1984 AHSME Problems

Problem 1

$\dfrac{1000^2}{252^2-248^2}$ equals

$\textbf{(A) }62,500\qquad\textbf{(B) }1,000\qquad\textbf{(C) }500\qquad\textbf{(D) }250\qquad\textbf{(E) }\frac{1}{2}$

Problem 2

If x, y, and $y-\frac{1}{x}$ are not 0, then

$\dfrac{x-\frac{1}{y}}{y-\frac{1}{x}}$ equals

$\textbf{(A) }1\qquad\textbf{(B) }\frac{x}{y}\qquad\textbf{(C) }\frac{y}{x}\qquad\textbf{(D) }\frac{x}{y}-\frac{y}{x}\qquad\textbf{(E) }xy-\frac{1}{xy}$

Problem 3

Let n be the smallest nonprime integer greater than 1 with no prime factor less than 10. Then

$\textbf{(A) }100<n\leq110\qquad\textbf{(B) }110<n\leq120\qquad\textbf{(C) }120<n\leq130\qquad\textbf{(D) }130<n\leq140\qquad\textbf{(E) }140<n\leq150$

Problem 4

Points B, C, F, E are picked on a circle such that $BC||EF$. When BC is extended to the left, point A is marked outside the circle such that $AB = 4$ and $BC = 5$. When EF is extended to the left, point D is marked outside the circle such that $DE = 3$. AD is perpendicular to both AC and DF. Find the length of EF.

This article is a stub. Help us out by expanding it.

Problem 5

The largest integer n for which $n^{200}<5^{300}$ is

$\textbf{(A) }8\qquad\textbf{(B) }9\qquad\textbf{(C) }10\qquad\textbf{(D) }11\qquad\textbf{(E) }12$

Problem 6

In a certain school, there are 3 times as many boys as girls and 9 times as many girls as teachers. Using the letters b, g, t to represent the number of boys, girls, and teachers, respectively, then the total number of boys, girls, and teachers can be represented by the expression

$\textbf{(A) }31b\qquad\textbf{(B) }\frac{37b}{27}\qquad\textbf{(C) }13g\qquad\textbf{(D) }\frac{37g}{27}\qquad\textbf{(E) }\frac{37t}{27}$

Problem 7

When Dave walks to school, he averages 90 steps per minute, and each of his steps is 75 cm long. It takes him 16 minutes to get to school. His brother, Jack, going to the same school by the same route, averages 100 steps per minute, but his steps are only 60 cm long. How long does it take Jack to get to school?

$\textbf{(A) }14\frac{2}{9}\text{ minutes}\qquad\textbf{(B) }15\text{ minutes}\qquad\textbf{(C) }18\text{ minutes}\qquad\textbf{(D) }20\text{ minutes}\qquad\textbf{(E) }22\frac{2}{9}\text{ minutes}$

Problem 8

Figure $ABCD$ is a trapezoid with $AB||DC$, $AB=5$, $BC=3\sqrt{2}$, $\angle BCD=45^\circ$, and $\angle CDA=60^\circ$. The length of DC is

$\mathrm{(A) \ }7+\frac{2}{3}\sqrt{3} \qquad \mathrm{(B) \ } 8 \qquad \mathrm{(C) \ } 9\frac{1}{2} \qquad \mathrm{(D) \ } 8+\sqrt{3} \qquad \mathrm{(E) \ } 8+3\sqrt{3}$

Problem 9

The number of digits in $4^{16}5^{25}$ (when written in the usual base 10 form) is

$\mathrm{(A) \ }31 \qquad \mathrm{(B) \ } 30 \qquad \mathrm{(C) \ } 29 \qquad \mathrm{(D) \ } 28 \qquad \mathrm{(E) \ } 27$

Problem 10

Four complex numbers lie at the vertices of a square in the complex plane. Three of the numbers are $1+2i, -2+i$, and $-1-2i$. The fourth number is

$\mathrm{(A) \ }2+i \qquad \mathrm{(B) \ } 2-i \qquad \mathrm{(C) \ } 1-2i \qquad \mathrm{(D) \ } -1+2i \qquad \mathrm{(E) \ } -2-i$

