1997 AHSME Problems

Problem 1

If a and b are digits for which

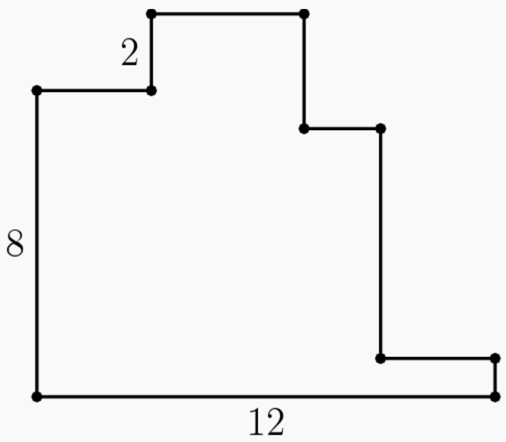
$$\begin{array}{ccc} & 2 & a \\ \times & b & 3 \\ \hline & 6 & 9 \\ 9 & 2 & \\ \hline 9 & 8 & 9 \end{array}$$

then a+b =

$\mathrm{(A)}\ 3 \qquad \mathrm{(B)}\ 4 \qquad \mathrm{(C)}\ 7 \qquad \mathrm{(D)}\ 9 \qquad \mathrm{(E)}\ 12$

Problem 2

The adjacent sides of the decagon shown meet at right angles. What is its perimeter?



$\mathrm{(A)}\ 22 \qquad \mathrm{(B)}\ 32 \qquad \mathrm{(C)}\ 34 \qquad \mathrm{(D)}\ 44 \qquad \mathrm{(E)}\ 50$

Problem 3

If x, y, and z are real numbers such that

$$(x-3)^2 + (y-4)^2 + (z-5)^2 = 0,$$

then $x + y + z =$

$\mathrm{(A)}\ -12 \qquad \mathrm{(B)}\ 0 \qquad \mathrm{(C)}\ 8 \qquad \mathrm{(D)}\ 12 \qquad \mathrm{(E)}\ 50$

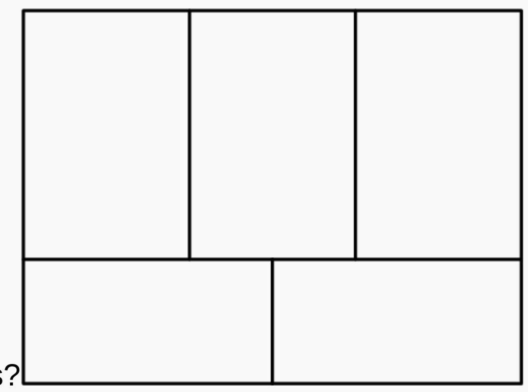
Problem 4

If a is 50% larger than c, and b is 25% larger than c, then a is what percent larger than b?

$\mathrm{(A)}\ 20\% \qquad \mathrm{(B)}\ 25\% \qquad \mathrm{(C)}\ 50\% \qquad \mathrm{(D)}\ 100\% \qquad \mathrm{(E)}\ 200\%$

Problem 5

A rectangle with perimeter 176 is divided into five congruent rectangles as shown in the diagram. What is the perimeter of one of the five congruent rectangles?



(A) 35.2 (B) 76 (C) 80 (D) 84 (E) 86

Problem 6

Consider the sequence

$1, -2, 3, -4, 5, -6, \ldots,$

whose nth term is $(-1)^{n+1} \cdot n$. What is the average of the first 200 terms of the sequence?

$\textbf{(A)}\ -1 \qquad \textbf{(B)}\ -0.5 \qquad \textbf{(C)}\ 0 \qquad \textbf{(D)}\ 0.5 \qquad \textbf{(E)}\ 1$

Problem 7

The sum of seven integers is -1. What is the maximum number of the seven integers that can be larger than 13?

$\textbf{(A)}\ 1 \qquad \textbf{(B)}\ 4 \qquad \textbf{(C)}\ 5 \qquad \textbf{(D)}\ 6 \qquad \textbf{(E)}\ 7$

Problem 8

Mientka Publishing Company prices its bestseller Where's Walter? as follows:

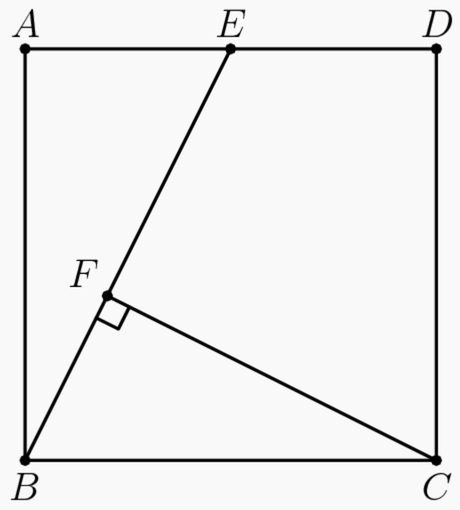
$$C(n) = \begin{cases} 12n, & \text{if } 1 \le n \le 24 \\ 11n, & \text{if } 25 \le n \le 48 \\ 10n, & \text{if } 49 \le n \end{cases}$$

where n is the number of books ordered, and $C(n)$ is the cost in dollars of n books. Notice that 25 books cost less than 24 books. For how many values of n is it cheaper to buy more than n books than to buy exactly n books?

$\textbf{(A)}\ 3 \qquad \textbf{(B)}\ 4 \qquad \textbf{(C)}\ 5 \qquad \textbf{(D)}\ 6 \qquad \textbf{(E)}\ 8$

Problem 9

In the figure, $ABCD$ is a 2×2 square, E is the midpoint of $\overline{AD}$, and F is on $\overline{BE}$. If $\overline{CF}$ is perpendicular to $\overline{BE}$, then the area of quadrilateral $CDEF$ is



$\textbf{(A)}\ 2 \qquad \textbf{(B)}\ 3-\frac{\sqrt{3}}{2} \qquad \textbf{(C)}\ \frac{11}{5} \qquad \textbf{(D)}\ \sqrt{5} \qquad \textbf{(E)}\ \frac{9}{4}$

Problem 10

Two six-sided dice are fair in the sense that each face is equally likely to turn up. However, one of the dice has the 4 replaced by 3 and the other die has the 3 replaced by 4 . When these dice are rolled, what is the probability that the sum is an odd number?

$\textbf{(A)}\ \frac{1}{3} \qquad \textbf{(B)}\ \frac{4}{9} \qquad \textbf{(C)}\ \frac{1}{2} \qquad \textbf{(D)}\ \frac{5}{9} \qquad \textbf{(E)}\ \frac{11}{18}$

Problem 11

In the sixth, seventh, eighth, and ninth basketball games of the season, a player scored 23,14, 11, and 20 points, respectively. Her points-per-game average was higher after nine games than it was after the first five games. If her average after ten games was greater than 18, what is the least number of points she could have scored in the tenth game?

$\textbf{(A)}\ 26 \qquad \textbf{(B)}\ 27 \qquad \textbf{(C)}\ 28 \qquad \textbf{(D)}\ 29 \qquad \textbf{(E)}\ 30$

Problem 12

If m and b are real numbers and $mb>0$, then the line whose equation is $y=mx+b$ *cannot* contain the point

$\textbf{(A)}\ (0,1997) \qquad \textbf{(B)}\ (0,-1997) \qquad \textbf{(C)}\ (19,97) \qquad \textbf{(D)}\ (19,-97) \qquad \textbf{(E)}\ (1997,0)$

Problem 13

How many two-digit positive integers N have the property that the sum of N and the number obtained by reversing the order of the digits of is a perfect square?

$\textbf{(A)}\ 4 \qquad \textbf{(B)}\ 5 \qquad \textbf{(C)}\ 6 \qquad \textbf{(D)}\ 7 \qquad \textbf{(E)}\ 8$

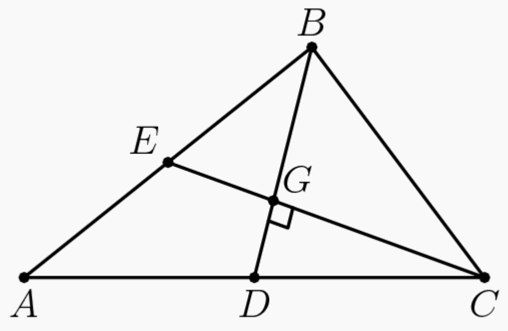
Problem 14

The number of geese in a flock increases so that the difference between the populations in year $n+2$ and year n is directly proportional to the population in year $n+1$. If the populations in the years 1994, 1995, and 1997 were 39, 60, and 123, respectively, then the population in 1996 was

$\textbf{(A)}\ 81 \qquad \textbf{(B)}\ 84 \qquad \textbf{(C)}\ 87 \qquad \textbf{(D)}\ 90 \qquad \textbf{(E)}\ 102$

Problem 15

Medians BD and AE of triangle ABC are perpendicular, $BD=8$, and $CE=12$. The area of triangle ABC is



$\textbf{(A)}\ 24 \qquad \textbf{(B)}\ 32 \qquad \textbf{(C)}\ 48 \qquad \textbf{(D)}\ 64 \qquad \textbf{(E)}\ 96$

Problem 16

The three row sums and the three column sums of the array

$$\begin{bmatrix} 4 & 9 & 2 \\ 8 & 1 & 6 \\ 3 & 5 & 7 \end{bmatrix}$$

are the same. What is the least number of entries that must be altered to make all six sums different from one another?

