ is:

(A) b^2-4ac **(B)** $\frac{b^2-4ac}{2a}$ **(C)** $\frac{b^2-4ac}{c^2}$ **(D)** $\frac{b^2-2ac}{c^2}$ **(E)** none of these

Problem 48

The area of a square inscribed in a semicircle is to the area of the square inscribed in the entire circle as:

(A) $1:2$ **(B)** $2:3$ **(C)** $2:5$ **(D)** $3:4$ **(E)** $3:5$

Problem 49

The medians of a right triangle which are drawn from the vertices of the acute angles are 5 and $\sqrt{40}$. The value of the hypotenuse is:

(A) 10 **(B)** $2\sqrt{40}$ **(C)** $\sqrt{13}$ **(D)** $2\sqrt{13}$ **(E)** none of these

Problem 50

Tom, Dick and Harry started out on a 100-mile journey. Tom and Harry went by automobile at the rate of 25 mph, while Dick walked at the rate of 5 mph. After a certain distance, Harry got off and walked on at 5 mph, while Tom went back for Dick and got him to the destination at the same time that Harry arrived. The number of hours required for the trip was:

(A) 5 **(B)** 6 **(C)** 7 **(D)** 8 **(E)** none of these answers