Problem 11

A calculator has a key that replaces the displayed entry with its square, and another key which replaces the displayed entry with its reciprocal. Let y be the final result when one starts with a number $x\ne 0$ and alternately squares and reciprocates n times each. Assuming the calculator is completely accurate (e.g. no roundoff or overflow), then y equals

$\mathrm{(A) \ }x^{((-2)^n)} \qquad \mathrm{(B) \ } x^{2n} \qquad \mathrm{(C) \ } x^{-2n} \qquad \mathrm{(D) \ } x^{-(2^n)} \qquad \mathrm{(E) \ } x^{((-1)^n2n)}$

Problem 12

If the sequence $\{a_n\}$ is defined by

$a_1=2$

$a_{n+1}=a_n+2n$

where $n\geq1$.

Then a_{100} equals

$\mathrm{(A) \ }9900 \qquad \mathrm{(B) \ } 9902 \qquad \mathrm{(C) \ } 9904 \qquad \mathrm{(D) \ } 10100 \qquad \mathrm{(E) \ } 10102$

Problem 13

$\frac{2\sqrt{6}}{\sqrt{2}+\sqrt{3}+\sqrt{5}}$ equals

$\mathrm{(A) \ }\sqrt{2}+\sqrt{3}-\sqrt{5} \qquad \mathrm{(B) \ } 4-\sqrt{2}-\sqrt{3} \qquad \mathrm{(C) \ } \sqrt{2}+\sqrt{3}+\sqrt{6}-5$

$\mathrm{(D) \ }\frac{1}{2}(\sqrt{2}+\sqrt{5}-\sqrt{3}) \qquad \mathrm{(E) \ } \frac{1}{3}(\sqrt{3}+\sqrt{5}-\sqrt{2})$

Problem 14

The product of all real roots of the equation $x^{\log_{10}{x}}=10$ is

$\mathrm{(A) \ }1 \qquad \mathrm{(B) \ } -1 \qquad \mathrm{(C) \ } 10 \qquad \mathrm{(D) \ } 10^{-1} \qquad \mathrm{(E) \ } \text{None of these}$

Problem 15

If $\sin 2x \sin 3x = \cos 2x \cos 3x$, then one value for x is

$\text{(A) } 18^\circ \qquad \text{(B) } 30^\circ \qquad \text{(C) } 36^\circ \qquad \text{(D) } 45^\circ \qquad \text{(E) } 60^\circ$

Problem 16

The function $f(x)$ satisfies $f(2+x) = f(2-x)$ for all real numbers x. If the equation $f(x) = 0$ has exactly four distinct real roots, then the sum of these roots is

$\text{(A) } 0 \qquad \text{(B) } 2 \qquad \text{(C) } 4 \qquad \text{(D) } 6 \qquad \text{(E) } 8$

Problem 17

A right triangle ABC with hypotenuse AB has side $AC = 15$. Altitude CH divides AB into segments AH and HB, with $HB = 16$. The area of $\triangle ABC$ is:

$\text{(A) } 120 \qquad \text{(B) } 144 \qquad \text{(C) } 150 \qquad \text{(D) } 216 \qquad \text{(E) } 144\sqrt{5}$

Problem 18

A point (x, y) is to be chosen in the coordinate plane so that it is equally distant from the x-axis, the y-axis, and the line $x + y = 2$. Then x is

$\text{(A) } \sqrt{2} - 1 \qquad \text{(B) } \frac{1}{2} \qquad \text{(C) } 2 - \sqrt{2} \qquad \text{(D) } 1 \qquad \text{(E) Not uniquely determined}$

Problem 19

A box contains 11 balls, numbered $1, 2, 3, ... 11$. If 6 balls are drawn simultaneously at random, what is the probability that the sum of the numbers on the balls drawn is odd?