$\textbf{(A)}\ 1 \qquad \textbf{(B)}\ 2 \qquad \textbf{(C)}\ 3 \qquad \textbf{(D)}\ 4 \qquad \textbf{(E)}\ 5$

Problem 17

A line $x = k$ intersects the graph of $y = \log_5 x$ and the graph of $y = \log_5(x+4)$. The distance between the points of intersection is 0.5. Given that $k = a + \sqrt{b}$, where a and b are integers, what is $a+b$?

$\textbf{(A)}\ 6 \qquad \textbf{(B)}\ 7 \qquad \textbf{(C)}\ 8 \qquad \textbf{(D)}\ 9 \qquad \textbf{(E)}\ 10$

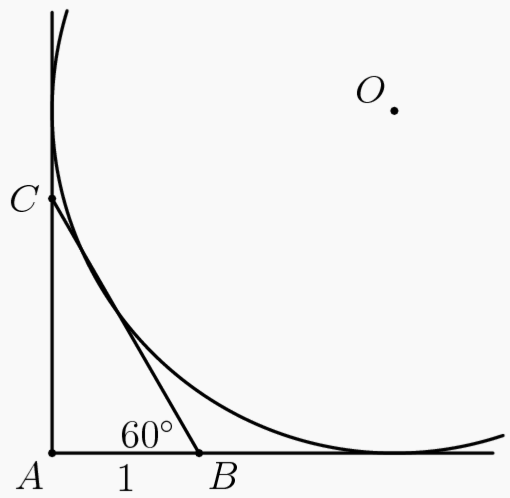
Problem 18

A list of integers has mode 32 and mean 22. The smallest number in the list is 10. The median m of the list is a member of the list. If the list member m were replaced by $m+10$, the mean and median of the new list would be 24 and $m+10$, respectively. If were m instead replaced by $m-8$, the median of the new list would be $m-4$. What is m?

$\textbf{(A)}\ 16 \qquad \textbf{(B)}\ 17 \qquad \textbf{(C)}\ 18 \qquad \textbf{(D)}\ 19 \qquad \textbf{(E)}\ 20$

Problem 19

A circle with center O is tangent to the coordinate axes and to the hypotenuse of the 30°-60°-90° triangle ABC as shown, where $AB = 1$. To the nearest hundredth, what is the radius of the circle?



$\textbf{(A)}\ 2.18 \qquad \textbf{(B)}\ 2.24 \qquad \textbf{(C)}\ 2.31 \qquad \textbf{(D)}\ 2.37 \qquad \textbf{(E)}\ 2.41$

Problem 20

Which one of the following integers can be expressed as the sum of 100 consecutive positive integers?

$\textbf{(A)}\ 1{,}627{,}384{,}950 \qquad \textbf{(B)}\ 2{,}345{,}678{,}910 \qquad \textbf{(C)}\ 3{,}579{,}111{,}300 \qquad \textbf{(D)}\ 4{,}692{,}581{,}470 \qquad \textbf{(E)}\ 5{,}815{,}937{,}260$

Problem 21

For any positive integer n, let

$$f(n) = \begin{cases} \log_8 n, & \text{if } \log_8 n \text{ is rational,} \\ 0, & \text{otherwise.} \end{cases}$$

What is $\sum_{n=1}^{1997} f(n)$?

