1969 AHSME Problems

Problem 1

When x is added to both the numerator and denominator of the fraction $\frac{a}{b}, a \neq b, b \neq 0$, the value of the fraction is changed to $\frac{c}{d}$. Then x equals:

$\text{(A) } \frac{1}{c-d}\quad \text{(B) } \frac{ad-bc}{c-d}\quad \text{(C) } \frac{ad-bc}{c+d}\quad \text{(D) } \frac{bc-ad}{c-d}\quad \text{(E) } \frac{bc+ad}{c-d}$

Problem 2

If an item is sold for x dollars, there is a loss of 15% based on the cost. If, however, the same item is sold for y dollars, there is a profit of 15% based on the cost. The ratio of $y : x$ is:

$\text{(A) } 23:17\quad \text{(B) } 17y:23\quad \text{(C) } 23x:17\quad$
$\text{(D) dependent upon the cost} \quad \text{(E) none of these.}$

Problem 3

If N, written in base 2, is 11000, the integer immediately preceding N, written in base 2, is:

$\text{(A) } 10001\quad \text{(B) } 10010\quad \text{(C) } 10011\quad \text{(D) } 10110\quad \text{(E) } 10111$

Problem 4

Let a binary operation $\star$ on ordered pairs of integers be defined by $(a,b)\star (c,d)=(a-c,b+d)$. Then, if $(3,3)\star (0,0)$ and $(x,y)\star (3,2)$ represent identical pairs, x equals:

$\text{(A) } -3\quad \text{(B) } 0\quad \text{(C) } 2\quad \text{(D) } 3\quad \text{(E) } 6$

Problem 5

If a number $N, N \ne 0$, diminished by four times its reciprocal, equals a given real constant R, then, for this given R, the sum of all such possible values of N is

$\text{(A) } \frac{1}{R}\quad \text{(B) } R\quad \text{(C) } 4\quad \text{(D) } \frac{1}{4}\quad \text{(E) } -R$

Problem 6

The area of the ring between two concentric circles is $12\tfrac{1}{2}\pi$ square inches. The length of a chord of the larger circle tangent to the smaller circle, in inches, is:

$\text{(A) } \frac{5}{\sqrt{2}}\quad \text{(B) } 5\quad \text{(C) } 5\sqrt{2}\quad \text{(D) } 10\quad \text{(E) } 10\sqrt{2}$

Problem 7

If the points $(1,y_1)$ and $(-1,y_2)$ lie on the graph of $y=ax^2+bx+c$, and $y_1-y_2=-6$, then b equals:

(A) -3 (B) 0 (C) 3 (D) $\sqrt{ac}$ (E) $\frac{a+c}{2}$

Problem 8

Triangle ABC is inscribed in a circle. The measure of the non-overlapping minor arcs AB, BC and CA are, respectively, $x+75^\circ, 2x+25^\circ, 3x-22^\circ$. Then one interior angle of the triangle is:

(A) $57\frac{1}{2}^\circ$ (B) 59° (C) 60° (D) 61° (E) 122°

Problem 9

The arithmetic mean (ordinary average) of the fifty-two successive positive integers beginning at 2 is:

(A) 27 (B) $27\frac{1}{4}$ (C) $27\frac{1}{2}$ (D) 28 (E) $27\frac{1}{2}$

Problem 10

The number of points equidistant from a circle and two parallel tangents to the circle is:

(A) 0 (B) 2 (C) 3 (D) 4 (E) ∞

Problem 11

Given points $P(-1,-2)$ and $Q(4,2)$ in the xy-plane; point $R(1,m)$ is taken so that $PR+RQ$ is a minimum. Then m equals:

(A) $-\frac{3}{5}$ (B) $-\frac{2}{5}$ (C) $-\frac{1}{5}$ (D) $\frac{1}{5}$ (E) either $-\frac{1}{5}$ or $\frac{1}{5}$.

