1994 AHSME Problems

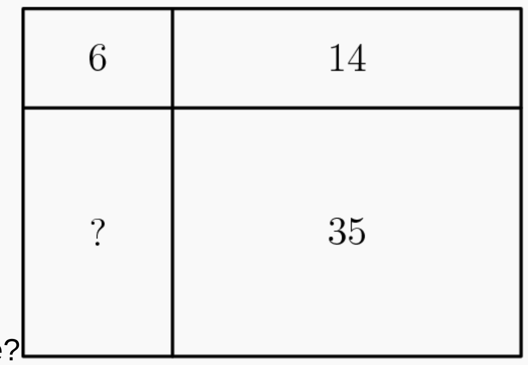
Problem 1

$4^4 \cdot 9^4 \cdot 4^9 \cdot 9^9 =$

$\textbf{(A)}\ 13^{13} \qquad \textbf{(B)}\ 13^{36} \qquad \textbf{(C)}\ 36^{13} \qquad \textbf{(D)}\ 36^{36} \qquad \textbf{(E)}\ 1296^{26}$

Problem 2

A large rectangle is partitioned into four rectangles by two segments parallel to its sides. The areas of three of the resulting rectangles are shown. What is the area of the fourth rectangle?



$\textbf{(A)}\ 10 \qquad \textbf{(B)}\ 15 \qquad \textbf{(C)}\ 20 \qquad \textbf{(D)}\ 21 \qquad \textbf{(E)}\ 25$

Problem 3

How many of the following are equal to $x^x + x^x$ for all $x > 0$?

$\textbf{I:}\ 2x^x \qquad \textbf{II:}\ x^{2x} \qquad \textbf{III:}\ (2x)^x \qquad \textbf{IV:}\ (2x)^{2x}$

$\textbf{(A)}\ 0 \qquad \textbf{(B)}\ 1 \qquad \textbf{(C)}\ 2 \qquad \textbf{(D)}\ 3 \qquad \textbf{(E)}\ 4$

Problem 4

In the xy-plane, the segment with endpoints $(-5,0)$ and $(25,0)$ is the diameter of a circle. If the point $(x,15)$ is on the circle, then $x =$

$\textbf{(A)}\ 10 \qquad \textbf{(B)}\ 12.5 \qquad \textbf{(C)}\ 15 \qquad \textbf{(D)}\ 17.5 \qquad \textbf{(E)}\ 20$

Problem 5

Pat intended to multiply a number by 6 but instead divided by 6. Pat then meant to add 14 but instead subtracted 14. After these mistakes, the result was 16. If the correct operations had been used, the value produced would have been

$\textbf{(A)}\ \text{less than 400} \qquad \textbf{(B)}\ \text{between 400 and 600} \qquad \textbf{(C)}\ \text{between 600 and 800}$
$\textbf{(D)}\ \text{between 800 and 1000} \qquad \textbf{(E)}\ \text{greater than 1000}$

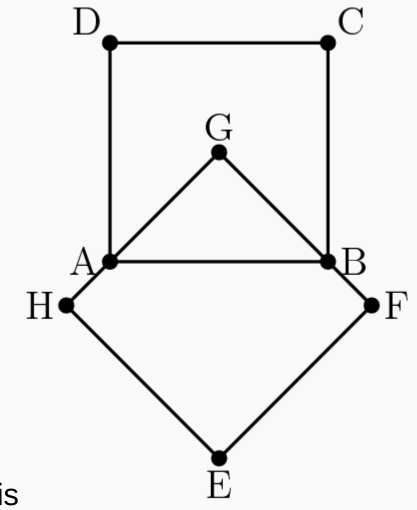
Problem 6

In the sequence $\ldots, a, b, c, d, 0, 1, 1, 2, 3, 5, 8, \ldots$ each term is the sum of the two terms to its left. Find a.

$\textbf{(A)}\ -3 \qquad \textbf{(B)}\ -1 \qquad \textbf{(C)}\ 0 \qquad \textbf{(D)}\ 1 \qquad \textbf{(E)}\ 3$