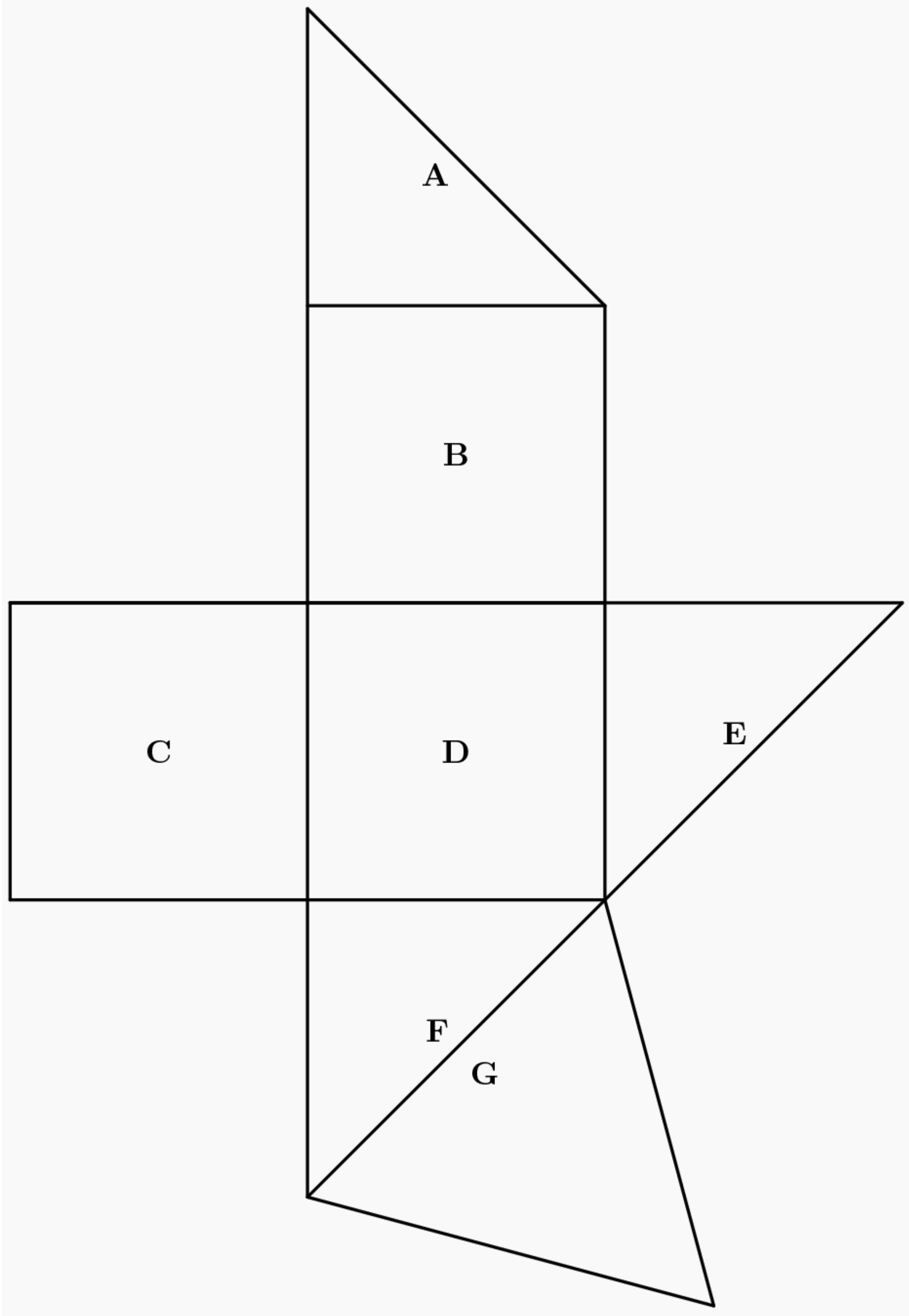
$\textbf{(A)}\ \log_8 2047 \qquad \textbf{(B)}\ 6 \qquad \textbf{(C)}\ \frac{55}{3} \qquad \textbf{(D)}\ \frac{58}{3} \qquad \textbf{(E)}\ 585$

Problem 22

Ashley, Betty, Carlos, Dick, and Elgin went shopping. Each had a whole number of dollars to spend, and together they had 56 dollars. The absolute difference between the amounts Ashley and Betty had to spend was 19 dollars. The absolute difference between the amounts Betty and Carlos had was 7 dollars, between Carlos and Dick was 5 dollars, between Dick and Elgin was 4 dollars, and between Elgin and Ashley was 11 dollars. How many dollars did Elgin have?

