1952 AHSME Problems

Problem 1

If the radius of a circle is a rational number, its area is given by a number which is:

$\textbf{(A)}\ \text{rational} \qquad \textbf{(B)}\ \text{irrational} \qquad \textbf{(C)}\ \text{integral} \qquad \textbf{(D)}\ \text{a perfect square} \qquad \textbf{(E)}\ \text{none of these}$

Problem 2

Two high school classes took the same test. One class of 20 students made an average grade of 80%; the other class of 30 students made an average grade of 70%. The average grade for all students in both classes is:

$\textbf{(A)}\ 75\% \qquad \textbf{(B)}\ 74\% \qquad \textbf{(C)}\ 72\% \qquad \textbf{(D)}\ 77\% \qquad \textbf{(E)}\ \text{none of these}$

Problem 3

The expression a^3-a^{-3} equals:

$\textbf{(A)}\ \left(a-\frac{1}{a}\right)\left(a^2+1+\frac{1}{a^2}\right) \qquad \textbf{(B)}\ \left(\frac{1}{a}-a\right)\left(a^2-1+\frac{1}{a^2}\right) \qquad \textbf{(C)}\ \left(a-\frac{1}{a}\right)\left(a^2-2+\frac{1}{a^2}\right)$

$\textbf{(D)}\ \left(\frac{1}{a}-a\right)\left(\frac{1}{a^2}+1+a^2\right) \qquad \textbf{(E)}\ \text{none of these}$

Problem 4

The cost C of sending a parcel post package weighing P pounds, P an integer, is 10 cents for the first pound and 3 cents for each additional pound. The formula for the cost is:

$\textbf{(A)}\ C=10+3P \qquad \textbf{(B)}\ C=10P+3 \qquad \textbf{(C)}\ C=10+3(P-1)$

$\textbf{(D)}\ C=9+3P \qquad \textbf{(E)}\ C=10P-7$

Problem 5

The points $(6,12)$ and $(0,-6)$ are connected by a straight line. Another point on this line is:

$\textbf{(A)}\ (3,3) \qquad \textbf{(B)}\ (2,1) \qquad \textbf{(C)}\ (7,16) \qquad \textbf{(D)}\ (-1,-4) \qquad \textbf{(E)}\ (-3,-8)$

Problem 6

The difference of the roots of $x^2-7x-9=0$ is:

$\textbf{(A)}\ +7 \qquad \textbf{(B)}\ +\frac{7}{2} \qquad \textbf{(C)}\ +9 \qquad \textbf{(D)}\ 2\sqrt{85} \qquad \textbf{(E)}\ \sqrt{85}$

Problem 7

When simplified, $(x^{-1}+y^{-1})^{-1}$ is equal to:

$\textbf{(A)}\ x+y \qquad \textbf{(B)}\ \frac{xy}{x+y} \qquad \textbf{(C)}\ xy \qquad \textbf{(D)}\ \frac{1}{xy} \qquad \textbf{(E)}\ \frac{x+y}{xy}$

Problem 8

Two equal circles in the same plane cannot have the following number of common tangents.

(A) 1 **(B)** 2 **(C)** 3 **(D)** 4 **(E)** none of these

Problem 9

If $m=\frac{cab}{a-b}$, then b equals:

(A) $\frac{m(a-b)}{ca}$ **(B)** $\frac{cab-ma}{-m}$ **(C)** $\frac{1}{1+c}$ **(D)** $\frac{ma}{m+ca}$ **(E)** $\frac{m+ca}{ma}$

Problem 10

An automobile went up a hill at a speed of 10 miles an hour and down the same distance at a speed of 20 miles an hour. The average speed for the round trip was:

(A) $12\frac{1}{2}$mph **(B)** $13\frac{1}{3}$mph **(C)** $14\frac{1}{2}$mph **(D)** 15mph **(E)** none of these

Problem 11

If $y=f(x)=\frac{x+2}{x-1}$, then it is incorrect to say:

(A) $x=\frac{y+2}{y-1}$ **(B)** $f(0)=-2$ **(C)** $f(1)=0$

(D) $f(-2)=0$ **(E)** $f(y)=x$

Problem 12

The sum to infinity of the terms of an infinite geometric progression is 6. The sum of the first two terms is $4\frac{1}{2}$. The first term of the progression is:

(A) 3 or $1\frac{1}{2}$ **(B)** 1 **(C)** $2\frac{1}{2}$ **(D)** 6 **(E)** 9 or 3

Problem 13

The function x^2+px+q with p and q greater than zero has its minimum value when:

(A) $x=-p$ **(B)** $x=\frac{p}{2}$ **(C)** $x=-2p$ **(D)** $x=\frac{p^2}{4q}$

(E) $x=\frac{-p}{2}$

Problem 14

A house and store were sold for $\$12,000$ each. The house was sold at a loss of 20% of the cost, and the store at a gain of 20% of the cost. The entire transaction resulted in:

(A) no loss or gain **(B)** loss of \$1000 **(C)** gain of \$1000 **(D)** gain of \$2000 **(E)** none of these

Problem 15

The sides of a triangle are in the ratio $6:8:9$. Then:

(A) the triangle is obtuse

(B) the angles are in the ratio $6:8:9$
(C) the triangle is acute
(D) the angle opposite the largest side is double the angle opposite the smallest side
(E) none of these

Problem 16

If the base of a rectangle is increased by 10% and the area is unchanged, then the altitude is decreased by:

$\textbf{(A)}\ 9\% \qquad \textbf{(B)}\ 10\% \qquad \textbf{(C)}\ 11\% \qquad \textbf{(D)}\ 11\frac{1}{9}\% \qquad \textbf{(E)}\ 9\frac{1}{11}\%$

Problem 17

A merchant bought some goods at a discount of 20% of the list price. He wants to mark them at such a price that he can give a discount of 20% of the marked price and still make a profit of 20% of the selling price.. The per cent of the list price at which he should mark them is:

$\textbf{(A)}\ 20 \qquad \textbf{(B)}\ 100 \qquad \textbf{(C)}\ 125 \qquad \textbf{(D)}\ 80 \qquad \textbf{(E)}\ 120$

Problem 18

$\log p+\log q=\log(p+q)$ only if:

$\textbf{(A)}\ p=q=\text{zero} \qquad \textbf{(B)}\ p=\frac{q^2}{1-q} \qquad \textbf{(C)}\ p=q=1$

$\textbf{(D)}\ p=\frac{q}{q-1} \qquad \textbf{(E)}\ p=\frac{q}{q+1}$

Problem 19

Angle B of triangle ABC is trisected by BD and BE which meet AC at D and E respectively. Then:

$\textbf{(A)}\ \frac{AD}{EC}=\frac{AE}{DC} \qquad \textbf{(B)}\ \frac{AD}{EC}=\frac{AB}{BC} \qquad \textbf{(C)}\ \frac{AD}{EC}=\frac{BD}{BE}$

$\textbf{(D)}\ \frac{AD}{EC}=\frac{(AB)(BD)}{(BE)(BC)} \qquad \textbf{(E)}\ \frac{AD}{EC}=\frac{(AE)(BD)}{(DC)(BE)}$

Problem 20

If $\frac{x}{y}=\frac{3}{4}$, then the incorrect expression in the following is:

$\textbf{(A)}\ \frac{x+y}{y}=\frac{7}{4} \qquad \textbf{(B)}\ \frac{y}{y-x}=\frac{4}{1} \qquad \textbf{(C)}\ \frac{x+2y}{x}=\frac{11}{3}$

$\textbf{(D)}\ \frac{x}{2y}=\frac{3}{8} \qquad \textbf{(E)}\ \frac{x-y}{y}=\frac{1}{4}$

