1957 AHSME Problems

Problem 1

The number of distinct lines representing the altitudes, medians, and interior angle bisectors of a triangle that is isosceles, but not equilateral, is:

$\textbf{(A)}\ 9 \qquad \textbf{(B)}\ 7 \qquad \textbf{(C)}\ 6 \qquad \textbf{(D)}\ 5 \qquad \textbf{(E)}\ 3$

Problem 2

In the equation $2x^2 - hx + 2k = 0$, the sum of the roots is 4 and the product of the roots is -3. Then h and k have the values, respectively:

$\textbf{(A)}\ 8\text{ and }{-6} \qquad \textbf{(B)}\ 4\text{ and }{-3} \qquad \textbf{(C)}\ {-3}\text{ and }4 \qquad \textbf{(D)}\ {-3}\text{ and }8 \qquad \textbf{(E)}\ 8\text{ and }{-3}$

Problem 3

The simplest form of $1 - \frac{1}{1 + \frac{a}{1 - a}}$ is:

$\textbf{(A)}\ a\text{ if }a\not= 0 \qquad \textbf{(B)}\ 1 \qquad \textbf{(C)}\ a\text{ if }a\not=-1 \qquad \textbf{(D)}\ 1-a\text{ with not restriction on }a \qquad \textbf{(E)}\ a\text{ if }a\not= 1$

Problem 4

The first step in finding the product $(3x + 2)(x - 5)$ by use of the distributive property in the form $a(b + c) = ab + ac$ is:

$\textbf{(A)}\ 3x^2 - 13x - 10 \qquad \textbf{(B)}\ 3x(x - 5) + 2(x - 5)$
$\textbf{(C)}\ (3x + 2)x + (3x + 2)(-5) \qquad \textbf{(D)}\ 3x^2 - 17x - 10 \qquad \textbf{(E)}\ 3x^2 + 2x - 15x - 10$

Problem 5

Through the use of theorems on logarithms $\log{\frac{a}{b}} + \log{\frac{b}{c}} + \log{\frac{c}{d}} - \log{\frac{ay}{dx}}$ can be reduced to:

$\textbf{(A)}\ \log{\frac{y}{x}}\qquad \textbf{(B)}\ \log{\frac{x}{y}}\qquad \textbf{(C)}\ 1\qquad$
$\textbf{(D)}\ 140x-24x^2+x^3\qquad \textbf{(E)}\ \text{none of these}$

Problem 6

An open box is constructed by starting with a rectangular sheet of metal 10 in. by 14 in. and cutting a square of side x inches from each corner. The resulting projections are folded up and the seams welded. The volume of the resulting box is:

$\textbf{(A)}\ 140x - 48x^2 + 4x^3 \qquad \textbf{(B)}\ 140x + 48x^2 + 4x^3$
$\textbf{(C)}\ 140x+24x^2+x^3 \qquad \textbf{(D)}\ 140x-24x^2+x^3 \qquad \textbf{(E)}\ \text{none of these}$

Problem 7

The area of a circle inscribed in an equilateral triangle is 48π. The perimeter of this triangle is:

$\textbf{(A)}\ 72\sqrt{3} \qquad \textbf{(B)}\ 48\sqrt{3} \qquad \textbf{(C)}\ 36 \qquad \textbf{(D)}\ 24 \qquad \textbf{(E)}\ 72$

Problem 8

The numbers $x,\, y,\, z$ are proportional to $2,\,3,\,5$. The sum of x, y, and z is 100. The number y is given by the equation $y = ax - 10$. Then a is:

$\textbf{(A) } 2 \qquad \textbf{(B) } \frac{3}{2} \qquad \textbf{(C) } 3 \qquad \textbf{(D) } \frac{5}{2} \qquad \textbf{(E) } 4$

Problem 9

The value of $x - y^{x-y}$ when $x = 2$ and $y = -2$ is:

$\textbf{(A) }-18 \qquad \textbf{(B) }-14 \qquad \textbf{(C) } 14 \qquad \textbf{(D) } 18 \qquad \textbf{(E) } 256$

Problem 10

The graph of $y = 2x^2 + 4x + 3$ has its:

$\textbf{(A)}$ lowest point at $(-1, 9)$ $\textbf{(B)}$ lowest point at $(1, 1)$
$\textbf{(C)}$ lowest point at $(-1, 1)$ $\textbf{(D)}$ highest point at $(-1, 9)$
$\textbf{(E)}$ highest point at $(-1, 1)$

Problem 11

The angle formed by the hands of a clock at $2:15$ is:

$\textbf{(A) } 30^\circ \qquad \textbf{(B) } 27\frac{1}{2}^\circ \qquad \textbf{(C) } 157\frac{1}{2}^\circ \qquad \textbf{(D) } 172\frac{1}{2}^\circ \qquad \textbf{(E) }$ none of these

Problem 12

Comparing the numbers 10^{-49} and $2 \cdot 10^{-50}$ we may say:

$\textbf{(A)}$ the first exceeds the second by $8 \cdot 10^{-1}$
$\textbf{(B)}$ the first exceeds the second by $2 \cdot 10^{-1}$
$\textbf{(C)}$ the first exceeds the second by $8 \cdot 10^{-50}$
$\textbf{(D)}$ the second is five times the first
$\textbf{(E)}$ the first exceeds the second by 5

Problem 13

A rational number between $\sqrt{2}$ and $\sqrt{3}$ is:

$\textbf{(A) } \frac{\sqrt{2} + \sqrt{3}}{2} \qquad \textbf{(B) } \frac{\sqrt{2} \cdot \sqrt{3}}{2} \qquad \textbf{(C) } 1.5 \qquad \textbf{(D) } 1.8 \qquad \textbf{(E) } 1.4$

Problem 14

If $y = \sqrt{x^2 - 2x + 1} + \sqrt{x^2 + 2x + 1}$, then y is:

$\textbf{(A) } 2x \qquad \textbf{(B) } 2(x+1) \qquad \textbf{(C) } 0 \qquad \textbf{(D) } |x-1| + |x+1| \qquad \textbf{(E) }$ none of these

Problem 15

The table below shows the distance s in feet a ball rolls down an inclined plane in t seconds.

t	0	1	2	3	4	5
s	0	10	40	90	160	250

The distance s for $t = 2.5$ is:

$\textbf{(A) } 45 \qquad \textbf{(B) } 62.5 \qquad \textbf{(C) } 70 \qquad \textbf{(D) } 75 \qquad \textbf{(E) } 82.5$

Problem 16

Goldfish are sold at 15 cents each. The rectangular coordinate graph showing the cost of 1 to 12 goldfish is:

$\textbf{(A)}$ a straight line segment
$\textbf{(B)}$ a set of horizontal parallel line segments
$\textbf{(C)}$ a set of vertical parallel line segments
$\textbf{(D)}$ a finite set of distinct points $\qquad \textbf{(E)}$ a straight line

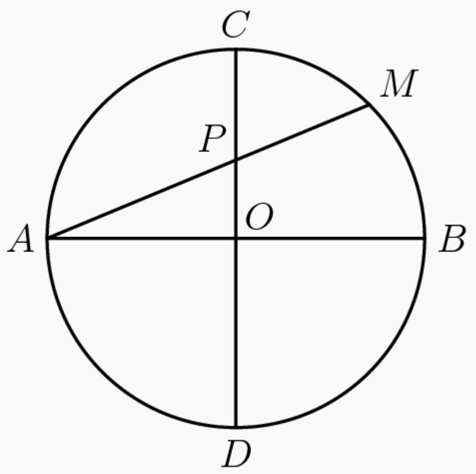
Problem 17

A cube is made by soldering twelve 3-inch lengths of wire properly at the vertices of the cube. If a fly alights at one of the vertices and then walks along the edges, the greatest distance it could travel before coming to any vertex a second time, without retracing any distance, is:

$\textbf{(A)}\ 24\text{ in.} \qquad \textbf{(B)}\ 12\text{ in.} \qquad \textbf{(C)}\ 30\text{ in.} \qquad \textbf{(D)}\ 18\text{ in.} \qquad \textbf{(E)}\ 36\text{ in.}$

Problem 18

Circle O has diameters AB and CD perpendicular to each other. AM is any chord intersecting CD at P. Then $AP\cdot AM$ is equal to:



$\textbf{(A)}\ AO\cdot OB \qquad \textbf{(B)}\ AO\cdot AB$
$\textbf{(C)}\ CP\cdot CD \qquad \textbf{(D)}\ CP\cdot PD \qquad \textbf{(E)}\ CO\cdot OP$

Problem 19

The base of the decimal number system is ten, meaning, for example, that $123=1\cdot 10^2+2\cdot 10+3$. In the binary system, which has base two, the first five positive integers are $1,\ 10,\ 11,\ 100,\ 101$. The numeral 10011 in the binary system would then be written in the decimal system as:

$\textbf{(A)}\ 19 \qquad \textbf{(B)}\ 40 \qquad \textbf{(C)}\ 10011 \qquad \textbf{(D)}\ 11 \qquad \textbf{(E)}\ 7$

Problem 20

A man makes a trip by automobile at an average speed of 50 mph. He returns over the same route at an average speed of 45 mph. His average speed for the entire trip is:

$\textbf{(A)}\ 47\frac{7}{19} \qquad \textbf{(B)}\ 47\frac{1}{4} \qquad \textbf{(C)}\ 47\frac{1}{2} \qquad \textbf{(D)}\ 47\frac{11}{19} \qquad \textbf{(E)}\ \text{none of these}$

Problem 21

Start with the theorem "If two angles of a triangle are equal, the triangle is isosceles," and the following four statements:

1. If two angles of a triangle are not equal, the triangle is not isosceles. 2. The base angles of an isosceles triangle are equal. 3. If a triangle is not isosceles, then two of its angles are not equal. 4. A necessary condition that two angles of a triangle be equal is that the triangle be isosceles.

Which combination of statements contains only those which are logically equivalent to the given theorem?

(A) $1, 2, 3, 4$ **(B)** $1, 2, 3$ **(C)** $2, 3, 4$ **(D)** $1, 2$ **(E)** $3, 4$

Problem 22

If $\sqrt{x-1}-\sqrt{x+1}+1=0$, then $4x$ equals:

(A) 5 **(B)** $4\sqrt{-1}$ **(C)** 0 **(D)** $1\frac{1}{4}$ **(E)** no real value

Problem 23

The graph of $x^2+y=10$ and the graph of $x+y=10$ meet in two points. The distance between these two points is:

(A) less than 1 **(B)** 1 **(C)** $\sqrt{2}$ **(D)** 2 **(E)** more than 2

Problem 24

If the square of a number of two digits is decreased by the square of the number formed by reversing the digits, then the result is not always divisible by:

(A) 9 **(B)** the product of the digits **(C)** the sum of the digits **(D)** the difference of the digits **(E)** 11

Problem 25

The vertices of $\triangle PQR$ have coordinates as follows: $P(0,a),\ Q(b,0),\ R(c,d)$, where $a,\ b,\ c$ and d are positive. The origin and point Rlie on opposite sides of PQ. The area of $\triangle PQR$ may be found from the expression:

(A) $\frac{ab+ac+bc+cd}{2}$ **(B)** $\frac{ac+bd-ab}{2}$ **(C)** $\frac{ab-ac-bd}{2}$ **(D)** $\frac{ac+bd+ab}{2}$ **(E)** $\frac{ac+bd-ab-cd}{2}$

Problem 26

From a point within a triangle, line segments are drawn to the vertices. A necessary and sufficient condition that the three triangles thus formed have equal areas is that the point be:

(A) the center of the inscribed circle
(B) the center of the circumscribed circle
(C) such that the three angles fromed at the point each be 120°
(D) the intersection of the altitudes of the triangle
(E) the intersection of the medians of the triangle

Problem 27

The sum of the reciprocals of the roots of the equation $x^2+px+q=0$ is:

(A) $-\frac{p}{q}$ **(B)** $\frac{q}{p}$ **(C)** $\frac{p}{q}$ **(D)** $-\frac{q}{p}$ **(E)** pq

Problem 28

If a and b are positive and $a\neq 1,\ b\neq 1$, then the value of $b^{\log_b a}$ is:

(A) dependent upon b **(B)** dependent upon a **(C)** dependent upon a and b **(D)** zero **(E)** one

Problem 29

The relation $x^2(x^2-1) \geq 0$ is true only for:

$\textbf{(A) } x \geq 1 \qquad \textbf{(B) } -1 \leq x \leq 1 \qquad \textbf{(C) } x=0,\ x=1,\ x=-1$
$\textbf{(D) } x=0,\ x \leq -1,\ x \geq 1 \qquad \textbf{(E) } x \geq 0$

Problem 30

The sum of the squares of the first n positive integers is given by the expression $\frac{n(n+c)(2n+k)}{6}$, if c and k are, respectively:

$\textbf{(A) } 1 \text{ and } 2 \qquad \textbf{(B) } 3 \text{ and } 5 \qquad \textbf{(C) } 2 \text{ and } 2 \qquad \textbf{(D) } 1 \text{ and } 1 \qquad \textbf{(E) } 2 \text{ and } 1$

Problem 31

A regular octagon is to be formed by cutting equal isosceles right triangles from the corners of a square. If the square has sides of one unit, the leg of each of the triangles has length:

$\textbf{(A) } \frac{2+\sqrt{2}}{3} \qquad \textbf{(B) } \frac{2-\sqrt{2}}{2} \qquad \textbf{(C) } \frac{1+\sqrt{2}}{2} \qquad \textbf{(D) } \frac{1+\sqrt{2}}{3} \qquad \textbf{(E) } \frac{2-\sqrt{2}}{3}$

Problem 32

The largest of the following integers which divides each of the numbers of the sequence $1^5-1,\ 2^5-2,\ 3^5-3,\ \cdots, n^5-n, \cdots$ is:

$\textbf{(A) } 1 \qquad \textbf{(B) } 60 \qquad \textbf{(C) } 15 \qquad \textbf{(D) } 120 \qquad \textbf{(E) } 30$

Problem 33

If $9^{x+2} = 240 + 9^x$, then the value of x is:

$\textbf{(A) } 0.1 \qquad \textbf{(B) } 0.2 \qquad \textbf{(C) } 0.3 \qquad \textbf{(D) } 0.4 \qquad \textbf{(E) } 0.5$

Problem 34

The points that satisfy the system $x+y=1,\ x^2+y^2<25$, constitute the following set:

$\textbf{(A) }$ only two points
$\textbf{(B) }$ an arc of a circle
$\textbf{(C) }$ a straight line segment not including the end-points
$\textbf{(D) }$ a straight line segment including the end-points
$\textbf{(E) }$ a single point

Problem 35

Side AC of right triangle ABC is divide into 8 equal parts. Seven line segments parallel to BC are drawn to AB from the points of division. If $BC = 10$, then the sum of the lengths of the seven line segments:

$\textbf{(A) }$ cannot be found from the given information $\qquad \textbf{(B) }$ is 33 $\qquad \textbf{(C) }$ is 34 $\qquad \textbf{(D) }$ is 35 $\qquad \textbf{(E) }$ is 45

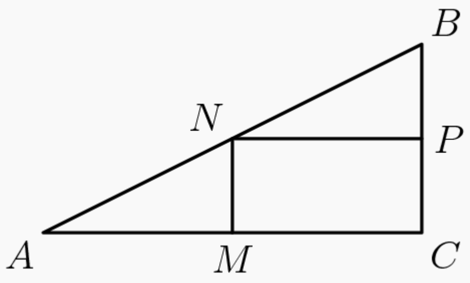
Problem 36

If $x+y=1$, then the largest value of xy is:

$\textbf{(A) } 1 \qquad \textbf{(B) } 0.5 \qquad \textbf{(C) }$ an irrational number about 0.4 $\qquad \textbf{(D) } 0.25 \qquad \textbf{(E) } 0$

Problem 37

In right triangle $ABC, BC = 5, AC = 12$, and $AM = x; \overline{MN} \perp \overline{AC}, \overline{NP} \perp \overline{BC}$; N is on AB. If $y = MN + NP$, one-half the perimeter of rectangle $MCPN$, then:



$\textbf{(A) } y = \frac{1}{2}(5 + 12) \qquad \textbf{(B) } y = \frac{5x}{12} + \frac{12}{5} \qquad \textbf{(C) } y = \frac{144 - 7x}{12}$
$\textbf{(D) } y = 12 \qquad \textbf{(E) } y = \frac{5x}{12} + 6$

