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1977 AHSME Problems

Problem 1

If $y = 2x$ and $z = 2y$, then $x + y + z$ equals

(A) x (B) $3x$ (C) $5x$ (D) $7x$ (E) $9x$

Problem 2

Which one of the following statements is false? All equilateral triangles are

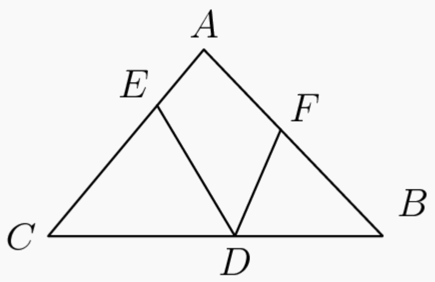
(A) equiangular **(B)** isosceles **(C)** regular polygons
(D) congruent to each other **(E)** similar to each other

Problem 3

A man has $2.73 in pennies, nickels, dimes, quarters and half dollars. If he has an equal number of coins of each kind, then the total number of coins he has is

(A) 3 (B) 5 (C) 9 (D) 10 (E) 15

Problem 4



In triangle $ABC, AB = AC$ and $\measuredangle A = 80^\circ$. If points D, E, and F lie on sides BC, AC and AB, respectively, and $CE = CD$ and $BF = BD$, then $\measuredangle EDF$ equals

(A) 30° **(B)** 40° **(C)** 50° **(D)** 65° **(E)** none of these

Problem 5

The set of all points P such that the sum of the (undirected) distances from P to two fixed points A and B equals the distance between A and B is

(A) the line segment from A to B **(B)** the line passing through A and B
(C) the perpendicular bisector of the line segment from A to B
(D) an ellipse having positive area **(E)** a parabola

Problem 6

If x, y and $2x + \frac{y}{2}$ are not zero, then $\left(2x + \frac{y}{2}\right)\left[(2x)^{-1} + \left(\frac{y}{2}\right)^{-1}\right]$ equals

(A) 1 **(B)** xy^{-1} **(C)** $x^{-1}y$ **(D)** $(xy)^{-1}$ **(E)** none of these

Problem 7

If $t = \frac{1}{1 - \sqrt[4]{2}}$, then t equals

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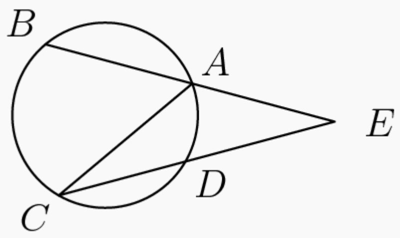
(A) $(1-\sqrt[4]{2})(2-\sqrt{2})$ (B) $(1-\sqrt[4]{2})(1+\sqrt{2})$ (C) $(1+\sqrt[4]{2})(1-\sqrt{2})$
(D) $(1+\sqrt[4]{2})(1+\sqrt{2})$ (E) $-(1+\sqrt[4]{2})(1+\sqrt{2})$

Problem 8

For every triple (a,b,c) of non-zero real numbers, form the number $\frac{a}{|a|}+\frac{b}{|b|}+\frac{c}{|c|}+\frac{abc}{|abc|}$. The set of all numbers formed is

$\textbf{(A)}\ 0 \qquad \textbf{(B)}\ \{-4,0,4\} \qquad \textbf{(C)}\ \{-4,-2,0,2,4\} \qquad \textbf{(D)}\ \{-4,-2,2,4\} \qquad \textbf{(E)}\ \text{none of these}$

Problem 9



In the adjoining figure $\measuredangle E=40^\circ$ and arc AB, arc BC, and arc CD all have equal length. Find the measure of $\measuredangle ACD$.