Problem 12

Let $F=\frac{6x^2+16x+3m}{6}$ be the square of an expression which is linear in x. Then m has a particular value between:

(A) 3 and 4 (B) 4 and 5 (C) 5 and 6 (D) -4 and -3 (E) -6 and -5

Problem 13

A circle with radius r is contained within the region bounded by a circle with radius R. The area bounded by the larger circle is $\frac{a}{b}$ times the area of the region outside the smaller circle and inside the larger circle. Then $R:r$ equals:

(A) $\sqrt{a}:\sqrt{b}$ (B) $\sqrt{a}:\sqrt{a-b}$ (C) $\sqrt{b}:\sqrt{a-b}$ (D) $a:\sqrt{a-b}$ (E) $b:\sqrt{a-b}$

Problem 14

The complete set of x-values satisfying the inequality $\frac{x^2-4}{x^2-1}>0$ is the set of all x such that:

(A) $x>2$ or $x<-2$ or $-1<x<1$ (B) $x>2$ or $x<-2$
(C) $x>1$ or $x<-2$ (D) $x>1$ or $x<-1$
(E) x is any real number except 1 or -1

Problem 15

In a circle with center O and radius r, chord AB is drawn with length equal to r (units). From O, a perpendicular to AB meets AB at M. From M a perpendicular to OA meets OA at D. In terms of r the area of triangle MDA, in appropriate square units, is:

$\text{(A) } \frac{3r^2}{16} \quad \text{(B) } \frac{\pi r^2}{16} \quad \text{(C) } \frac{\pi r^2\sqrt{2}}{8} \quad \text{(D) } \frac{r^2\sqrt{3}}{32} \quad \text{(E) } \frac{r^2\sqrt{6}}{48}$

Problem 16

When $(a-b)^n, n \geq 2, ab \neq 0$, is expanded by the binomial theorem, it is found that when $a = kb$, where k is a positive integer, the sum of the second and third terms is zero. Then n equals:

$\text{(A) } \frac{1}{2}k(k-1) \quad \text{(B) } \frac{1}{2}k(k+1) \quad \text{(C) } 2k-1 \quad \text{(D) } 2k \quad \text{(E) } 2k+1$

Problem 17

The equation $2^{2x} - 8 \cdot 2^x + 12 = 0$ is satisfied by:

$\text{(A) } log(3) \quad \text{(B) } \frac{1}{2}log(6) \quad \text{(C) } 1+log(\frac{3}{2}) \quad \text{(D) } 1+\frac{log(3)}{log(2)} \quad \text{(E) none of these}$

Problem 18

The number of points common to the graphs of $(x-y+2)(3x+y-4)=0 \text{ and } (x+y-2)(2x-5y+7)=0$ is:

$\text{(A) } 2 \quad \text{(B) } 4 \quad \text{(C) } 6 \quad \text{(D) } 16 \quad \text{(E) } \infty$

Problem 19

The number of distinct ordered pairs (x,y) where x and y have positive integral values satisfying the equation $x^4y^4 - 10x^2y^2 + 9 = 0$is:

$\text{(A) } 0 \quad \text{(B) } 3 \quad \text{(C) } 4 \quad \text{(D) } 12 \quad \text{(E) } \infty$

Problem 20

Let P equal the product of 3,659,893,456,789,325,678 and 342,973,489,379,256. The number of digits in P is:

$\text{(A) } 36 \quad \text{(B) } 35 \quad \text{(C) } 34 \quad \text{(D) } 33 \quad \text{(E) } 32$

Problem 21

If the graph of $x^2+y^2=m$ is tangent to that of $x+y=\sqrt{2m}$, then:

$\text{(A) m must equal } \frac{1}{2} \quad \text{(B) m must equal } \frac{1}{\sqrt{2}}$
$\text{(C) m must equal } \sqrt{2} \quad \text{(D) m must equal } 2$
$\text{(E) m may be an non-negative real number}$

Problem 22

Let K be the measure of the area bounded by the x-axis, the line $x = 8$, and the curve defined by

$f = (x, y) \quad | \quad y = x$ when $0 \le x \le 5, y = 2x - 5$ when $5 \le x \le 8$.

