1981 AHSME Problems

Problem 1

If $\sqrt{x+2}=2$, then $(x+2)^2$ equals

$\textbf{(A)}\ \sqrt{2}\qquad\textbf{(B)}\ 2\qquad\textbf{(C)}\ 4\qquad\textbf{(D)}\ 8\qquad\textbf{(E)}\ 16$

Problem 2

Point E is on side AB of square $ABCD$. If EB has length one and EC has length two, then the area of the square is

$\textbf{(A)}\ \sqrt{3}\qquad\textbf{(B)}\ \sqrt{5}\qquad\textbf{(C)}\ 3\qquad\textbf{(D)}\ 2\sqrt{3}\qquad\textbf{(E)}\ 5$

Problem 3

For $x\neq 0$, $\frac{1}{x}+\frac{1}{2x}+\frac{1}{3x}$ equals

$\textbf{(A)}\ \frac{1}{2x}\qquad\textbf{(B)}\ \frac{1}{6}\qquad\textbf{(C)}\ \frac{5}{6x}\qquad\textbf{(D)}\ \frac{11}{6x}\qquad\textbf{(E)}\ \frac{1}{6x^3}$

Problem 4

If three times the larger of two numbers is four times the smaller and the difference between the numbers is 8, the the larger of two numbers is

$\textbf{(A)}\ 16\qquad\textbf{(B)}\ 24\qquad\textbf{(C)}\ 32\qquad\textbf{(D)}\ 44\qquad\textbf{(E)}\ 52$

Problem 5

In trapezoid $ABCD$, sides AB and CD are parallel, and diagonal BD and side AD have equal length.
If $m\angle DCB=110^\circ$ and $m\angle CBD=30^\circ$, then $m\angle ADB=$

$\textbf{(A)}\ 80^\circ\qquad\textbf{(B)}\ 90^\circ\qquad\textbf{(C)}\ 100^\circ\qquad\textbf{(D)}\ 110^\circ\qquad\textbf{(E)}\ 120^\circ$

Problem 6

If $\frac{x}{x-1}=\frac{y^2+2y-1}{y^2+2y-2}$, then x equals

$\textbf{(A)}\ y^2+2y-1\qquad\textbf{(B)}\ y^2+2y-2\qquad\textbf{(C)}\ y^2+2y+2$
$\textbf{(D)}\ y^2+2y+1\qquad\textbf{(E)}\ -y^2-2y+1$

Problem 7

How many of the first one hundred positive integers are divisible by all of the numbers $2, 3, 4$, and 5?

$\textbf{(A)}\ 0\qquad\textbf{(B)}\ 1\qquad\textbf{(C)}\ 2\qquad\textbf{(D)}\ 3\qquad\textbf{(E)}\ 4$

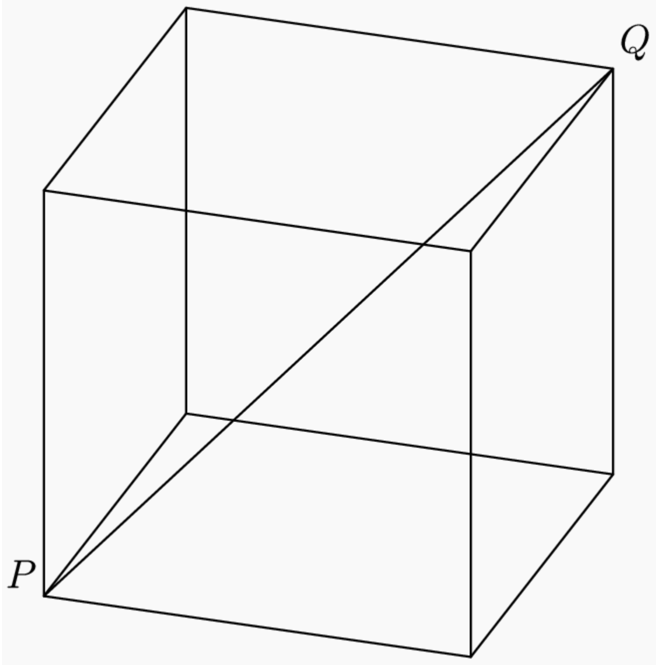
Problem 8

For all positive numbers x, y, z, the product $(x+y+z)^{-1}(x^{-1}+y^{-1}+z^{-1})(xy+yz+xz)^{-1}[(xy)^{-1}+(yz)^{-1}+(xz)^{-1}]$ equals