$\textbf{(A)}\ 10^\circ \qquad \textbf{(B)}\ 15^\circ \qquad \textbf{(C)}\ 20^\circ \qquad \textbf{(D)}\ \left(\frac{45}{2}\right)^\circ \qquad \textbf{(E)}\ 30^\circ$

Problem 10

If $(3x-1)^7=a_7x^7+a_6x^6+\cdots+a_0$, then $a_7+a_6+\cdots+a_0$ equals

(A) 0 (B) 1 (C) 64 (D) -64 (E) 128

Problem 11

For each real number x, let $[x]$ be the largest integer not exceeding x (i.e., the integer n such that $n\le x<n+1$). Which of the following statements is (are) true?

$\textbf{I.}\ [x+1]=[x]+1 \text{ for all } x$
$\textbf{II.}\ [x+y]=[x]+[y] \text{ for all } x \text{ and } y$
$\textbf{III.}\ [xy]=[x][y] \text{ for all } x \text{ and } y$

$\textbf{(A)}\ \text{none} \qquad \textbf{(B)}\ \textbf{I} \text{ only} \qquad \textbf{(C)}\ \textbf{I} \text{ and } \textbf{II} \text{ only} \qquad \textbf{(D)}\ \textbf{III} \text{ only} \qquad \textbf{(E)}\ \text{all}$

Problem 12

Al's age is 16 more than the sum of Bob's age and Carl's age, and the square of Al's age is 1632 more than the square of the sum of Bob's age and Carl's age. What is the sum of the ages of Al, Bob, and Carl?

(A) 64 (B) 94 (C) 96 (D) 102 (E) 140

Problem 13

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If $a_1, a_2, a_3, \cdots$ is a sequence of positive numbers such that $a_{n+2} = a_n a_{n+1}$ for all positive integers n, then the sequence $a_1, a_2, a_3, \cdots$ is a geometric progression

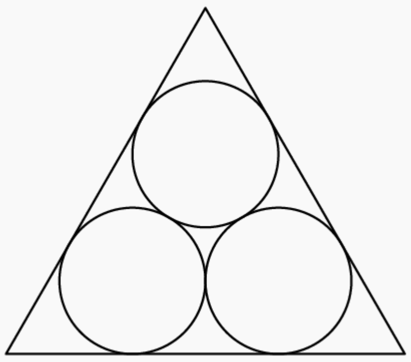
$\textbf{(A)}$ for all positive values of a_1 and a_2
$\textbf{(B)}$ if and only if $a_1 = a_2$
$\textbf{(C)}$ if and only if $a_1 = 1$
$\textbf{(D)}$ if and only if $a_2 = 1$
$\textbf{(E)}$ if and only if $a_1 = a_2 = 1$

Problem 14

How many pairs (m, n) of integers satisfy the equation $m + n = mn$?

$\textbf{(A)}\ 1 \qquad \textbf{(B)}\ 2 \qquad \textbf{(C)}\ 3 \qquad \textbf{(D)}\ 4 \qquad \textbf{(E)}\ \text{more than } 4$

Problem 15



Each of the three circles in the adjoining figure is externally tangent to the other two, and each side of the triangle is tangent to two of the circles. If each circle has radius three, then the perimeter of the triangle is

$\textbf{(A)}\ 36+9\sqrt{2} \qquad \textbf{(B)}\ 36+6\sqrt{3} \qquad \textbf{(C)}\ 36+9\sqrt{3} \qquad \textbf{(D)}\ 18+18\sqrt{3} \qquad \textbf{(E)}\ 45$

Problem 16

If $i^2 = -1$, then the sum $\cos 45^\circ + i\cos 135^\circ + \cdots + i^n \cos(45+90n)^\circ + \cdots + i^{40} \cos 3645^\circ$ equals

$\text{(A)}\ \frac{\sqrt{2}}{2} \qquad \text{(B)}\ -10i\sqrt{2} \qquad \text{(C)}\ \frac{21\sqrt{2}}{2}$
$\text{(D)}\ \frac{\sqrt{2}}{2}(21-20i) \qquad \text{(E)}\ \frac{\sqrt{2}}{2}(21+20i)$

Problem 17

Three fair dice are tossed at random (i.e., all faces have the same probability of coming up). What is the probability that the three numbers turned up can be arranged to form an arithmetic progression with common difference one?