Then K is:

(A) 21.5 (B) 36.4 (C) 36.5 (D) 44 (E) less than 44 but arbitrarily close to it

Problem 23

For any integer $n > 1$, the number of prime numbers greater than $n! + 1$ and less than $n! + n$ is:

(A) 0 (B) 1

(C) $\frac{n}{2}$ for n even, $\frac{n+1}{2}$ for n odd

(D) $n - 1$ (E) n

Problem 24

When the natural numbers P and P', with $P > P'$, are divided by the natural number D, the remainders are R and R', respectively. When PP' and RR' are divided by D, the remainders are r and r', respectively. Then:

(A) $r > r'$ always (B) $r < r'$ always

(C) $r > r'$ sometimes and $r < r'$ sometimes

(D) $r > r'$ sometimes and $r = r'$ sometimes

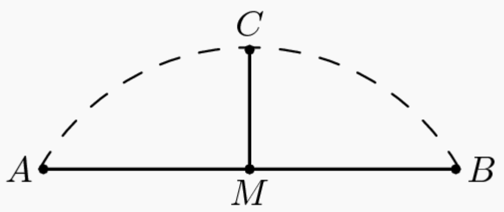
(E) $r = r'$ always

Problem 25

If it is known that $log_2(a) + log_2(b) \ge 6$, then the least value that can be taken on by $a + b$ is:

(A) $2\sqrt{6}$ (B) 6 (C) $8\sqrt{2}$ (D) 16 (E) none of these

Problem 26



A parabolic arch has a height of 16 inches and a span of 40 inches. The height, in inches, of the arch at the point 5 inches from the center M is:

(A) 1 (B) 15 (C) $15\frac{1}{3}$ (D) $15\frac{1}{2}$ (E) $15\frac{3}{4}$

Problem 27

A particle moves so that its speed for the second and subsequent miles varies inversely as the integral number of miles already traveled. For each subsequent mile the speed is constant. If the second mile is traversed in 2 hours, then the time, in hours, needed to traverse the nth mile is:

(A) $\frac{2}{n-1}$ (B) $\frac{n-1}{2}$ (C) $\frac{2}{n}$ (D) $2n$ (E) $2(n-1)$

Problem 28

Let n be the number of points P interior to the region bounded by a circle with radius 1, such that the sum of squares of the distances from P to the endpoints of a given diameter is 3. Then n is:

(A) 0 (B) 1 (C) 2 (D) 4 (E) ∞

Problem 29

If $x = t^{1/(t-1)}$ and $y = t^{t/(t-1)}, t > 0, t \neq 1$, a relation between x and y is:

(A) $y^x = x^{1/y}$ (B) $y^{1/x} = x^y$ (C) $y^x = x^y$ (D) $x^x = y^y$ (E) none of these

Problem 30

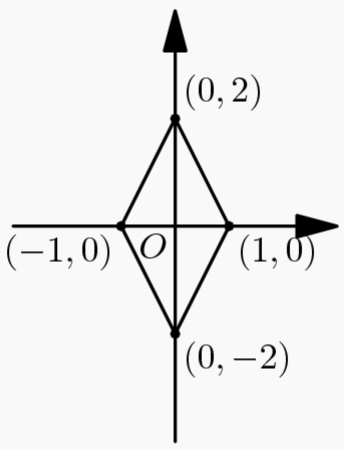
Let P be a point of hypotenuse AB (or its extension) of isosceles right triangle ABC. Let $s = AP^2 + PB^2$. Then:

(A) $s < 2CP^2$ for a finite number of positions of P
(B) $s < 2CP^2$ for an infinite number of positions of P
(C) $s = 2CP^2$ only if P is the midpoint or an endpoint of AB
(D) $s = 2CP^2$ always
(E) $s > 2CP^2$ if P is a trisection point of AB