Problem 7

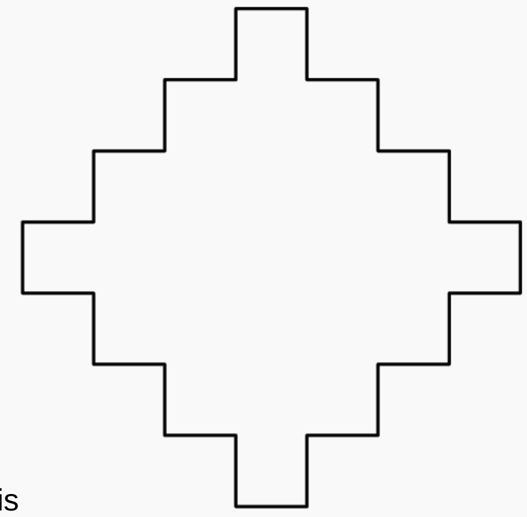
Squares $ABCD$ and $EFGH$ are congruent, $AB = 10$, and G is the center of square $ABCD$. The area of the region in the plane covered by these squares is



$\textbf{(A)}\ 75 \qquad \textbf{(B)}\ 100 \qquad \textbf{(C)}\ 125 \qquad \textbf{(D)}\ 150 \qquad \textbf{(E)}\ 175$

Problem 8

In the polygon shown, each side is perpendicular to its adjacent sides, and all 28 of the sides are congruent. The perimeter of the polygon is 56. The area of the region bounded by the polygon is



$\textbf{(A)}\ 84 \qquad \textbf{(B)}\ 96 \qquad \textbf{(C)}\ 100 \qquad \textbf{(D)}\ 112 \qquad \textbf{(E)}\ 196$

Problem 9

If $\angle A$ is four times $\angle B$, and the complement of $\angle B$ is four times the complement of $\angle A$, then $\angle B =$

$\textbf{(A)}\ 10^{\circ} \qquad \textbf{(B)}\ 12^{\circ} \qquad \textbf{(C)}\ 15^{\circ} \qquad \textbf{(D)}\ 18^{\circ} \qquad \textbf{(E)}\ 22.5^{\circ}$

Problem 10

For distinct real numbers x and y, let $M(x,y)$ be the larger of x and y and let $m(x,y)$ be the smaller of x and y.

If $a<b<c<d<e$, then $M(M(a,m(b,c)),m(d,m(a,e))) =$ $\textbf{(A)}\ a \qquad \textbf{(B)}\ b \qquad \textbf{(C)}\ c \qquad \textbf{(D)}\ d \qquad \textbf{(E)}\ e$

Problem 11

Three cubes of volume $1, 8$ and 27 are glued together at their faces. The smallest possible surface area of the resulting configuration is

$\textbf{(A)}\ 36 \qquad \textbf{(B)}\ 56 \qquad \textbf{(C)}\ 70 \qquad \textbf{(D)}\ 72 \qquad \textbf{(E)}\ 74$

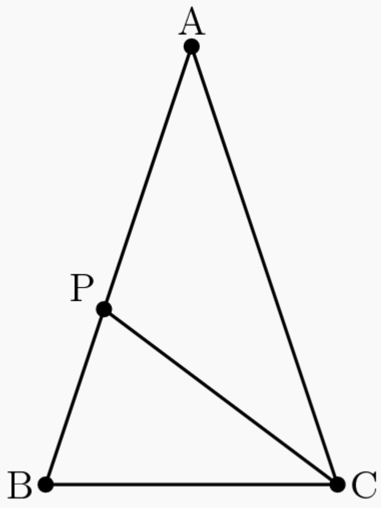
Problem 12

If $i^2 = -1$, then $(i-i^{-1})^{-1} =$

$\textbf{(A)}\ 0 \qquad \textbf{(B)}\ -2i \qquad \textbf{(C)}\ 2i \qquad \textbf{(D)}\ -\frac{i}{2} \qquad \textbf{(E)}\ \frac{i}{2}$

Problem 13

In triangle ABC, $AB = AC$. If there is a point P strictly between A and B such that $AP = PC = CB$, then $\angle A =$



$\textbf{(A)}\ 30^{\circ} \qquad \textbf{(B)}\ 36^{\circ} \qquad \textbf{(C)}\ 48^{\circ} \qquad \textbf{(D)}\ 60^{\circ} \qquad \textbf{(E)}\ 72^{\circ}$

Problem 14

Find the sum of the arithmetic series $$20+20\frac{1}{5}+20\frac{2}{5}+\cdots+40$$
$\textbf{(A)}\ 3000 \qquad \textbf{(B)}\ 3030 \qquad \textbf{(C)}\ 3150 \qquad \textbf{(D)}\ 4100 \qquad \textbf{(E)}\ 6000$

Problem 15

For how many n in $\{1, 2, 3, ..., 100\}$ is the tens digit of n^2 odd?

