1970 AHSME Problems

Problem 1

The fourth power of $\sqrt{1+\sqrt{1+\sqrt{1}}}$ is:

$\textbf{(A) } \sqrt{2}+\sqrt{3} \qquad \textbf{(B) } \frac{1}{2}(7+3\sqrt{5}) \qquad \textbf{(C) } 1+2\sqrt{3} \qquad \textbf{(D) } 3 \qquad \textbf{(E) } 3+2\sqrt{2}$

Problem 2

A square and a circle have equal perimeters. The ratio of the area of the circle to the area of the square is:

$\textbf{(A) } \frac{4}{\pi} \qquad \textbf{(B) } \frac{\pi}{\sqrt{2}} \qquad \textbf{(C) } \frac{4}{1} \qquad \textbf{(D) } \frac{\sqrt{2}}{\pi} \qquad \textbf{(E) } \frac{\pi}{4}$

Problem 3

If $x=1+2^p$ and $y=1+2^{-p}$, then y in terms of x is:

$\textbf{(A) } \frac{x+1}{x-1} \qquad \textbf{(B) } \frac{x+2}{x-1} \qquad \textbf{(C) } \frac{x}{x-1} \qquad \textbf{(D) } 2-x \qquad \textbf{(E) } \frac{x-1}{x}$

Problem 4

Let S be the set of all numbers which are the sum of the squares of three consecutive integers. Then we can say that:

$\textbf{(A) }$ No member of S is divisible by 2
$\textbf{(B) }$ No member of S is divisible by 3 but some member is divisible by 11
$\textbf{(C) }$ No member of S is divisible by 3 or 5
$\textbf{(D) }$ No member of S is divisible by 3 or 7
$\textbf{(E) }$ None of these

Problem 5

If $f(x)=\frac{x^4+x^2}{x+1}$, then $f(i)$, where $i=\sqrt{-1}$, is equal to:

$\textbf{(A) } 1+i \qquad \textbf{(B) } 1 \qquad \textbf{(C) } -1 \qquad \textbf{(D) } 0 \qquad \textbf{(E) } -1-i$

Problem 6

The smallest value of x^2+8x for real values of x is:

$\textbf{(A) } -16.25 \qquad \textbf{(B) } -16 \qquad \textbf{(C) } -15 \qquad \textbf{(D) } -8 \qquad \textbf{(E) }$ None of these

Problem 7

Inside square $ABCD$ with side s, quarter-circle arcs with radii s and centers at A and B are drawn. These arcs intersect at point X inside the square. How far is X from side CD?

$\textbf{(A) } \frac{1}{2}s(\sqrt{3}+4) \qquad \textbf{(B) } \frac{1}{2}s\sqrt{3} \qquad \textbf{(C) } \frac{1}{2}s(1+\sqrt{3})$
$\textbf{(D) } \frac{1}{2}s(\sqrt{3}-1) \qquad \textbf{(E) } \frac{1}{2}s(2-\sqrt{3})$

Problem 8

If $a = \log_8 225$ and $b = \log_2 15$, then

$\textbf{(A)}\ a=\frac{1}{2}b \qquad \textbf{(B)}\ a=\frac{2b}{3} \qquad \textbf{(C)}\ a=b \qquad \textbf{(D)}\ b=\frac{1}{2}a \qquad \textbf{(E)}\ a=\frac{3b}{2}$

Problem 9

Points P and Q are on line segment AB, and both points are on the same side of the midpoint of AB. Point P divides AB in the ratio $2:3$ and Q divides AB in the ratio $3:4$. If $PQ=2$, then the length of segment AB is

$\textbf{(A)}\ 12 \qquad \textbf{(B)}\ 28 \qquad \textbf{(C)}\ 70 \qquad \textbf{(D)}\ 75 \qquad \textbf{(E)}\ 105$

Problem 10

Let $F=.48181\cdots$ be an infinite repeating decimal with the digits 8 and 1 repeating. When F is written as a fraction in lowest terms, the denominator exceeds the numerator by