$\textbf{(A)}\ \frac{1}{6} \qquad \textbf{(B)}\ \frac{1}{9} \qquad \textbf{(C)}\ \frac{1}{27} \qquad \textbf{(D)}\ \frac{1}{54} \qquad \textbf{(E)}\ \frac{7}{36}$

Problem 18

If $y = (\log_2 3)(\log_3 4)\cdots(\log_n[n+1])\cdots(\log_{31} 32)$ then

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(A) $4 < y < 5$ **(B)** $y = 5$ **(C)** $5 < y < 6$ **(D)** $y = 6$
(E) $6 < y < 7$

Problem 19

Let E be the point of intersection of the diagonals of convex quadrilateral $ABCD$, and let P, Q, R, and S be the centers of the circles circumscribing triangles ABE, BCE, CDE, and ADE, respectively. Then

(A) $PQRS$ is a parallelogram
(B) $PQRS$ is a parallelogram if an only if $ABCD$ is a rhombus
(C) $PQRS$ is a parallelogram if an only if $ABCD$ is a rectangle
(D) $PQRS$ is a parallelogram if an only if $ABCD$ is a parallelogram
(E) none of the above are true

Problem 20

						C						
					C	O	C					
				C	O	N	O	C				
			C	O	N	T	N	O	C			
		C	O	N	T	E	T	N	O	C		
	C	O	N	T	E	S	E	T	N	O	C	
C	O	N	T	E	S	T	S	E	T	N	O	C

For how many paths consisting of a sequence of horizontal and/or vertical line segments, with each segment connecting a pair of adjacent letters in the diagram above, is the word CONTEST spelled out as the path is traversed from beginning to end?

(A) 63 **(B)** 128 **(C)** 129 **(D)** 255 **(E)** none of these

Problem 21

For how many values of the coefficient a do the equations
$$x^2 + ax + 1 = 0$$
$$x^2 - x - a = 0$$
have a common real solution?

(A) 0 **(B)** 1 **(C)** 2 **(D)** 3 **(E)** ∞

Problem 22

If $f(x)$ is a real valued function of the real variable x, and $f(x)$ is not identically zero, and for

all a and b $f(a+b) + f(a-b) = 2f(a) + 2f(b)$, then for all x and y

(A) $f(0) = 1$ **(B)** $f(-x) = -f(x)$ **(C)** $f(-x) = f(x)$
(D) $f(x+y) = f(x) + f(y)$
(E) there is a positive real number T such that $f(x+T) = f(x)$

Problem 23

If the solutions of the equation $x^2 + px + q = 0$ are the cubes of the solutions of the equation $x^2 + mx + n = 0$, then

(A) $p = m^3 + 3mn$ **(B)** $p = m^3 - 3mn$ **(C)** $p + q = m^3$
(D) $\left(\frac{m}{n}\right)^2 = \frac{p}{q}$ **(E)** none of these

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Problem 24

Find the sum $\frac{1}{1(3)}+\frac{1}{3(5)}+\cdots+\frac{1}{(2n-1)(2n+1)}+\cdots+\frac{1}{255(257)}$.

$\textbf{(A) } \frac{127}{255} \qquad \textbf{(B) } \frac{128}{255} \qquad \textbf{(C) } \frac{1}{2} \qquad \textbf{(D) } \frac{128}{257} \qquad \textbf{(E) } \frac{129}{257}$

Problem 25

Determine the largest positive integer n such that $1005!$ is divisible by 10^n.