$\textbf{(A)}\ 6 \qquad \textbf{(B)}\ 7 \qquad \textbf{(C)}\ 8 \qquad \textbf{(D)}\ 9 \qquad \textbf{(E)}\ 10$

Problem 23



In the figure, polygons A, E, and F are isosceles right triangles; B, C, and D are squares with sides of length 1; and G is an equilateral triangle. The figure can be folded along its edges to form a polyhedron having the polygons as faces. The volume of this polyhedron is

$\textbf{(A)}\ 1/2 \qquad \textbf{(B)}\ 2/3 \qquad \textbf{(C)}\ 3/4 \qquad \textbf{(D)}\ 5/6 \qquad \textbf{(E)}\ 4/3$

Problem 24

A rising number, such as 34689, is a positive integer each digit of which is larger than each of the digits to its left. There are $\binom{9}{5} = 126$ five-digit rising numbers. When these numbers are arranged from smallest to largest, the 97^{th} number in the list does not contain the digit

$\textbf{(A)}\ 4 \qquad \textbf{(B)}\ 5 \qquad \textbf{(C)}\ 6 \qquad \textbf{(D)}\ 7 \qquad \textbf{(E)}\ 8$

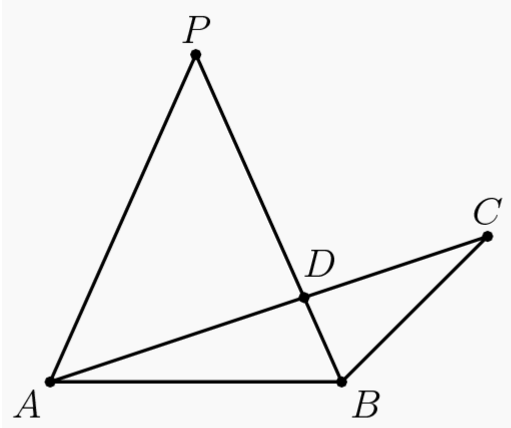
Problem 25

Let $ABCD$ be a parallelogram and let $\overrightarrow{AA^\prime}$, $\overrightarrow{BB^\prime}$, $\overrightarrow{CC^\prime}$, and $\overrightarrow{DD^\prime}$ be parallel rays in space on the same side of the plane determined by $ABCD$. If $AA^{\prime} = 10$, $BB^{\prime}= 8$, $CC^\prime = 18$, and $DD^\prime = 22$ and M and N are the midpoints of $A^{\prime} C^{\prime}$ and $B^{\prime}D^{\prime}$, respectively, then $MN =$

$\textbf{(A)}\ 0 \qquad \textbf{(B)}\ 1 \qquad \textbf{(C)}\ 2 \qquad \textbf{(D)}\ 3 \qquad \textbf{(E)}\ 4$

Problem 26

Triangle ABC and point P in the same plane are given. Point P is equidistant from A and B, angle APB is twice angle ACB, and $\overline{AC}$ intersects $\overline{BP}$ at point D. If $PB = 3$ and $PD = 2$, then $AD \cdot CD =$



$\textbf{(A)}\ 5 \qquad \textbf{(B)}\ 6 \qquad \textbf{(C)}\ 7 \qquad \textbf{(D)}\ 8 \qquad \textbf{(E)}\ 9$

Problem 27

Consider those functions f that satisfy $f(x+4)+f(x-4) = f(x)$ for all real x. Any such function is periodic, and there is a least common positive period p for all of them. Find p.

$\textbf{(A)}\ 8 \qquad \textbf{(B)}\ 12 \qquad \textbf{(C)}\ 16 \qquad \textbf{(D)}\ 24 \qquad \textbf{(E)}\ 32$

Problem 28

How many ordered triples of integers (a,b,c) satisfy $|a+b|+c = 19$ and $ab+|c| = 97$?

$\textbf{(A)}\ 0 \qquad \textbf{(B)}\ 4 \qquad \textbf{(C)}\ 6 \qquad \textbf{(D)}\ 10 \qquad \textbf{(E)}\ 12$

Problem 29

Call a positive real number special if it has a decimal representation that consists entirely of digits 0 and 7. For example, $\frac{700}{99} = 7.\overline{07} = 7.070707\cdots$ and 77.007 are special numbers. What is the smallest n such that 1 can be written as a sum of n special numbers?

$\textbf{(A)}\ 7 \qquad \textbf{(B)}\ 8 \qquad \textbf{(C)}\ 9 \qquad \textbf{(D)}\ 10$
$\textbf{(E)}\ \text{The number 1 cannot be represented as a sum of finitely many special numbers.}$

Problem 30

For positive integers n, denote $D(n)$ by the number of pairs of different adjacent digits in the binary (base two) representation of n. For example, $D(3) = D(11_{2}) = 0$, $D(21) = D(10101_{2}) = 4$, and $D(97) = D(1100001_{2}) = 2$. For how many positive integers less than or equal 97 to does $D(n) = 2$?

$\textbf{(A)}\ 16 \qquad \textbf{(B)}\ 20 \qquad \textbf{(C)}\ 26 \qquad \textbf{(D)}\ 30 \qquad \textbf{(E)}\ 35$