$\textbf{(A)}\ 13 \qquad \textbf{(B)}\ 14 \qquad \textbf{(C)}\ 29 \qquad \textbf{(D)}\ 57 \qquad \textbf{(E)}\ 126$

Problem 11

If two factors of $2x^3-hx+k$ are $x+2$ and $x-1$, the value of $|2h-3k|$ is

$\textbf{(A)}\ 4 \qquad \textbf{(B)}\ 3 \qquad \textbf{(C)}\ 2 \qquad \textbf{(D)}\ 1 \qquad \textbf{(E)}\ 0$

Problem 12

A circle with radius r is tangent to sides AB, AD, and CD of rectangle $ABCD$ and passes through the midpoint of diagonal AC. The area of the rectangle in terms of r, is

$\textbf{(A)}\ 4r^2 \qquad \textbf{(B)}\ 6r^2 \qquad \textbf{(C)}\ 8r^2 \qquad \textbf{(D)}\ 12r^2 \qquad \textbf{(E)}\ 20r^2$

Problem 13

Given the binary operation $*$ defined by $a*b=a^b$ for all positive numbers a and b. The for all positive a,b,c,n, we have

$\textbf{(A)}\ a*b=b*a \qquad \textbf{(B)}\ a*(b*c)=(a*b)*c$
$\textbf{(C)}\ (a*b^n)=(a*n)*b \qquad \textbf{(D)}\ (a*b)^n=a*(bn) \qquad \textbf{(E)}\ \text{None of these}$

Problem 14

Consider $x^2+px+q=0$ where p and q are positive numbers. If the roots of this equation differ by 1, then p equals

$\textbf{(A)}\ \sqrt{4q+1} \qquad \textbf{(B)}\ q-1 \qquad \textbf{(C)}\ -\sqrt{4q+1}$
$\textbf{(D)}\ q+1 \qquad \textbf{(E)}\ \sqrt{4q-1}$

Problem 15

Lines in the xy-plane are drawn through the point $(3,4)$ and the trisection points of the line segment joining the points $(-4,5)$ and $(5,-1)$. One of these lines has the equation

$\textbf{(A)}\ 3x-2y-1=0 \qquad \textbf{(B)}\ 4x-5y+8=0 \qquad \textbf{(C)}\ 5x+2y-23=0$
$\textbf{(D)}\ x+7y-31=0 \qquad \textbf{(E)}\ x-4y+13=0$

Problem 16

If $F(n)$ is a function such that $F(1) = F(2) = F(3) = 1$, and such that $F(n+1) = \dfrac{F(n) \cdot F(n-1) + 1}{F(n-2)}$ for $n \geq 3$, then $F(6)$ is equal to

$\textbf{(A)}\ 2 \qquad \textbf{(B)}\ 3 \qquad \textbf{(C)}\ 7 \qquad \textbf{(D)}\ 11 \qquad \textbf{(E)}\ 26$

Problem 17

If $r \geq 0$, then for all p and q such that $pq \neq 0$ and $pr > qr$, we have

$\textbf{(A)}\ -p > -q \qquad \textbf{(B)}\ -p > q \qquad \textbf{(C)}\ 1 > -q/p$
$\textbf{(D)}\ 1 < q/p \qquad \textbf{(E)}\ \text{None of These}$

Problem 18

$\sqrt{3+2\sqrt{2}} - \sqrt{3-2\sqrt{2}}$ is equal to

$\textbf{(A)}\ 2 \qquad \textbf{(B)}\ 2\sqrt{3} \qquad \textbf{(C)}\ 4\sqrt{2} \qquad \textbf{(D)}\ \sqrt{6} \qquad \textbf{(D)}\ \sqrt{6} \qquad \textbf{(E)}\ 2\sqrt{2}$

Problem 19

The sum of an infinite geometric series with common ratio r such that $|r| < 1$, is 15, and the sum of the squares of the terms of this series is 45. The first term of the series is