$\textbf{(A) } 102 \qquad \textbf{(B) } 112 \qquad \textbf{(C) } 249 \qquad \textbf{(D) } 502 \qquad \textbf{(E) } \text{none of these}$

Problem 26

Let a, b, c, and d be the lengths of sides MN, NP, PQ, and QM, respectively, of quadrilateral $MNPQ$. If A is the area of $MNPQ$, then

$\textbf{(A) } A=\left(\frac{a+c}{2}\right)\left(\frac{b+d}{2}\right) \text{ if and only if } MNPQ \text{ is convex}$
$\textbf{(B) } A=\left(\frac{a+c}{2}\right)\left(\frac{b+d}{2}\right) \text{ if and only if } MNPQ \text{ is a rectangle}$
$\textbf{(C) } A\le\left(\frac{a+c}{2}\right)\left(\frac{b+d}{2}\right) \text{ if and only if } MNPQ \text{ is a rectangle}$
$\textbf{(D) } A\le\left(\frac{a+c}{2}\right)\left(\frac{b+d}{2}\right) \text{ if and only if } MNPQ \text{ is a parallelogram}$
$\textbf{(E) } A\ge\left(\frac{a+c}{2}\right)\left(\frac{b+d}{2}\right) \text{ if and only if } MNPQ \text{ is a parallelogram}$

Problem 27

There are two spherical balls of different sizes lying in two corners of a rectangular room, each touching two walls and the floor. If there is a point on each ball which is 5 inches from each wall which that ball touches and 10 inches from the floor, then the sum of the diameters of the balls is

$\textbf{(A) } 20 \text{ inches} \qquad \textbf{(B) } 30 \text{ inches} \qquad \textbf{(C) } 40 \text{ inches} \qquad \textbf{(D) } 60 \text{ inches}$
$\textbf{(E) } \text{not determined by the given information}$

Problem 28

Let $g(x)=x^5+x^4+x^3+x^2+x+1$. What is the remainder when the polynomial $g(x^{12})$ is divided by the polynomial $g(x)$?

$\textbf{(A) } 6 \qquad \textbf{(B) } 5-x \qquad \textbf{(C) } 4-x+x^2 \qquad \textbf{(D) } 3-x+x^2-x^3$
$\textbf{(E) } 2-x+x^2-x^3+x^4$

Problem 29

Find the smallest integer n such that $(x^2+y^2+z^2)^2\le n(x^4+y^4+z^4)$ for all real numbers x, y, and z.

$\textbf{(A) } 2 \qquad \textbf{(B) } 3 \qquad \textbf{(C) } 4 \qquad \textbf{(D) } 6 \qquad \textbf{(E) } \text{There is no such integer n}$

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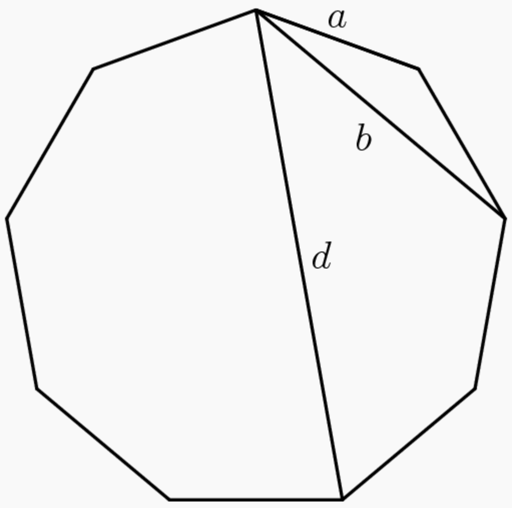
Problem 30



If a, b, and d are the lengths of a side, a shortest diagonal and a longest diagonal, respectively, of a regular nonagon (see adjoining figure), then

$\textbf{(A)}\ d = a + b \qquad \textbf{(B)}\ d^2 = a^2 + b^2 \qquad \textbf{(C)}\ d^2 = a^2 + ab + b^2$
$\textbf{(D)}\ b = \dfrac{a + d}{2} \qquad \textbf{(E)}\ b^2 = ad$