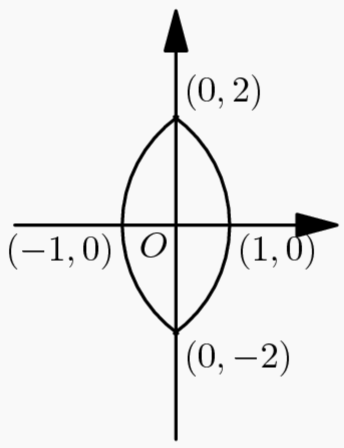
Problem 31

Let $OABC$ be a unit square in the xy-plane with $O(0,0), A(1,0), B(1,1)$ and $C(0,1)$. Let $u = x^2 - y^2$, and $v = xy$ be a transformation of the xy-plane into the uv-plane. The transform (or image) of the square is:

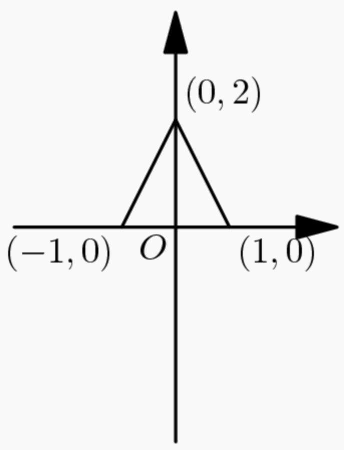
(A)



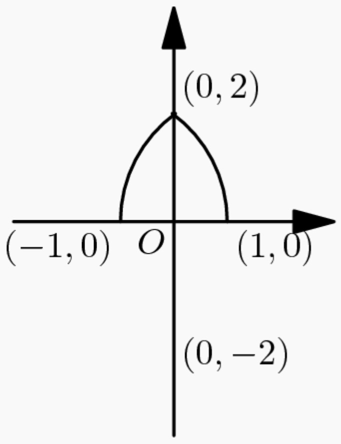
(B)



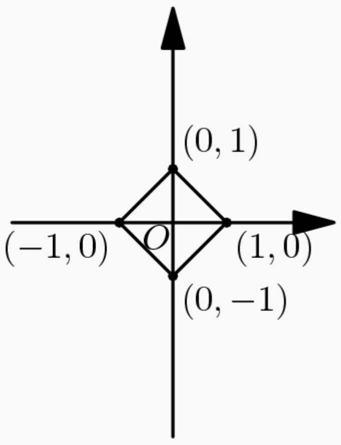
(C)



(D)



(E)



Problem 32

Let a sequence $\{u_n\}$ be defined by $u_1 = 5$ and the relationship $u_{n+1} - u_n = 3 + 4(n-1), n = 1, 2, 3 \cdots$. If u_n is expressed as a polynomial in n, the algebraic sum of its coefficients is:

$\text{(A) } 3 \quad \text{(B) } 4 \quad \text{(C) } 5 \quad \text{(D) } 6 \quad \text{(E) } 11$

Problem 33

Let S_n and T_n be the respective sums of the first n terms of two arithmetic series. If $S_n : T_n = (7n+1) : (4n+27)$ for all n, the ratio of the eleventh term of the first series to the eleventh term of the second series is:

$\text{(A) } 4:3 \quad \text{(B) } 3:2 \quad \text{(C) } 7:4 \quad \text{(D) } 78:71 \quad \text{(E) undetermined}$

Problem 34

The remainder R obtained by dividing x^{100} by $x^2 - 3x + 2$ is a polynomial of degree less than 2. Then R may be written as:

$\text{(A) } 2^{100} - 1 \quad \text{(B) } 2^{100}(x-1) - (x-2) \quad \text{(C) } 2^{200}(x-3)$
$\text{(D) } x(2^{100} - 1) + 2(2^{99} - 1) \quad \text{(E) } 2^{100}(x+1) - (x+2)$

Problem 35

Let $L(m)$ be the x coordinate of the left end point of the intersection of the graphs of $y = x^2 - 6$ and $y = m$, where $-6 < m < 6$. Let $r = [L(-m) - L(m)]/m$. Then, as m is made arbitrarily close to zero, the value of r is:

$\text{(A) arbitrarily close to } 0$

$\text{(B) arbitrarily close to } \frac{1}{\sqrt{6}} \quad \text{(C) arbitrarily close to } \frac{2}{\sqrt{6}} \quad \text{(D) arbitrarily large} \quad \text{(E) undetermined}$