$\textbf{(A) } x^{-2}y^{-2}z^{-2} \qquad \textbf{(B) } x^{-2}+y^{-2}+z^{-2} \qquad \textbf{(C) } (x+y+z)^{-1} \qquad \textbf{(D) } \frac{1}{xyz} \qquad \textbf{(E) } \frac{1}{xy+yz+xz}$

Problem 9

In the adjoining figure, PQ is a diagonal of the cube. If PQ has length a, then the surface area of the cube is



$\textbf{(A) } 2a^2 \qquad \textbf{(B) } 2\sqrt{2}a^2 \qquad \textbf{(C) } 2\sqrt{3}a^2 \qquad \textbf{(D) } 3\sqrt{3}a^2 \qquad \textbf{(E) } 6a^2$

Problem 10

The lines L and K are symmetric to each other with respect to the line $y=x$. If the equation of the line L is $y=ax+b$ with $a\neq 0$and $b\neq 0$, then the equation of K is $y=$

$\textbf{(A) } \frac{1}{a}x+b \qquad \textbf{(B) } -\frac{1}{a}x+b \qquad \textbf{(C) } \frac{1}{a}x-\frac{b}{a} \qquad \textbf{(D) } \frac{1}{a}x+\frac{b}{a} \qquad \textbf{(E) } \frac{1}{a}x-\frac{b}{a}$

Problem 11

The three sides of a right triangle have integral lengths which form an arithmetic progression. One of the sides could have length

$\textbf{(A) } 22 \qquad \textbf{(B) } 58 \qquad \textbf{(C) } 81 \qquad \textbf{(D) } 91 \qquad \textbf{(E) } 361$

Problem 12

If p, q, and M are positive numbers and $q<100$, then the number obtained by increasing M by $p\%$ and decreasing the result by $q\%$exceeds M if and only if

$\textbf{(A) } p>q \qquad \textbf{(B) } p>\frac{q}{100-q} \qquad \textbf{(C) } p>\frac{q}{1-q} \qquad \textbf{(D) } p>\frac{100q}{100+q} \qquad \textbf{(E) } p>\frac{100q}{100-q}$

Problem 13

Suppose that at the end of any year, a unit of money has lost 10% of the value it had at the beginning of that year. Find the smallest integer n such that after n years, the money will have lost at least 90% of its value (To the nearest thousandth $\log_{10} 3 = 0.477$).

$\textbf{(A)}\ 14 \qquad \textbf{(B)}\ 16 \qquad \textbf{(C)}\ 18 \qquad \textbf{(D)}\ 20 \qquad \textbf{(E)}\ 22$

Problem 14

In a geometric sequence of real numbers, the sum of the first 2 terms is 7, and the sum of the first 6 terms is 91. The sum of the first 4terms is

$\textbf{(A)}\ 28 \qquad \textbf{(B)}\ 32 \qquad \textbf{(C)}\ 35 \qquad \textbf{(D)}\ 49 \qquad \textbf{(E)}\ 84$

Problem 15

If $b > 1$, $x > 0$, and $(2x)^{\log_b 2} - (3x)^{\log_b 3} = 0$, then x is

$\textbf{(A)}\ \frac{1}{216} \qquad \textbf{(B)}\ \frac{1}{6} \qquad \textbf{(C)}\ 1 \qquad \textbf{(D)}\ 6 \qquad \textbf{(E)}\ \text{not uniquely determined}$

Problem 16

The base three representation of x is12112211122211112222The first digit (on the left) of the base nine representation of x is

$\textbf{(A)}\ 1 \qquad \textbf{(B)}\ 2 \qquad \textbf{(C)}\ 3 \qquad \textbf{(D)}\ 4 \qquad \textbf{(E)}\ 5$

Problem 17

The function f is not defined for $x = 0$, but, for all non-zero real numbers x, $f(x) + f\left(\frac{1}{x}\right) = x$. The equation $f(x) = f(-x)$ is satisfied by

$\textbf{(A)}\ \text{exactly one real number} \qquad \textbf{(B)}\ \text{exactly two real numbers} \qquad \textbf{(C)}\ \text{no real numbers}$
$\textbf{(D)}\ \text{infinitely many, but not all, non-zero real numbers} \qquad \textbf{(E)}\ \text{all non-zero real numbers}$