$\textbf{(A)}\ 12 \qquad \textbf{(B)}\ 10 \qquad \textbf{(C)}\ 5 \qquad \textbf{(D)}\ 3 \qquad \textbf{(E)}\ 2$

Problem 20

Lines HK and BC lie in a plane. M is the midpoint of line segment BC, and BH and CK are perpendicular to HK. Then we

$\textbf{(A)}\ \text{always have } MH = MK$
$\textbf{(B)}\ \text{always have } MH > BK$
$\textbf{(C)}\ \text{sometimes have } MH = MK \text{ but not always}$
$\textbf{(D)}\ \text{always have } MH > MB$
$\textbf{(E)}\ \text{always have } BH < BC$

Problem 21

On an auto trip, the distance read from the instrument panel was 450 miles. With snow tires on for the return trip over the same route, the reading was 440 miles. Find, to the nearest hundredth of an inch, the increase in radius of the wheels if the original radius was 15inches.

$\textbf{(A)}\ .33 \qquad \textbf{(B)}\ .34 \qquad \textbf{(C)}\ .35 \qquad \textbf{(D)}\ .38 \qquad \textbf{(E)}\ .66$

Problem 22

If the sum of the first $3n$ positive integers is 150 more than the sum of the first n positive integers, then the sum of the first $4n$ positive integers is

$\textbf{(A)}\ 300 \qquad \textbf{(B)}\ 350 \qquad \textbf{(C)}\ 400 \qquad \textbf{(D)}\ 450 \qquad \textbf{(E)}\ 600$

Problem 23

The number $10!$ (10 is written in base 10), when written in the base 12 system, ends in exactly k zeroes. The value of k is

$\textbf{(A) }1\qquad\textbf{(B) }2\qquad\textbf{(C) }3\qquad\textbf{(D) }4\qquad\textbf{(E) }5$

Problem 24

An equilateral triangle and a regular hexagon have equal perimeters. If the area of the triangle is 2, then the area of the hexagon is

$\textbf{(A) }2\qquad\textbf{(B) }3\qquad\textbf{(C) }4\qquad\textbf{(D) }6\qquad\textbf{(E) }12$

Problem 25

For every real number x, let $[x]$ be the greatest integer less than or equal to x. If the postal rate for first class mail is six cents for every ounce or portion thereof, then the cost in cents of first-class postage on a letter weighing W ounces is always

$\textbf{(A) }6W\qquad\textbf{(B) }6[W]\qquad\textbf{(C) }6([W]-1)\qquad\textbf{(D) }6([W]+1)\qquad\textbf{(E) }-6[-W]$

Problem 26

The number of distinct points in the xy-plane common to the graphs of $(x+y-5)(2x-3y+5)=0$ and $(x-y+1)(3x+2y-12)=0$ is

$\textbf{(A) }0\qquad\textbf{(B) }1\qquad\textbf{(C) }2\qquad\textbf{(D) }3\qquad\textbf{(E) }4$

Problem 27

In a triangle, the area is numerically equal to the perimeter. What is the radius of the inscribed circle?

$\textbf{(A) }2\qquad\textbf{(B) }3\qquad\textbf{(C) }4\qquad\textbf{(D) }5\qquad\textbf{(E) }6$

Problem 28

In triangle ABC, the median from vertex A is perpendicular to the median from vertex B. If the lengths of sides AC and BC are 6 and 7 respectively, then the length of side AB is

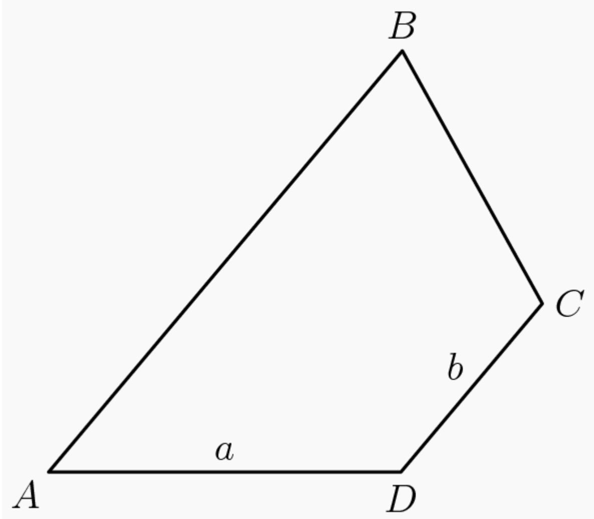
$\textbf{(A) }\sqrt{17}\qquad\textbf{(B) }4\qquad\textbf{(C) }4\frac{1}{2}\qquad\textbf{(D) }2\sqrt{5}\qquad\textbf{(E) }4\frac{1}{4}$