Problem 38

From a two-digit number N we subtract the number with the digits reversed and find that the result is a positive perfect cube. Then:

$\textbf{(A) }$ N cannot end in 5
$\textbf{(B) }$ N can end in any digit other than 5
$\textbf{(C) }$ N does not exist
$\textbf{(D) }$ there are exactly 7 values for N
$\textbf{(E) }$ there are exactly 10 values for N

Problem 39

Two men set out at the same time to walk towards each other from M and N, 72 miles apart. The first man walks at the rate of 4 mph. The second man walks 2 miles the first hour, $2\frac{1}{2}$ miles the second hour, 3 miles the third hour, and so on in arithmetic progression. Then the men will meet:

$\textbf{(A) }$ in 7 hours $\qquad \textbf{(B) }$ in $8\frac{1}{4}$ hours $\qquad \textbf{(C) }$ nearer M than N
$\textbf{(D) }$ nearer N than M $\qquad \textbf{(E) }$ midway between M and N

Problem 40

If the parabola $y = -x^2 + bx - 8$ has its vertex on the x-axis, then b must be:

$\textbf{(A) }$ a positive integer
$\textbf{(B) }$ a positive or a negative rational number
$\textbf{(C) }$ a positive rational number
$\textbf{(D) }$ a positive or a negative irrational number
$\textbf{(E) }$ a negative irrational number

Problem 41

Given the system of equations $ax + (a-1)y = 1(a+1)x - ay = 1$. For which one of the following values of a is there no solution x and y?

$\textbf{(A) } 1 \qquad \textbf{(B) } 0 \qquad \textbf{(C) } -1 \qquad \textbf{(D) } \frac{\pm\sqrt{2}}{2} \qquad \textbf{(E) } \pm\sqrt{2}$

Problem 42

If $S = i^n + i^{-n}$, where $i = \sqrt{-1}$ and n is an integer, then the total number of possible distinct values for S is:

$\textbf{(A) } 1 \qquad \textbf{(B) } 2 \qquad \textbf{(C) } 3 \qquad \textbf{(D) } 4 \qquad \textbf{(E) } \text{more than } 4$

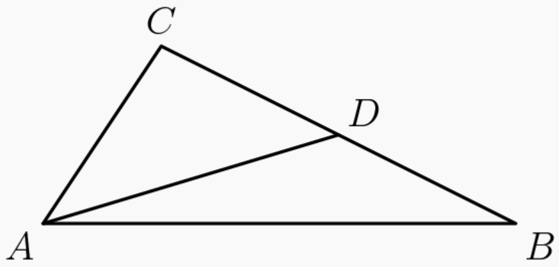
Problem 43

We define a lattice point as a point whose coordinates are integers, zero admitted. Then the number of lattice points on the boundary and inside the region bounded by the x-axis, the line $x = 4$, and the parabola $y = x^2$ is:

$\textbf{(A) } 24 \qquad \textbf{(B) } 35 \qquad \textbf{(C) } 34 \qquad \textbf{(D) } 30 \qquad \textbf{(E) } \infty$

Problem 44

In $\triangle ABC, AC = CD$ and $\angle CAB - \angle ABC = 30^\circ$. Then $\angle BAD$ is:



$\textbf{(A) } 30^\circ \qquad \textbf{(B) } 20^\circ \qquad \textbf{(C) } 22\frac{1}{2}^\circ \qquad \textbf{(D) } 10^\circ \qquad \textbf{(E) } 15^\circ$

Problem 45

If two real numbers x and y satisfy the equation $\frac{x}{y} = x - y$, then:

$\textbf{(A) } x \ge 4 \text{ and } x \le 0$
$\textbf{(B) } y \text{ can equal } 1$
$\textbf{(C) } \text{both } x \text{ and } y \text{ must be irrational}$
$\textbf{(D) } x \text{ and } y \text{ cannot both be integers}$
$\textbf{(E) } \text{both } x \text{ and } y \text{ must be rational}$