$\textbf{(A)}\ 10 \qquad \textbf{(B)}\ 20 \qquad \textbf{(C)}\ 30 \qquad \textbf{(D)}\ 40 \qquad \textbf{(E)}\ 50$

Problem 16

Some marbles in a bag are red and the rest are blue. If one red marble is removed, then one-seventh of the remaining marbles are red. If two blue marbles are removed instead of one red, then one-fifth of the remaining marbles are red. How many marbles were in the bag originally?

$\textbf{(A)}\ 8 \qquad \textbf{(B)}\ 22 \qquad \textbf{(C)}\ 36 \qquad \textbf{(D)}\ 57 \qquad \textbf{(E)}\ 71$

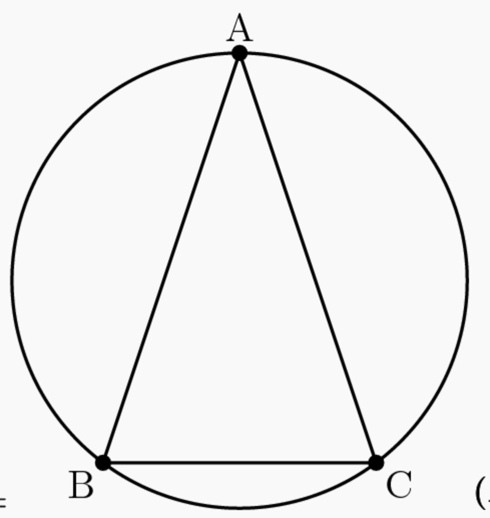
Problem 17

An 8 by $2\sqrt{2}$ rectangle has the same center as a circle of radius 2. The area of the region common to both the rectangle and the circle is

$\textbf{(A)}\ 2\pi \qquad \textbf{(B)}\ 2\pi+2 \qquad \textbf{(C)}\ 4\pi-4 \qquad \textbf{(D)}\ 2\pi+4 \qquad \textbf{(E)}\ 4\pi-2$

Problem 18

Triangle ABC is inscribed in a circle, and $\angle B = \angle C = 4\angle A$. If B and C are adjacent vertices of a regular polygon of n sides inscribed in this circle, then $n =$



$\textbf{(A)}\ 5 \qquad \textbf{(B)}\ 7 \qquad \textbf{(C)}\ 9 \qquad \textbf{(D)}\ 15 \qquad \textbf{(E)}\ 18$

Problem 19

Label one disk "1", two disks "2", three disks "3", $\cdots$, fifty disks "50". Put these $1+2+3+\cdots+50=1275$ labeled disks in a box. Disks are then drawn from the box at random without replacement. The minimum number of disks that must be drawn to guarantee drawing at least ten disks with the same label is

(A) 10 **(B)** 51 **(C)** 415 **(D)** 451 **(E)** 501

Problem 20

Suppose x, y, z is a geometric sequence with common ratio r and $x \neq y$. If $x, 2y, 3z$ is an arithmetic sequence, then r is

(A) $\frac{1}{4}$ **(B)** $\frac{1}{3}$ **(C)** $\frac{1}{2}$ **(D)** 2 **(E)** 4

Problem 21

Find the number of counter examples to the statement:
"If N is an odd positive integer the sum of whose digits is 4 and none of whose digits is 0, then N is prime."
(A) 0 **(B)** 1 **(C)** 2 **(D)** 3 **(E)** 4