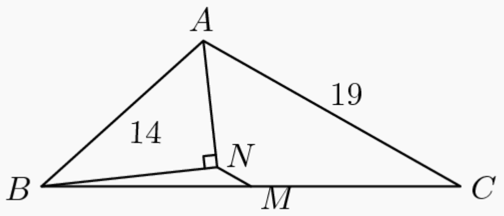
Problem 18

The number of real solutions to the equation $\frac{x}{100} = \sin x$ is

$\textbf{(A)}\ 61 \qquad \textbf{(B)}\ 62 \qquad \textbf{(C)}\ 63 \qquad \textbf{(D)}\ 64 \qquad \textbf{(E)}\ 65$

Problem 19

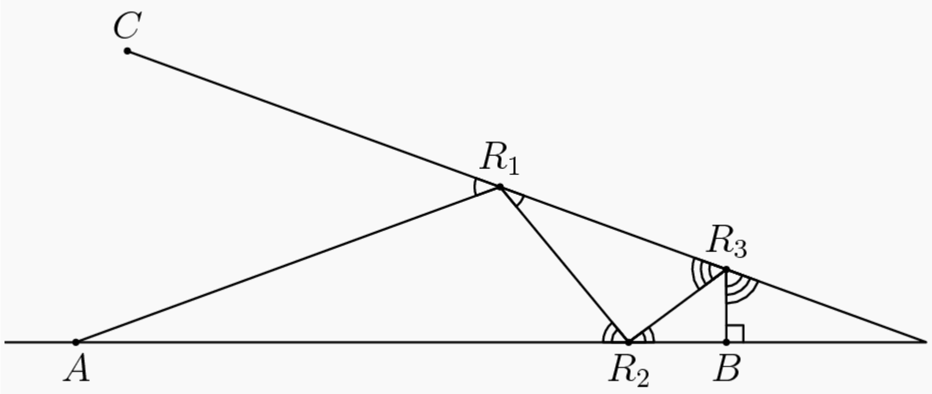
In $\triangle ABC$, M is the midpoint of side BC, AN bisects $\angle BAC$, and $BN \perp AN$. If sides AB and AC have lengths 14 and 19, respectively, then find MN.



$\textbf{(A)}\ 2 \qquad \textbf{(B)}\ \frac{5}{2} \qquad \textbf{(C)}\ \frac{5}{2} - \sin\theta \qquad \textbf{(D)}\ \frac{5}{2} - \frac{1}{2}\sin\theta \qquad \textbf{(E)}\ \frac{5}{2} - \frac{1}{2}\sin\left(\frac{1}{2}\theta\right)$

Problem 20

A ray of light originates from point A and travels in a plane, being reflected n times between lines AD and CD before striking a point B (which may be on AD or CD) perpendicularly and retracing its path back to A (At each point of reflection the light makes two equal angles as indicated in the adjoining figure. The figure shows the light path for $n = 3$). If $\measuredangle CDA = 8^\circ$, what is the largest value n can have?



$\textbf{(A)}\ 6\qquad \textbf{(B)}\ 10\qquad \textbf{(C)}\ 38\qquad \textbf{(D)}\ 98\qquad \textbf{(E)}\ \text{There is no largest value.}$

Problem 21

In a triangle with sides of lengths a, b, and c, $(a+b+c)(a+b-c) = 3ab$. The measure of the angle opposite the side length c is

$\textbf{(A)}\ 15^\circ\qquad \textbf{(B)}\ 30^\circ\qquad \textbf{(C)}\ 45^\circ\qquad \textbf{(D)}\ 60^\circ\qquad \textbf{(E)}\ 150^\circ$

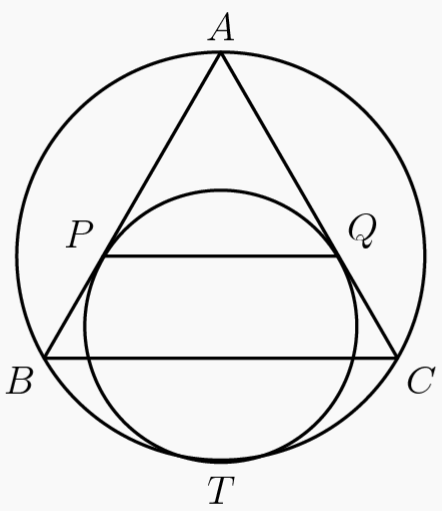
Problem 22

How many lines in a three dimensional rectangular coordiante system pass through four distinct points of the form (i, j, k), where i, j, and k are positive integers not exceeding four?