Problem 21

The sides of a regular polygon of n sides, $n>4$, are extended to form a star. The number of degrees at each point of the star is:

$\textbf{(A)}\ \frac{360}{n} \qquad \textbf{(B)}\ \frac{(n-4)180}{n} \qquad \textbf{(C)}\ \frac{(n-2)180}{n}$

$\textbf{(D) }180-\frac{90}{n} \qquad \textbf{(E) }\frac{180}{n}$

Problem 22

On hypotenuse AB of a right triangle ABC a second right triangle ABD is constructed with hypotenuse AB. If $\overline{BC}=1$, $\overline{AC}=b$, and $\overline{AD}=2$, then $\overline{BD}$ equals:

$\textbf{(A) }\sqrt{b^2+1} \qquad \textbf{(B) }\sqrt{b^2-3} \qquad \textbf{(C) }\sqrt{b^2+1}+2$

$\textbf{(D) }b^2+5 \qquad \textbf{(E) }\sqrt{b^2+3}$

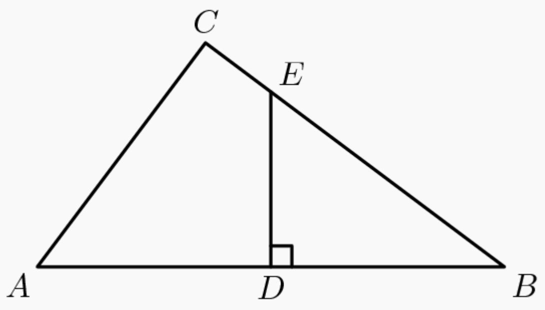
Problem 23

If $\frac{x^2-bx}{ax-c}=\frac{m-1}{m+1}$ has roots which are numerically equal but of opposite signs, the value of m must be:

$\textbf{(A) }\frac{a-b}{a+b} \qquad \textbf{(B) }\frac{a+b}{a-b} \qquad \textbf{(C) }c \qquad \textbf{(D) }\frac{1}{c} \qquad \textbf{(E) }1$

Problem 24

In the figure, it is given that angle $C=90^\circ$,$\overline{AD}=\overline{DB}$,$DE\perp AB$, $\overline{AB}=20$, and $\overline{AC}=12$. The area of quadrilateral $ADEC$ is:



$\textbf{(A) }75 \qquad \textbf{(B) }58\frac{1}{2} \qquad \textbf{(C) }48 \qquad \textbf{(D) }37\frac{1}{2} \qquad \textbf{(E) }\text{none of these}$

Problem 25

A powderman set a fuse for a blast to take place in 30 seconds. He ran away at a rate of 8 yards per second. Sound travels at the rate of 1080 feet per second. When the powderman heard the blast, he had run approximately:

$\textbf{(A) }200\text{ yd.} \qquad \textbf{(B) }352\text{ yd.} \qquad \textbf{(C) }300\text{ yd.} \qquad \textbf{(D) }245\text{ yd.} \qquad \textbf{(E) }512\text{ yd.}$

Problem 26

If $\left(r+\frac{1}{r}\right)^2=3$, then $r^3+\frac{1}{r^3}$ equals

$\textbf{(A) }1 \qquad \textbf{(B) }2 \qquad \textbf{(C) }0 \qquad \textbf{(D) }3 \qquad \textbf{(E) }6$

Problem 27

The ratio of the perimeter of an equilateral triangle having an altitude equal to the radius of a circle, to the perimeter of an equilateral triangle inscribed in the circle is:

$\textbf{(A) }1:2 \qquad \textbf{(B) }1:3 \qquad \textbf{(C) }1:\sqrt{3} \qquad \textbf{(D) }\sqrt{3}:2 \qquad \textbf{(E) }2:3$