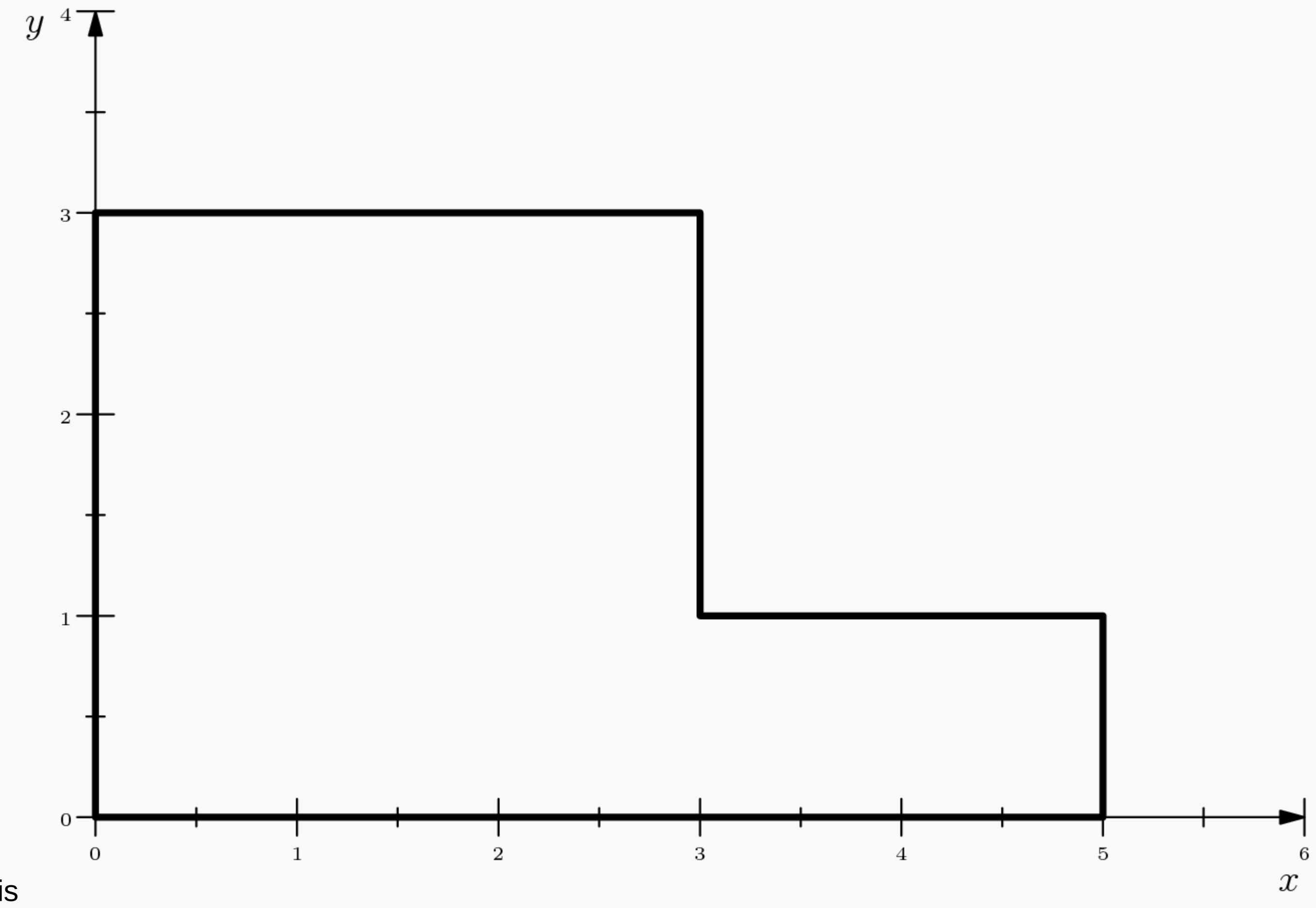
Problem 22

Nine chairs in a row are to be occupied by six students and Professors Alpha, Beta and Gamma. These three professors arrive before the six students and decide to choose their chairs so that each professor will be between two students. In how many ways can Professors Alpha, Beta and Gamma choose their chairs?

(A) 12 **(B)** 36 **(C)** 60 **(D)** 84 **(E)** 630

Problem 23

In the xy-plane, consider the L-shaped region bounded by horizontal and vertical segments with vertices at $(0,0)$, $(0,3)$, $(3,3)$, $(3,1)$, $(5,1)$ and $(5,0)$. The slope of the line through the origin that divides the area of this region exactly



in half is

(A) $\frac{2}{7}$ **(B)** $\frac{1}{3}$ **(C)** $\frac{2}{3}$ **(D)** $\frac{3}{4}$ **(E)** $\frac{7}{9}$

Problem 24

A sample consisting of five observations has an arithmetic mean of 10 and a median of 12. The smallest value that the range (largest observation minus smallest) can assume for such a sample is

$\textbf{(A)}\ 2 \qquad \textbf{(B)}\ 3 \qquad \textbf{(C)}\ 5 \qquad \textbf{(D)}\ 7 \qquad \textbf{(E)}\ 10$