Problem 29

It is now between $10:00$ and $11:00$ o'clock, and six minutes form now, the minute hand of the watch will be exactly opposite the place where the hour hand was three minutes ago. What is the exact time now?

$\textbf{(A) }10:05\frac{5}{11}\qquad\textbf{(B) }10:07\frac{1}{2}\qquad\textbf{(C) }10:10$
$\textbf{(D) }10:15\qquad\textbf{(E) }10:17\frac{1}{2}$

Problem 30



In the accompanying figure, segments AB and CD are parallel, the measure of $\angle D$ is twice the measure of $\angle B$, and the measures of segments AB and CD are a and b respectively. Then the measure of AB is equal to

$\textbf{(A)}\ \frac{1}{2}a+2b \qquad \textbf{(B)}\ \frac{3}{2}b+\frac{3}{4}a \qquad \textbf{(C)}\ 2a-b \qquad \textbf{(D)}\ 4b-\frac{1}{2}a \qquad \textbf{(E)}\ a+b$

Problem 31

If a number is selected at random from the set of all five-digit numbers in which the sum of the digits is equal to 43, what is the probability that this number is divisible by 11?

$\textbf{(A)}\ 2/5 \qquad \textbf{(B)}\ 1/5 \qquad \textbf{(C)}\ 1/6 \qquad \textbf{(D)}\ 1/11 \qquad \textbf{(E)}\ 1/15$

Problem 32

A and B travel around a circular track at uniform speeds in opposite directions, starting from diametrically opposite points. If they start at the same time, meet first after B has travelled 100 yards, and meet a second time 60 yards before A completes one lap, then the circumference of the track in yards is

$\textbf{(A)}\ 400 \qquad \textbf{(B)}\ 440 \qquad \textbf{(C)}\ 480 \qquad \textbf{(D)}\ 560 \qquad \textbf{(E)}\ 880$

Problem 33

Find the sum of the digits of all numerals in the sequence $1,2,3,4,\cdots ,10000$.

$\textbf{(A)}\ 180,001 \qquad \textbf{(B)}\ 154,756 \qquad \textbf{(C)}\ 45,001 \qquad \textbf{(D)}\ 154,755 \qquad \textbf{(E)}\ 270,001$

Problem 34

The greatest integer that will divide $13511, 13903$, and 14589 and leave the same remainder is

$\textbf{(A)}\ 28 \qquad \textbf{(B)}\ 49 \qquad \textbf{(C)}\ 98$
$\textbf{(D)}\ \text{an odd multiple of } 7 \text{ greater than } 49$
$\textbf{(E)}\ \text{an even multiple of } 7 \text{ greater than } 98$

Problem 35

A retiring employee receives and annual pension proportional to the square root of the number of years of his service. Had he served a years more, his pension would have been p dollars greater, whereas, had he served b years more $b\neq a$, his pension would have been $q$dollars greater than the original annual pension. Find his annual pension in terms of a,b,p, and q.

$\textbf{(A)}\ \frac{p^2-q^2}{2(a-b)} \qquad \textbf{(B)}\ \frac{(p-q)^2}{2\sqrt{ab}} \qquad \textbf{(C)}\ \frac{ap^2-bq^2}{2(ap-bq)} \qquad \textbf{(D)}\ \frac{aq^2-bp^2}{2(bp-aq)} \qquad \textbf{(E)}\ \sqrt{(a-b)(p-q)}$