# 1956 AHSME Problems

## Problem 1

The value of $x + x(x^x)$ when $x = 2$ is:

$\textbf{(A)}\ 10 \qquad \textbf{(B)}\ 16 \qquad \textbf{(C)}\ 18 \qquad \textbf{(D)}\ 36 \qquad \textbf{(E)}\ 64$

## Problem 2

Mr. Jones sold two pipes at $\$1.20$ each. Based on the cost, his profit one was $20\%$ and his loss on the other was $20\%$. On the sale of the pipes, he:

$\textbf{(A)}\ \text{broke even} \qquad \textbf{(B)}\ \text{lost 4 cents} \qquad \textbf{(C)}\ \text{gained 4 cents}$
$\textbf{(D)}\ \text{lost 10 cents} \qquad \textbf{(E)}\ \text{gained 10 cents}$

## Problem 3

The distance light travels in one year is approximately $5,870,000,000,000$ miles. The distance light travels in $100$ years is:

$\textbf{(A)}\ 587 * 10^8\ \text{miles} \qquad \textbf{(B)}\ 587 * 10^{10}\ \text{miles} \qquad \textbf{(C)}\ 587 * 10^{-10}\ \text{miles}$
$\textbf{(D)}\ 587 * 10^{12}\ \text{miles} \qquad \textbf{(E)}\ 587 * 10^{-12}\ \text{miles}$

## Problem 4

A man has $\$10,000$ to invest. He invests $\$4000$ at 5% and $\$3500$ at 4%. In order to have a yearly income of $\$500$, he must invest the remainder at:

$\textbf{(A)}\ 6\% \qquad \textbf{(B)}\ 6.1\% \qquad \textbf{(C)}\ 6.2\% \qquad \textbf{(D)}\ 6.3\% \qquad \textbf{(E)}\ 6.4\%$

## Problem 5

A nickel is placed on a table. The number of nickels which can be placed around it, each tangent to it and to two others is:

$\textbf{(A)}\ 4 \qquad \textbf{(B)}\ 5 \qquad \textbf{(C)}\ 6 \qquad \textbf{(D)}\ 8 \qquad \textbf{(E)}\ 12$

## Problem 6

In a group of cows and chickens, the number of legs was 14 more than twice the number of heads. The number of cows was:

$\textbf{(A)}\ 5 \qquad \textbf{(B)}\ 7 \qquad \textbf{(C)}\ 10 \qquad \textbf{(D)}\ 12 \qquad \textbf{(E)}\ 14$

## Problem 7

The roots of the equation $ax^2 + bx + c = 0$ will be reciprocal if:

$\textbf{(A)}\ a = b \qquad \textbf{(B)}\ a = bc \qquad \textbf{(C)}\ c = a \qquad \textbf{(D)}\ c = b \qquad \textbf{(E)}\ c = ab$

## Problem 8

If $8 \cdot 2^x = 5^{y+8}$, then when $y = -8, x =$

$\textbf{(A)}\ -4 \qquad \textbf{(B)}\ -3 \qquad \textbf{(C)}\ 0 \qquad \textbf{(D)}\ 4 \qquad \textbf{(E)}\ 8$

## Problem 9

Simplify $\left[\sqrt[3]{\sqrt[6]{a^9}}\right]^4 \left[\sqrt[6]{\sqrt[3]{a^9}}\right]^4$; the result is:

$\textbf{(A)}\ a^{16} \qquad \textbf{(B)}\ a^{12} \qquad \textbf{(C)}\ a^8 \qquad \textbf{(D)}\ a^4 \qquad \textbf{(E)}\ a^2$

## Problem 10

A circle of radius $10$ inches has its center at the vertex $C$ of an equilateral $\triangle ABC$ and passes through the other two vertices. The side $AC$ extended through $C$ intersects the circle at $D$. The number of degrees of $\angle ADB$ is:

$\textbf{(A)}\ 15^\circ \qquad \textbf{(B)}\ 30^\circ \qquad \textbf{(C)}\ 60^\circ \qquad \textbf{(D)}\ 90^\circ \qquad \textbf{(E)}\ 120^\circ$

## Problem 11

The expression $1-\frac{1}{1+\sqrt{3}}+\frac{1}{1-\sqrt{3}}$ equals:

$\textbf{(A)}\ 1-\sqrt{3} \qquad \textbf{(B)}\ 1 \qquad \textbf{(C)}\ -\sqrt{3} \qquad \textbf{(D)}\ \sqrt{3} \qquad \textbf{(E)}\ 1+\sqrt{3}$

## Problem 12

If $x^{-1}-1$ is divided by $x-1$ the quotient is:

$\textbf{(A)}\ 1 \qquad \textbf{(B)}\ \frac{1}{x-1} \qquad \textbf{(C)}\ \frac{-1}{x-1} \qquad \textbf{(D)}\ \frac{1}{x} \qquad \textbf{(E)}\ -\frac{1}{x}$

## Problem 13

Given two positive integers $x$ and $y$ with $x<y$. The percent that $x$ is less than $y$ is:

$\textbf{(A)}\ \frac{100(y-x)}{x} \qquad \textbf{(B)}\ \frac{100(x-y)}{x} \qquad \textbf{(C)}\ \frac{100(y-x)}{y}$
$\textbf{(D)}\ 100(y-x) \textbf{(E)}\ 100(x-y)$

## Problem 14

The points $A, B, C$ are on a circle $O$. The tangent line at $A$ and the secant $BC$ intersect at $P$, $B$ lying between $C$ and $P$.
If $\overline{BC}=20$and $\overline{PA}=10\sqrt{3}$, then $\overline{PB}$ equals:

$\textbf{(A)}\ 5 \qquad \textbf{(B)}\ 10 \qquad \textbf{(C)}\ 10\sqrt{3} \qquad \textbf{(D)}\ 20 \qquad \textbf{(E)}\ 30$

## Problem 15

The root(s) of $\frac{15}{x^2-4}-\frac{2}{x-2}=1$ is (are):

$\textbf{(A)}\ -5 \text{ and } 3 \qquad \textbf{(B)}\ \pm 2 \qquad \textbf{(C)}\ 2 \text{ only} \qquad \textbf{(D)}\ -3 \text{ and } 5 \qquad \textbf{(E)}\ 3 \text{ only}$

## Problem 16

The sum of three numbers is $98$. The ratio of the first to the second is $\frac{2}{3}$, and the ratio of the second to the third is $\frac{5}{8}$. The second number is:

$\textbf{(A)}\ 15 \qquad \textbf{(B)}\ 20 \qquad \textbf{(C)}\ 30 \qquad \textbf{(D)}\ 32 \qquad \textbf{(E)}\ 33$

## Problem 17

The fraction $\frac{5x-11}{2x^2+x-6}$ was obtained by adding the two fractions $\frac{A}{x+2}$ and $\frac{B}{2x-3}$. The values of $A$ and $B$ must be, respectively:

$\textbf{(A)}\ 5x, -11 \qquad \textbf{(B)}\ -11, 5x \qquad \textbf{(C)}\ -1, 3 \qquad \textbf{(D)}\ 3, -1 \qquad \textbf{(E)}\ 5, -11$

## Problem 18

If $10^{2y} = 25$, then $10^{-y}$ equals:

$\textbf{(A)}\ -\frac{1}{5} \qquad \textbf{(B)}\ \frac{1}{625} \qquad \textbf{(C)}\ \frac{1}{50} \qquad \textbf{(D)}\ \frac{1}{25} \qquad \textbf{(E)}\ \frac{1}{5}$

## Problem 19

Two candles of the same height are lighted at the same time. The first is consumed in $4$ hours and the second in $3$ hours. Assuming that each candle burns at a constant rate, in how many hours after being lighted was the first candle twice the height of the second?

$\textbf{(A)}\ \frac{3}{4} \qquad \textbf{(B)}\ 1\frac{1}{2} \qquad \textbf{(C)}\ 2 \qquad \textbf{(D)}\ 2\frac{2}{5} \qquad \textbf{(E)}\ 2\frac{1}{2}$

## Problem 20

If $(0.2)^x = 2$ and $\log 2 = 0.3010$, then the value of $x$ to the nearest tenth is:

$\textbf{(A)}\ -10.0 \qquad \textbf{(B)}\ -0.5 \qquad \textbf{(C)}\ -0.4 \qquad \textbf{(D)}\ -0.2 \qquad \textbf{(E)}\ 10.0$

## Problem 21

If each of two intersecting lines intersects a hyperbola and neither line is tangent to the hyperbola, then the possible number of points of intersection with the hyperbola is:

$\textbf{(A)}\ 2 \qquad \textbf{(B)}\ 2 \text{ or } 3 \qquad \textbf{(C)}\ 2 \text{ or } 4 \qquad \textbf{(D)}\ 3 \text{ or } 4 \qquad \textbf{(E)}\ 2, 3, \text{ or } 4$

## Problem 22

Jones covered a distance of $50$ miles on his first trip. On a later trip he traveled $300$ miles while going three times as fast. His new time compared with the old time was:

$\textbf{(A)}$ three times as much $\qquad \textbf{(B)}$ twice as much $\qquad \textbf{(C)}$ the same
$\textbf{(D)}$ half as much $\qquad \textbf{(E)}$ a third as much

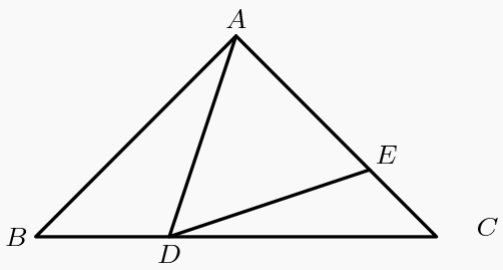
## Problem 23

About the equation $ax^2 - 2x\sqrt{2} + c = 0$, with $a$ and $c$ real constants, we are told that the discriminant is zero. The roots are necessarily:

$\textbf{(A)}$ equal and integral $\qquad \textbf{(B)}$ equal and rational $\qquad \textbf{(C)}$ equal and real
$\textbf{(D)}$ equal and irrational $\qquad \textbf{(E)}$ equal and imaginary

## Problem 24

In the figure $\overline{AB} = \overline{AC}$, $\angle BAD = 30^\circ$, and $\overline{AE} = \overline{AD}$.




$\textbf{(A)}\ 7\frac{1}{2}^\circ \qquad \textbf{(B)}\ 10^\circ \qquad \textbf{(C)}\ 12\frac{1}{2}^\circ \qquad \textbf{(D)}\ 15^\circ \qquad \textbf{(E)}\ 20^\circ$

## Problem 25

The sum of all numbers of the form $2k+1$, where $k$ takes on integral values from $1$ to $n$ is:

$\textbf{(A)}\ n^2 \qquad \textbf{(B)}\ n(n+1) \qquad \textbf{(C)}\ n(n+2) \qquad \textbf{(D)}\ (n+1)^2 \qquad \textbf{(E)}\ (n+1)(n+2)$

## Problem 26

Which one of the following combinations of given parts does not determine the indicated triangle?

$\textbf{(A)}$ base angle and vertex angle; isosceles triangle
$\textbf{(B)}$ vertex angle and the base; isosceles triangle
$\textbf{(C)}$ the radius of the circumscribed circle; equilateral triangle
$\textbf{(D)}$ one arm and the radius of the inscribed circle; right triangle
$\textbf{(E)}$ two angles and a side opposite one of them; scalene triangle

## Problem 27

If an angle of a triangle remains unchanged but each of its two including sides is doubled, then the area is multiplied by:

$\textbf{(A)}\ 2 \qquad \textbf{(B)}\ 3 \qquad \textbf{(C)}\ 4 \qquad \textbf{(D)}\ 6 \qquad \textbf{(E)}\ \text{more than } 6$

## Problem 28

Mr. J left his entire estate to his wife, his daughter, his son, and the cook. His daughter and son got half the estate, sharing in the ratio of $4$ to $3$. His wife got twice as much as the son. If the cook received a bequest of $\$500$, then the entire estate was:

$\textbf{(A)}\ \$3500 \qquad \textbf{(B)}\ \$5500 \qquad \textbf{(C)}\ \$6500 \qquad \textbf{(D)}\ \$7000 \qquad \textbf{(E)}\ \$7500$

## Problem 29

The points of intersection of $xy=12$ and $x^2+y^2=25$ are joined in succession. The resulting figure is:

$\textbf{(A)}$ a straight line $\qquad \textbf{(B)}$ an equilateral triangle $\qquad \textbf{(C)}$ a parallelogram
$\textbf{(D)}$ a rectangle $\qquad \textbf{(E)}$ a square

## Problem 30

If the altitude of an equilateral triangle is $\sqrt{6}$, then the area is:

$\textbf{(A)}\ 2\sqrt{2} \qquad \textbf{(B)}\ 2\sqrt{3} \qquad \textbf{(C)}\ 3\sqrt{3} \qquad \textbf{(D)}\ 6\sqrt{2} \qquad \textbf{(E)}\ 12$

## Problem 31

In our number system the base is ten. If the base were changed to four you would count as follows:
$1, 2, 3, 10, 11, 12, 13, 20, 21, 22, 23, 30, \ldots$ The twentieth number would be:

$\textbf{(A)}\ 20 \qquad \textbf{(B)}\ 38 \qquad \textbf{(C)}\ 44 \qquad \textbf{(D)}\ 104 \qquad \textbf{(E)}\ 110$

## Problem 32

George and Henry started a race from opposite ends of the pool. After a minute and a half, they passed each other in the center of the pool. If they lost no time in turning and maintained their respective speeds, how many minutes after starting did they pass each other the second time?

$\textbf{(A)}\ 3 \qquad \textbf{(B)}\ 4\frac{1}{2} \qquad \textbf{(C)}\ 6 \qquad \textbf{(D)}\ 7\frac{1}{2} \qquad \textbf{(E)}\ 9$

## Problem 33

The number $\sqrt{2}$ is equal to:

$\textbf{(A)}\ \text{a rational fraction} \qquad \textbf{(B)}\ \text{a finite decimal} \qquad \textbf{(C)}\ 1.41421$
$\textbf{(D)}\ \text{an infinite repeating decimal} \qquad \textbf{(E)}\ \text{an infinite non - repeating decimal}$

## Problem 34

If $n$ is any whole number, $n^2(n^2-1)$ is always divisible by

$\textbf{(A)}\ 12 \qquad \textbf{(B)}\ 24 \qquad \textbf{(C)}\ \text{any multiple of } 12 \qquad \textbf{(D)}\ 12-n \qquad \textbf{(E)}\ 12 \text{ and } 24$

## Problem 35

A rhombus is formed by two radii and two chords of a circle whose radius is $16$ feet. The area of the rhombus in square feet is:

$\textbf{(A)}\ 128 \qquad \textbf{(B)}\ 128\sqrt{3} \qquad \textbf{(C)}\ 256 \qquad \textbf{(D)}\ 512 \qquad \textbf{(E)}\ 512\sqrt{3}$

## Problem 36

If the sum $1+2+3+\cdots+K$ is a perfect square $N^2$ and if $N$ is less than $100$, then the possible values for $K$ are:

$\textbf{(A)}\ \text{only } 1 \qquad \textbf{(B)}\ 1 \text{ and } 8 \qquad \textbf{(C)}\ \text{only } 8 \qquad \textbf{(D)}\ 8 \text{ and } 49 \qquad \textbf{(E)}\ 1, 8, \text{ and } 49$

## Problem 37

On a map whose scale is $400$ miles to an inch and a half, a certain estate is represented by a rhombus having a $60^\circ$ angle. The diagonal opposite $60^\circ$ is $\overline{16}$ in. The area of the estate in square miles is:

$\textbf{(A)}\ \frac{2500}{\sqrt{3}} \qquad \textbf{(B)}\ \frac{1250}{\sqrt{3}} \qquad \textbf{(C)}\ 1250 \qquad \textbf{(D)}\ \frac{5625\sqrt{3}}{2} \qquad \textbf{(E)}\ 1250\sqrt{3}$

## Problem 38

In a right triangle with sides $a$ and $b$, and hypotenuse $c$, the altitude drawn on the hypotenuse is $x$. Then:

$\textbf{(A)}\ ab=x^2 \qquad \textbf{(B)}\ \frac{1}{a}+\frac{1}{b}=\frac{1}{x} \qquad \textbf{(C)}\ a^2+b^2=2x^2$
$\textbf{(D)}\ \frac{1}{x^2}=\frac{1}{a^2}+\frac{1}{b^2} \qquad \textbf{(E)}\ \frac{1}{x}=\frac{b}{a}$

## Problem 39

The hypotenuse $c$ and one arm a of a right triangle are consecutive integers. The square of the second arm is:

$\textbf{(A)}\ ca \qquad \textbf{(B)}\ \frac{c}{a} \qquad \textbf{(C)}\ c+a \qquad \textbf{(D)}\ c-a \qquad \textbf{(E)}\ \text{none of these}$