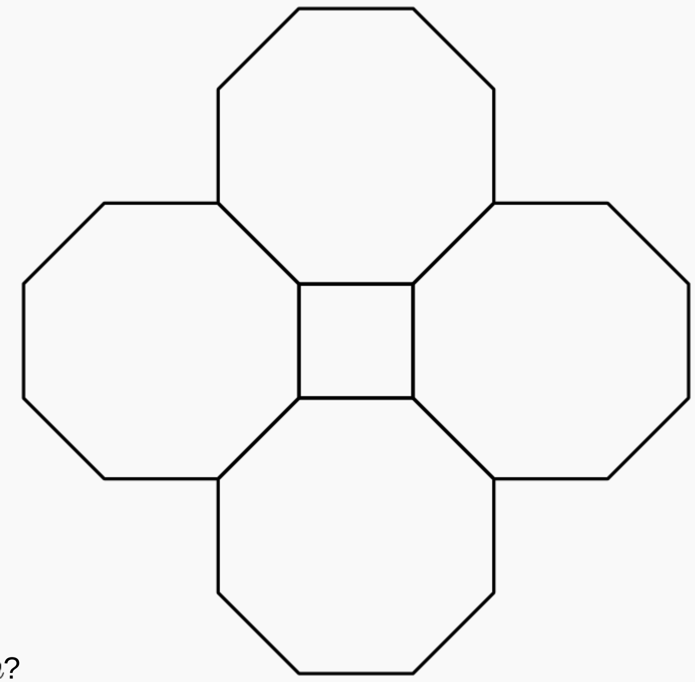
Problem 25

If x and y are non-zero real numbers such that $|x| + y = 3 \qquad \text{and} \qquad |x|y + x^3 = 0$, then the integer nearest to $x - y$ is

$\textbf{(A)}\ -3 \qquad \textbf{(B)}\ -1 \qquad \textbf{(C)}\ 2 \qquad \textbf{(D)}\ 3 \qquad \textbf{(E)}\ 5$

Problem 26

A regular polygon of m sides is exactly enclosed (no overlaps, no gaps) by m regular polygons of n sides each. (Shown here



for $m = 4, n = 8$.) If $m = 10$, what is the value of n?

$\textbf{(A)}\ 5 \qquad \textbf{(B)}\ 6 \qquad \textbf{(C)}\ 14 \qquad \textbf{(D)}\ 20 \qquad \textbf{(E)}\ 26$

Problem 27

A bag of popping corn contains $\frac{2}{3}$ white kernels and $\frac{1}{3}$ yellow kernels. Only $\frac{1}{2}$ of the white kernels will pop, whereas $\frac{2}{3}$ of the yellow ones will pop. A kernel is selected at random from the bag, and pops when placed in the popper. What is the probability that the kernel selected was white?

$\textbf{(A)}\ \frac{1}{2} \qquad \textbf{(B)}\ \frac{5}{9} \qquad \textbf{(C)}\ \frac{4}{7} \qquad \textbf{(D)}\ \frac{3}{5} \qquad \textbf{(E)}\ \frac{2}{3}$

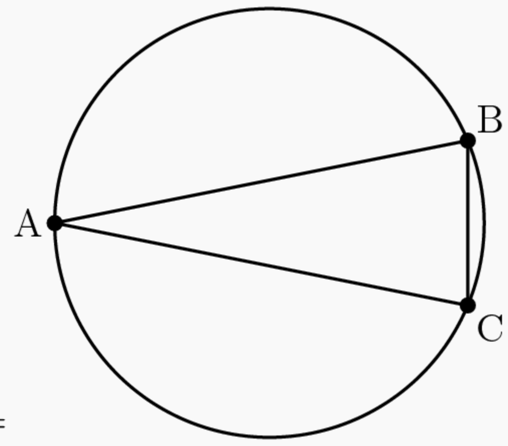
Problem 28

In the xy-plane, how many lines whose x-intercept is a positive prime number and whose y-intercept is a positive integer pass through the point $(4,3)$?

$\textbf{(A)}\ 0 \qquad \textbf{(B)}\ 1 \qquad \textbf{(C)}\ 2 \qquad \textbf{(D)}\ 3 \qquad \textbf{(E)}\ 4$

Problem 29

Points A, B and C on a circle of radius r are situated so that $AB = AC, AB > r$, and the length of minor arc BC is r. If



angles are measured in radians, then $AB/BC =$

$\textbf{(A)}\ \frac{1}{2}\csc\frac{1}{4} \qquad \textbf{(B)}\ 2\cos\frac{1}{2} \qquad \textbf{(C)}\ 4\sin\frac{1}{2} \qquad \textbf{(D)}\ \csc\frac{1}{2} \qquad \textbf{(E)}\ 2\sec\frac{1}{2}$

Problem 30

When n standard 6-sided dice are rolled, the probability of obtaining a sum of 1994 is greater than zero and is the same as the probability of obtaining a sum of S. The smallest possible value of S is

$\textbf{(A)}\ 333 \qquad \textbf{(B)}\ 335 \qquad \textbf{(C)}\ 337 \qquad \textbf{(D)}\ 339 \qquad \textbf{(E)}\ 341$