$\text{(A) } \frac{100}{231} \qquad \text{(B) } \frac{115}{231} \qquad \text{(C) } \frac{1}{2} \qquad \text{(D) } \frac{118}{231} \qquad \text{(E) } \frac{6}{11}$

Problem 20

The number of the distinct solutions to the equation

$|x - |2x + 1|| = 3$ is

$\text{(A) } 0 \qquad \text{(B) } 1 \qquad \text{(C) } 2 \qquad \text{(D) } 3 \qquad \text{(E) } 4$

Problem 21

The number of triples (a, b, c) of positive integers which satisfy the simultaneous equations

$ab + bc = 44$

$ac + bc = 23$

is

$\text{(A) } 0 \qquad \text{(B) } 1 \qquad \text{(C) } 2 \qquad \text{(D) } 3 \qquad \text{(E) } 4$

Problem 22

Let a and c be fixed positive numbers. For each real number t let (x_t, y_t) be the vertex of the parabola $y = ax^2 + bx + c$. If the set of the vertices (x_t, y_t) for all real numbers of t is graphed on the plane, the graph is

(A) a straight line (B) a parabola (C) part, but not all, of a parabola (D) one branch of a hyperbola (E) None of these

Problem 23

$\frac{\sin 10^\circ + \sin 20^\circ}{\cos 10^\circ + \cos 20^\circ}$ equals

(A) $\tan 10^\circ + \tan 20^\circ$ (B) $\tan 30^\circ$ (C) $\frac{1}{2}(\tan 10^\circ + \tan 20^\circ)$ (D) $\tan 15^\circ$ (E) $\frac{1}{4}\tan 60^\circ$

Problem 24

If a and b are positive real numbers and each of the equations $x^2 + ax + 2b = 0$ and $x^2 + 2bx + a = 0$ has real roots, then the smallest possible value of $a + b$ is

(A) 2 (B) 3 (C) 4 (D) 5 (E) 6

Problem 25

The total area of all the faces of a rectangular solid is 22cm^2, and the total length of all its edges is 24cm. Then the length in cm of any one of its interior diagonals is

(A) $\sqrt{11}$ (B) $\sqrt{12}$ (C) $\sqrt{13}$ (D) $\sqrt{14}$ (E) Not uniquely determined

Problem 26

In the obtuse triangle ABC with $\angle C > 90^\circ$, $AM = MB$, $MD \perp BC$, and $EC \perp BC$ (D is on BC, E is on AB, and M is on EB). If the area of $\triangle ABC$ is 24, then the area of $\triangle BED$ is

(A) 9 (B) 12 (C) 15 (D) 18 (E) Not uniquely determined

Problem 27

In $\triangle ABC$, D is on AC and F is on BC. Also, $AB \perp AC$, $AF \perp BC$, and $BD = DC = FC = 1$. Find AC.

(A) $\sqrt{2}$ (B) $\sqrt{3}$ (C) $\sqrt[3]{2}$ (D) $\sqrt[3]{3}$ (E) $\sqrt[4]{3}$

Problem 28

The number of distinct pairs of integers (x, y) such that $0 < x < y$ and $\sqrt{1984} = \sqrt{x} + \sqrt{y}$ is

(A) 0 (B) 1 (C) 3 (D) 4 (E) 7

Problem 29

Find the largest value for $\frac{y}{x}$ for pairs of real numbers (x, y) which satisfy $(x-3)^2 + (y-3)^2 = 6$.

(A) $3 + 2\sqrt{2}$ (B) $2 + \sqrt{3}$ (C) $3\sqrt{3}$ (D) 6 (E) $6 + 2\sqrt{3}$

Problem 30

For any complex number $w = a + bi$, $|w|$ is defined to be the real number $\sqrt{a^2 + b^2}$. If $w = \cos 40^\circ + i \sin 40^\circ$, then

$|w + 2w^2 + 3w^3 + ... + 9w^9|^{-1}$

equals

$\text{(A) } \frac{1}{9}\sin 40^\circ \qquad \text{(B) } \frac{2}{9}\sin 20^\circ \qquad \text{(C) } \frac{1}{9}\cos 40^\circ \qquad \text{(D) } \frac{1}{18}\cos 20^\circ \qquad \text{(E) None of these}$