Problem 28

In the table shown, the formula relating x and y is:

x	1	2	3	4	5
y	3	7	13	21	31

$\textbf{(A)}\ y = 4x - 1 \qquad \textbf{(B)}\ y = x^3 - x^2 + x + 2 \qquad \textbf{(C)}\ y = x^2 + x + 1$
$\textbf{(D)}\ y = (x^2 + x + 1)(x - 1) \qquad \textbf{(E)}\ \text{None of these}$

Problem 29

In a circle of radius 5 units, CD and AB are perpendicular diameters. A chord cutting CH cutting AB at K is 8 units long. The diameter AB is divided into two segments whose dimensions are:

$\textbf{(A)}\ 1.25, 8.75 \qquad \textbf{(B)}\ 2.75, 7.25 \qquad \textbf{(C)}\ 2, 8 \qquad \textbf{(D)}\ 4, 6 \qquad \textbf{(E)}\text{None of these}$

Problem 30

When the sum of the first ten terms of an arithmetic progression is four times the sum of the first five terms, the ratio of the first term to the common difference is:

$\textbf{(A)}\ 1:2 \qquad \textbf{(B)}\ 2:1 \qquad \textbf{(C)}\ 1:4 \qquad \textbf{(D)}\ 4:1 \qquad \textbf{(E)}\ 1:1$

Problem 31

Given 12 points in a plane no three of which are collinear, the number of lines they determine is:

$\textbf{(A)}\ 24 \qquad \textbf{(B)}\ 54 \qquad \textbf{(C)}\ 120 \qquad \textbf{(D)}\ 66 \qquad \textbf{(E)}\ \text{none of these}$

Problem 32

K takes 30 minutes less time than M to travel a distance of 30 miles. K travels $\frac {1}{3}$ mile per hour faster than M. If x is K's rate of speed in miles per hours, then K's time for the distance is:

$\textbf{(A)}\ \dfrac{x + \frac {1}{3}}{30} \qquad \textbf{(B)}\ \dfrac{x - \frac {1}{3}}{30} \qquad \textbf{(C)}\ \frac{30}{x+\frac{1}{3}}\qquad \textbf{(D)}\ \frac{30}{x}\qquad \textbf{(E)}\ \frac{x}{30}$

Problem 33

A circle and a square have the same perimeter. Then:

(A) their areas are equal
(B) the area of the circle is the greater
(C) the area of the square is the greater
(D) the area of the circle is π times the area of the square
(E) none of these

Problem 34

The price of an article was increased $p\%$. Later the new price was decreased $p\%$. If the last price was one dollar, the original price was:

$\textbf{(A)}\ \frac{1-p^2}{200}\qquad \textbf{(B)}\ \frac{\sqrt{1-p^2}}{100}\qquad \textbf{(C)}\ \text{one dollar}$
$\textbf{(D)}\ 1-\frac{p^2}{10000-p^2}\qquad \textbf{(E)}\ \frac{10000}{10000-p^2}$

Problem 35

With a rational denominator, the expression $\frac{\sqrt{2}}{\sqrt{2}+\sqrt{3}-\sqrt{5}}$ is equivalent to:

$\textbf{(A)}\ \frac{3+\sqrt{6}+\sqrt{15}}{6} \qquad \textbf{(B)}\ \frac{\sqrt{6}-2+\sqrt{10}}{6} \qquad \textbf{(C)}\ \frac{2+\sqrt{6}+\sqrt{10}}{10}$
$\textbf{(D)}\ \frac{2+\sqrt{6}-\sqrt{10}}{6} \qquad \textbf{(E)}\ \text{none of these}$

Problem 36

To be continuous at $x=-1$, the value of $\frac{x^3+1}{x^2-1}$ is taken to be:

$\textbf{(A)}\ -2 \qquad \textbf{(B)}\ 0 \qquad \textbf{(C)}\ \frac{3}{2} \qquad \textbf{(D)}\ \infty \qquad \textbf{(E)}\ -\frac{3}{2}$

Problem 37

Two equal parallel chords are drawn 8 inches apart in a circle of radius 8 inches. The area of that part of the circle that lies between the chords is:

$\textbf{(A)}\ 21\frac{1}{3}\pi-32\sqrt{3} \qquad \textbf{(B)}\ 32\sqrt{3}+21\frac{1}{3}\pi \qquad \textbf{(C)}\ 32\sqrt{3}+42\frac{2}{3}\pi$
$\textbf{(D)}\ 16\sqrt{3}+42\frac{2}{3}\pi \qquad \textbf{(E)}\ 42\frac{2}{3}\pi$

Problem 38

The area of a trapezoidal field is 1400 square yards. Its altitude is 50 yards. Find the two bases, if the number of yards in each base is an integer divisible by 8. The number of solutions to this problem is:

$\textbf{(A)}\ \text{none} \qquad \textbf{(B)}\ \text{one} \qquad \textbf{(C)}\ \text{two} \qquad \textbf{(D)}\ \text{three} \qquad \textbf{(E)}\ \text{more than three}$

Problem 39

If the perimeter of a rectangle is p and its diagonal is d, the difference between the length and width of the rectangle is:

$\textbf{(A)}\ \frac{\sqrt{8d^2-p^2}}{2} \qquad \textbf{(B)}\ \frac{\sqrt{8d^2+p^2}}{2} \qquad \textbf{(C)}\ \frac{\sqrt{6d^2-p^2}}{2}$
$\textbf{(D)}\ \frac{\sqrt{6d^2+p^2}}{2} \qquad \textbf{(E)}\ \frac{8d^2-p^2}{4}$

Problem 40

In order to draw a graph of ax^2+bx+c, a table of values was constructed. These values of the function for a set of equally spaced increasing values of x were $3844, 3969, 4096, 4227, 4356, 4489, 4624$, and 4761. The one which is incorrect is:

$\text{(A) } 4096 \qquad \text{(B) } 4356 \qquad \text{(C) } 4489 \qquad \text{(D) } 4761 \qquad \text{(E) } \text{none of these}$

Problem 41

Increasing the radius of a cylinder by 6 units increased the volume by y cubic units. Increasing the altitude of the cylinder by 6 units also increases the volume by y cubic units. If the original altitude is 2, then the original radius is:

$\text{(A) } 2 \qquad \text{(B) } 4 \qquad \text{(C) } 6 \qquad \text{(D) } 6\pi \qquad \text{(E) } 8$

Problem 42

Let D represent a repeating decimal. If P denotes the r figures of D which do not repeat themselves, and Q denotes the s figures of Dwhich do repeat themselves, then the incorrect expression is:

(A) $D = .PQQQ\dots$
(B) $10^rD = P.QQQ\dots$
(C) $10^{r+s}D = PQ.QQQ\dots$
(D) $10^r(10^s-1)D = Q(P-1)$
(E) $10^r\cdot 10^{2s}D = PQQ.QQQ\dots$

Problem 43

The diameter of a circle is divided into n equal parts. On each part a semicircle is constructed. As n becomes very large, the sum of the lengths of the arcs of the semicircles approaches a length:

$\textbf{(A)}$ equal to the semi-circumference of the original circle $\textbf{(B)}$ equal to the diameter of the original circle $\textbf{(C)}$ greater than the diameter, but less than the semi-circumference of the original circle $\textbf{(D)}$ that is infinite $\textbf{(E)}$ greater than the semi-circumference

Problem 44

If an integer of two digits is k times the sum of its digits, the number formed by interchanging the the digits is the sum of the digits multiplied by

$\textbf{(A)}\ 9-k \qquad \textbf{(B)}\ 10-k \qquad \textbf{(C)}\ 11-k \qquad \textbf{(D)}\ k-1 \qquad \textbf{(E)}\ k+1$

Problem 45

If a and b are two unequal positive numbers, then:

(A) $\dfrac{2ab}{a+b} > \sqrt{ab} > \dfrac{a+b}{2}$ (B) $\sqrt{ab} > \dfrac{2ab}{a+b} > \dfrac{a+b}{2}$
(C) $\dfrac{2ab}{a+b} > \dfrac{a+b}{2} > \sqrt{ab}$ (D) $\dfrac{a+b}{2} > \dfrac{2ab}{a+b} > \sqrt{ab}$
(E) $\dfrac{a+b}{2} > \sqrt{ab} > \dfrac{2ab}{a+b}$

Problem 46

The base of a new rectangle equals the sum of the diagonal and the greater side of a given rectangle, while the altitude of the new rectangle equals the difference of the diagonal and the greater side of the given rectangle. The area of the new rectangle is:

(A) greater than the area of the given rectangle
(B) equal to the area of the given rectangle
(C) equal to the area of a square with its side equal to the smaller side of the given rectangle
(D) equal to the area of a square with its side equal to the greater side of the given rectangle
(E) equal to the area of a rectangle whose dimensions are the diagonal and the shorter side of the given rectangle

Problem 47

In the set of equations $z^x = y^{2x},\quad 2^z = 2\cdot 4^x,\quad x+y+z=16$, the integral roots in the order x, y, z are:

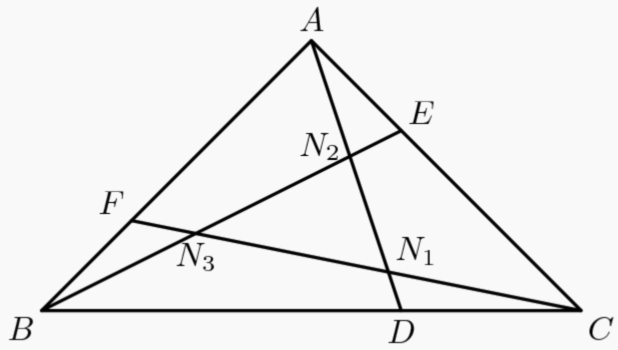
$\textbf{(A)}\ 3,4,9 \qquad \textbf{(B)}\ 9,-5,-12 \qquad \textbf{(C)}\ 12,-5,9 \qquad \textbf{(D)}\ 4,3,9 \qquad \textbf{(E)}\ 4,9,3$

Problem 48

Two cyclists, k miles apart, and starting at the same time, would be together in r hours if they traveled in the same direction, but would pass each other in t hours if they traveled in opposite directions. The ratio of the speed of the faster cyclist to that of the slower is:

(A) $\frac{r+t}{r-t}$ (B) $\frac{r}{r-t}$ (C) $\frac{r+t}{r}$ (D) $\frac{r}{t}$ (E) $\frac{r+k}{t-k}$

Problem 49



In the figure, $\overline{CD}$, $\overline{AE}$ and $\overline{BF}$ are one-third of their respective sides. It follows that $\overline{AN_2} : \overline{N_2N_1} : \overline{N_1D} = 3:3:1$, and similarly for lines BE and CF. Then the area of triangle $N_1N_2N_3$ is:

(A) $\frac{1}{10}\triangle ABC$ (B) $\frac{1}{9}\triangle ABC$ (C) $\frac{1}{7}\triangle ABC$ (D) $\frac{1}{6}\triangle ABC$ (E) none of these

Problem 50

A line initially 1 inch long grows according to the following law, where the first term is the initial length.

$$1+\frac{1}{4}\sqrt{2}+\frac{1}{4}+\frac{1}{16}\sqrt{2}+\frac{1}{16}+\frac{1}{64}\sqrt{2}+\frac{1}{64}+\cdots$$

If the growth process continues forever, the limit of the length of the line is:

$\textbf{(A)}\ \infty \qquad \textbf{(B)}\ \frac{4}{3} \qquad \textbf{(C)}\ \frac{8}{3} \qquad \textbf{(D)}\ \frac{1}{3}(4+\sqrt{2}) \qquad \textbf{(E)}\ \frac{2}{3}(4+\sqrt{2})$