$\textbf{(A)}\ 60\qquad \textbf{(B)}\ 64\qquad \textbf{(C)}\ 72\qquad \textbf{(D)}\ 76\qquad \textbf{(E)}\ 100$

Problem 23

Equilateral $\triangle ABC$ is inscribed in a circle. A second circle is tangent internally to the circumcircle at T and tangent to sides AB and ACat points P and Q. If side BC has length 12, then segment PQ has length



$\textbf{(A)}\ 6\qquad \textbf{(B)}\ 6\sqrt{3}\qquad \textbf{(C)}\ 8\qquad \textbf{(D)}\ 8\sqrt{3}\qquad \textbf{(E)}\ 9$

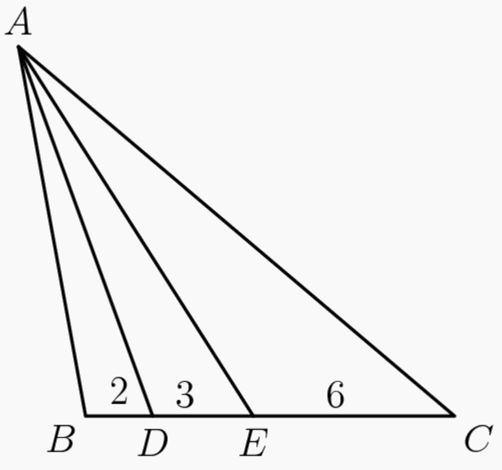
Problem 24

If θ is a constant such that $0 < \theta < \pi$ and $x + \dfrac{1}{x} = 2\cos\theta$, then for each positive integer n, $x^n + \dfrac{1}{x^n}$ equals

$\textbf{(A)}\ 2\cos\theta\qquad \textbf{(B)}\ 2^n\cos\theta\qquad \textbf{(C)}\ 2\cos^n\theta\qquad \textbf{(D)}\ 2\cos n\theta\qquad \textbf{(E)}\ 2^n\cos^n\theta$

Problem 25

In $\triangle ABC$ in the adjoining figure, AD and AE trisect $\angle BAC$. The lengths of BD, DE and EC are 2, 3, and 6, respectively. The length of the shortest side of $\triangle ABC$ is



$\textbf{(A) }2\sqrt{10}\qquad \textbf{(B) }11\qquad \textbf{(C) }6\sqrt{6}\qquad \textbf{(D) }6\qquad \textbf{(E) }\text{not uniquely determined by the given information}$