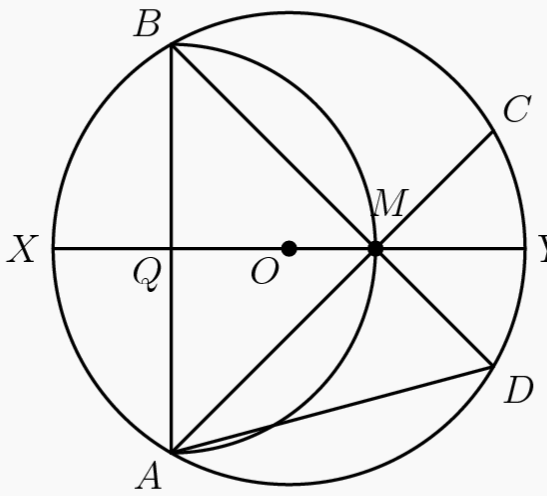
Problem 46

Two perpendicular chords intersect in a circle. The segments of one chord are 3 and 4; the segments of the other are 6 and 2. Then the diameter of the circle is:

$\textbf{(A) } \sqrt{89} \qquad \textbf{(B) } \sqrt{56} \qquad \textbf{(C) } \sqrt{61} \qquad \textbf{(D) } \sqrt{75} \qquad \textbf{(E) } \sqrt{65}$

Problem 47

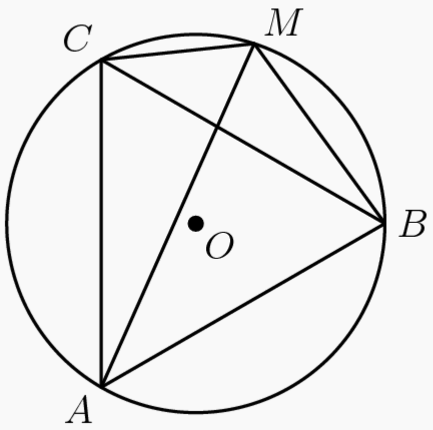
In circle O, the midpoint of radius OX is Q; at Q, $\overline{AB} \perp \overline{XY}$. The semi-circle with $\overline{AB}$ as diameter intersects $\overline{XY}$ in M. Line $\overline{AM}$ intersects circle O in C, and line $\overline{BM}$ intersects circle O in D. Line $\overline{AD}$ is drawn. Then, if the radius of circle O is r, AD is:



(A) $r\sqrt{2}$ **(B)** r **(C)** not a side of an inscribed regular polygon **(D)** $\frac{r\sqrt{3}}{2}$ **(E)** $r\sqrt{3}$

Problem 48

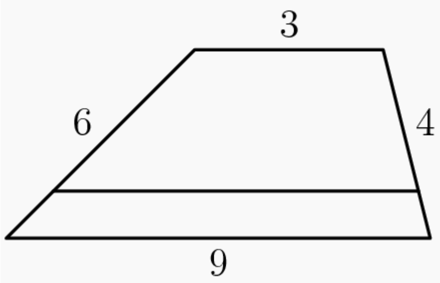
Let ABC be an equilateral triangle inscribed in circle O. M is a point on arc BC. Lines $\overline{AM}$, $\overline{BM}$, and $\overline{CM}$ are drawn. Then AM is:



(A) equal to $BM + CM$ **(B)** less than $BM + CM$
(C) greater than $BM + CM$
(D) equal, less than, or greater than $BM + CM$, depending upon the position of M
(E) none of these

Problem 49

The parallel sides of a trapezoid are 3 and 9. The non-parallel sides are 4 and 6. A line parallel to the bases divides the trapezoid into two trapezoids of equal perimeters. The ratio in which each of the non-parallel sides is divided is:



(A) $4:3$ **(B)** $3:2$ **(C)** $4:1$ **(D)** $3:1$ **(E)** $6:1$

Problem 50

In circle O, G is a moving point on diameter $\overline{AB}$. $\overline{AA'}$ is drawn perpendicular to $\overline{AB}$ and equal to $\overline{AG}$. $\overline{BB'}$ is drawn perpendicular to $\overline{AB}$, on the same side of diameter $\overline{AB}$ as $\overline{AA'}$, and equal to BG. Let O' be the midpoint of $\overline{A'B'}$. Then, as G moves from A to B, point O':

(A) moves on a straight line parallel to AB
(B) remains stationary
(C) moves on a straight line perpendicular to AB
(D) moves in a small circle intersecting the given circle
(E) follows a path which is neither a circle nor a straight line