## Problem 40

If $V = gt + V_0$ and $S = \frac{1}{2}gt^2 + V_0 t$, then $t$ equals:

$\textbf{(A)}\ \frac{2S}{V+V_0} \qquad \textbf{(B)}\ \frac{2S}{V-V_0} \qquad \textbf{(C)}\ \frac{2S}{V_0-V} \qquad \textbf{(D)}\ \frac{2S}{V} \qquad \textbf{(E)}\ 2S-V$

## Problem 41

The equation $3y^2 + y + 4 = 2(6x^2 + y + 2)$ where $y = 2x$ is satisfied by:

$\textbf{(A)}\ \text{no value of } x \qquad \textbf{(B)}\ \text{all values of } x \qquad \textbf{(C)}\ x = 0 \text{ only}$
$\textbf{(D)}\ \text{all integral values of } x \text{ only} \qquad \textbf{(E)}\ \text{all rational values of } x \text{ only}$

## Problem 42

The equation $\sqrt{x+4} - \sqrt{x-3} + 1 = 0$ has:

$\textbf{(A)}\ \text{no root} \qquad \textbf{(B)}\ \text{one real root}$
$\textbf{(C)}\ \text{one real root and one imaginary root}$
$\textbf{(D)}\ \text{two imaginary roots} \qquad \textbf{(E)}\ \text{two real roots}$

## Problem 43

The number of scalene triangles having all sides of integral lengths, and perimeter less than $13$ is:

$\textbf{(A)}\ 1 \qquad \textbf{(B)}\ 2 \qquad \textbf{(C)}\ 3 \qquad \textbf{(D)}\ 4 \qquad \textbf{(E)}\ 18$

## Problem 44

If $x < a < 0$ means that $x$ and $a$ are numbers such that $x$ is less than $a$ and $a$ is less than zero, then:

$\textbf{(A)}\ x^2 < ax < 0 \qquad \textbf{(B)}\ x^2 > ax > a^2 \qquad \textbf{(C)}\ x^2 < a^2 < 0$
$\textbf{(D)}\ x^2 > ax \text{ but } ax < 0 \qquad \textbf{(E)}\ x^2 > a^2 \text{ but } a^2 < 0$

## Problem 45

A wheel with a rubber tire has an outside diameter of $25$ in. When the radius has been decreased a quarter of an inch, the number of revolutions in one mile will:

$\text{(A) be increased about } 2\%$
$\text{(B) be increased about } 1\%$
$\text{(C) be increased about } 20\%$
$\text{(D) be increased about } \frac{1}{2}\%$
$\text{(E) remain the same}$

## Problem 46

For the equation $\frac{1+x}{1-x} = \frac{N+1}{N}$ to be true where $N$ is positive, $x$ can have:

**(A)** any positive value less than 1
**(B)** any value less than 1
**(C)** the value zero only
**(D)** any non-negative value
**(E)** any value

## Problem 47

An engineer said he could finish a highway section in $3$ days with his present supply of a certain type of machine. However, with $3$ more of these machines the job could be done in $2$ days. If the machines all work at the same rate, how many days would it take to do the job with one machine?

**(A)** $6$ **(B)** $12$ **(C)** $15$ **(D)** $18$ **(E)** $36$

## Problem 48

If $p$ is a positive integer, then $\dfrac{3p+25}{2p-5}$ can be a positive integer, if and only if $p$ is:

**(A)** at least $3$ **(B)** at least $3$ and no more than $35$
**(C)** no more than $35$ **(D)** equal to $35$ **(E)** equal to $3$ or $35$

## Problem 49

Triangle $PAB$ is formed by three tangents to circle $O$ and $\angle APB = 40^\circ$; then $\angle AOB$ equals:

**(A)** $45^\circ$ **(B)** $50^\circ$ **(C)** $55^\circ$ **(D)** $60^\circ$ **(E)** $70^\circ$

## Problem 50

In $\triangle ABC, \overline{CA} = \overline{CB}$. On $CB$ square $BCDE$ is constructed away from the triangle. If $x$ is the number of degrees in $\angle DAB$, then

**(A)** $x$ depends upon $\triangle ABC$ **(B)** $x$ is independent of the triangle
**(C)** $x$ may equal $\angle CAD$
**(D)** $x$ can never equal $\angle CAB$
**(E)** $x$ is greater than $45^\circ$ but less than $90^\circ$