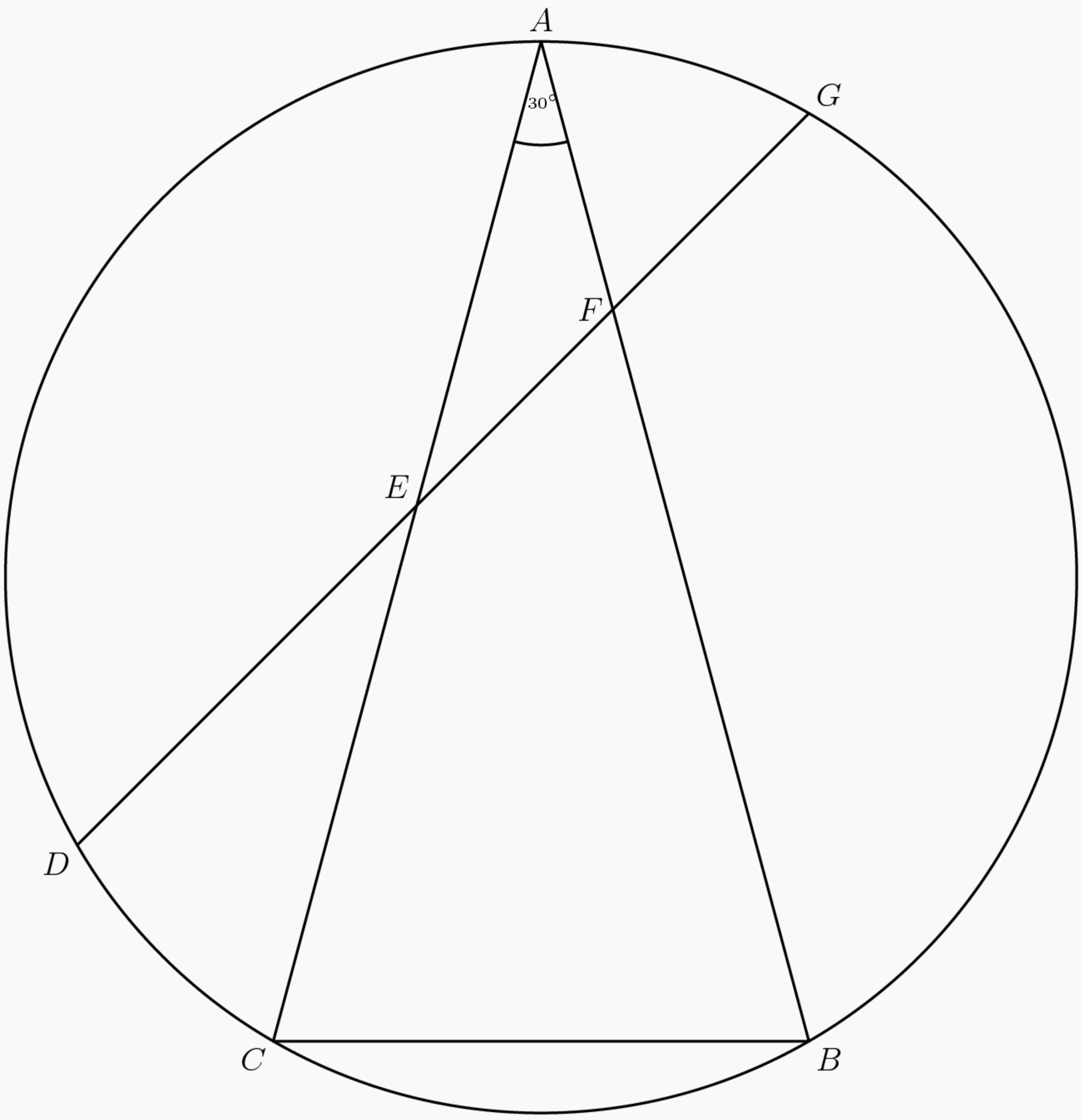
Problem 26

Alice, Bob, and Carol repeatedly take turns tossing a die. Alice begins; Bob always follows Alice; Carol always follows Bob; and Alice always follows Carol. Find the probability that Carol will be the first one to toss a six. (The probability of obtaining a six on any toss is $\frac{1}{6}$, independent of the outcome of any other toss.)

$\textbf{(A) } \frac{1}{3} \qquad \textbf{(B) } \frac{2}{9} \qquad \textbf{(C) } \frac{5}{18} \qquad \textbf{(D) } \frac{25}{91} \qquad \textbf{(E) } \frac{36}{91}$

Problem 27

In the adjoining figure triangle ABC is inscribed in a circle. Point D lies on $\overset{\frown}{AC}$ with $\overset{\frown}{DC} = 30^\circ$, and point G lies on $\overset{\frown}{BA}$ with $\overset{\frown}{BG} > \overset{\frown}{GA}$. Side AB and side AC each have length equal to the length of chord DG, and $\angle CAB = 30^\circ$. Chord DG intersects sides AC and AB at E and F, respectively. The ratio of the area of $\triangle AFE$ to the area of $\triangle ABC$ is



(A) $\dfrac{2-\sqrt{3}}{3}$ (B) $\dfrac{2\sqrt{3}-3}{3}$ (C) $7\sqrt{3}-12$ (D) $3\sqrt{3}-5$
(E) $\dfrac{9-5\sqrt{3}}{3}$

Problem 28

Consider the set of all equations $x^3+a_2x^2+a_1x+a_0=0$, where a_2, a_1, a_0 are real constants and $|a_i|<2$ for $i=0,1,2$. Let r be the largest positive real number which satisfies at least one of these equations. Then

(A) $1<r<\dfrac{3}{2}$ (B) $\dfrac{3}{2}<r<2$ (C) $2<r<\dfrac{5}{2}$ (D) $\dfrac{5}{2}<r<3$
(E) $3<r<\dfrac{7}{2}$

Problem 29

If $a>1$, then the sum of the real solutions of

$$\sqrt{a-\sqrt{a+x}}=x$$

is equal to

(A) $\sqrt{a}-1$ (B) $\dfrac{\sqrt{a}-1}{2}$ (C) $\sqrt{a-1}$ (D) $\dfrac{\sqrt{a-1}}{2}$ (E) $\dfrac{\sqrt{4a-3}-1}{2}$

Problem 30

If a, b, c, and d are the solutions of the equation $x^4 - bx - 3 = 0$, then an equation whose solutions are $\frac{a+b+c}{d^2}, \frac{a+b+d}{c^2}, \frac{a+c+d}{b^2}, \frac{b+c+d}{a^2}$ is

(A) $3x^4 + bx + 1 = 0$ **(B)** $3x^4 - bx + 1 = 0$ **(C)** $3x^4 + bx^3 - 1 = 0$
(D) $3x^4 - bx^3 - 1 = 0$ **(E)** none of these