1963 AHSME Problems

Problem 1

Which one of the following points is not on the graph of $y=\frac{x}{x+1}$?

$\textbf{(A)}\ (0,0)\qquad \textbf{(B)}\ \left(-\frac{1}{2},-1\right)\qquad \textbf{(C)}\ \left(\frac{1}{2},\frac{1}{3}\right)\qquad \textbf{(D)}\ (-1,1)\qquad \textbf{(E)}\ (-2,2)$

Problem 2

let $n=x-y^{x-y}$. Find n when $x=2$ and $y=-2$.

$\textbf{(A)}\ -14 \qquad \textbf{(B)}\ 0 \qquad \textbf{(C)}\ 1 \qquad \textbf{(D)}\ 18 \qquad \textbf{(E)}\ 256$

Problem 3

If the reciprocal of $x+1$ is $x-1$, then x equals:

$\textbf{(A)}\ 0\qquad \textbf{(B)}\ 1\qquad \textbf{(C)}\ -1\qquad \textbf{(D)}\ \pm 1\qquad \textbf{(E)}\ \text{none of these}$

Problem 4

For what value(s) of k does the pair of equations $y=x^2$ and $y=3x+k$ have two identical solutions?

$\textbf{(A)}\ \frac{4}{9}\qquad \textbf{(B)}\ -\frac{4}{9}\qquad \textbf{(C)}\ \frac{9}{4}\qquad \textbf{(D)}\ -\frac{9}{4}\qquad \textbf{(E)}\ \pm\frac{9}{4}$

Problem 5

If x and $\log_{10} x$ are real numbers and $\log_{10} x<0$, then:

$\textbf{(A)}\ x<0 \qquad \textbf{(B)}\ -1<x<1 \qquad \textbf{(C)}\ 0<x\le 1 \\ \textbf{(D)}\ -1<x<0 \qquad \textbf{(E)}\ 0<x<1$

Problem 6

$\triangle BAD$ is right-angled at B. On AD there is a point C for which $AC=CD$ and $AB=BC$. The magnitude of $\angle DAB$ is:

$\textbf{(A)}\ 67\tfrac{1}{2}^{\circ}\qquad \textbf{(B)}\ 60^{\circ}\qquad \textbf{(C)}\ 45^{\circ}\qquad \textbf{(D)}\ 30^{\circ}\qquad \textbf{(E)}\ 22\tfrac{1}{2}^{\circ}$

Problem 7

Given the four equations:

$\textbf{(1)}\ 3y-2x=12 \qquad\textbf{(2)}\ -2x-3y=10 \qquad\textbf{(3)}\ 3y+2x=12 \qquad\textbf{(4)}\ 2y+3x=10$

The pair representing the perpendicular lines is:

$\textbf{(A)}\ \text{(1) and (4)}\qquad \textbf{(B)}\ \text{(1) and (3)}\qquad \textbf{(C)}\ \text{(1) and (2)}\qquad \textbf{(D)}\ \text{(2) and (4)}\qquad \textbf{(E)}\ \text{(2) and (3)}$

Problem 8

The smallest positive integer x for which $1260x=N^3$, where N is an integer, is:

(A) 1050 **(B)** 1260 **(C)** 1260^2 **(D)** 7350 **(E)** 44100

Problem 9

In the expansion of $\left(a-\frac{1}{\sqrt{a}}\right)^7$ the coefficient of $a^{-\frac{1}{2}}$ is:

(A) -7 **(B)** 7 **(C)** -21 **(D)** 21 **(E)** 35

Problem 10

Point P is taken interior to a square with side-length a and such that is it equally distant from two consecutive vertices and from the side opposite these vertices. If d represents the common distance, then d equals:

(A) $\frac{3a}{5}$ **(B)** $\frac{5a}{8}$ **(C)** $\frac{3a}{8}$ **(D)** $\frac{a\sqrt{2}}{2}$ **(E)** $\frac{a}{2}$

Problem 12

The arithmetic mean of a set of 50 numbers is 38. If two numbers of the set, namely 45 and 55, are discarded, the arithmetic mean of the remaining set of numbers is:

(A) 38.5 **(B)** 37.5 **(C)** 37 **(D)** 36.5 **(E)** 36

Problem 12

Three vertices of parallelogram $PQRS$ are $P(-3,-2), Q(1,-5), R(9,1)$ with P and R diagonally opposite. The sum of the coordinates of vertex S is:

(A) 13 **(B)** 12 **(C)** 11 **(D)** 10 **(E)** 9

Problem 13

If $2^a+2^b=3^c+3^d$, the number of integers a,b,c,d which can possibly be negative, is, at most:

(A) 4 **(B)** 3 **(C)** 2 **(D)** 1 **(E)** 0

Problem 14

Given the equations $x^2+kx+6=0$ and $x^2-kx+6=0$. If, when the roots of the equation are suitably listed, each root of the second equation is 5 more than the corresponding root of the first equation, then k equals:

(A) 5 **(B)** -5 **(C)** 7 **(D)** -7 **(E)** none of these

Problem 15

A circle is inscribed in an equilateral triangle, and a square is inscribed in the circle. The ratio of the area of the triangle to the area of the square is:

(A) $\sqrt{3}:1$ **(B)** $\sqrt{3}:\sqrt{2}$ **(C)** $3\sqrt{3}:2$ **(D)** $3:\sqrt{2}$ **(E)** $3:2\sqrt{2}$

Problem 16

Three numbers a, b, c, none zero, form an arithmetic progression. Increasing a by 1 or increasing c by 2 results in a geometric progression. Then b equals:

$\textbf{(A)}\ 16 \qquad \textbf{(B)}\ 14 \qquad \textbf{(C)}\ 12 \qquad \textbf{(D)}\ 10 \qquad \textbf{(E)}\ 8$

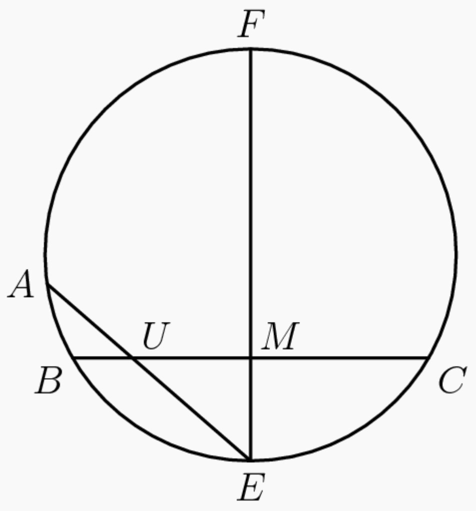
Problem 17

The expression $\frac{\frac{a}{a+y}+\frac{y}{a-y}}{\frac{y}{a+y}-\frac{a}{a-y}}$, a real, $a \neq 0$, has the value -1 for:

$\textbf{(A)}\ \text{all but two real values of }y$
$\textbf{(B)}\ \text{only two real values of }y$
$\textbf{(C)}\ \text{all real values of }y$
$\textbf{(D)}\ \text{only one real value of }y$
$\textbf{(E)}\ \text{no real values of }y$

Problem 18

Chord EF is the perpendicular bisector of chord BC, intersecting it in M. Between B and M point U is taken, and EU extended meets the circle in A. Then, for any selection of U, as described, $\triangle EUM$ is similar to:



$\textbf{(A)}\ \triangle EFA \qquad \textbf{(B)}\ \triangle EFC \qquad \textbf{(C)}\ \triangle ABM \qquad \textbf{(D)}\ \triangle ABU \qquad \textbf{(E)}\ \triangle FMC$

Problem 19

In counting n colored balls, some red and some black, it was found that 49 of the first 50 counted were red. Thereafter, 7 out of every 8counted were red. If, in all, 90 % or more of the balls counted were red, the maximum value of n is:

$\textbf{(A)}\ 225 \qquad \textbf{(B)}\ 210 \qquad \textbf{(C)}\ 200 \qquad \textbf{(D)}\ 180 \qquad \textbf{(E)}\ 175$

Problem 20

Two men at points R and S, 76 miles apart, set out at the same time to walk towards each other. The man at R walks uniformly at the rate of $4\tfrac{1}{2}$ miles per hour; the man at S walks at the constant rate of $3\tfrac{1}{4}$ miles per hour for the first hour, at $3\tfrac{3}{4}$ miles per hour for the second hour, and so on, in arithmetic progression. If the men meet x miles nearer R than S in an integral number of hours, then x is:

$\textbf{(A)}\ 10 \qquad \textbf{(B)}\ 8 \qquad \textbf{(C)}\ 6 \qquad \textbf{(D)}\ 4 \qquad \textbf{(E)}\ 2$

Problem 21

The expression $x^2-y^2-z^2+2yz+x+y-z$ has:

$\textbf{(A)}$ no linear factor with integer coefficients and integer exponents
$\textbf{(B)}$ the factor $-x+y+z$
$\textbf{(C)}$ the factor $x-y-z+1$
$\textbf{(D)}$ the factor $x+y-z+1$
$\textbf{(E)}$ the factor $x-y+z+1$

Problem 22

Acute-angled $\triangle ABC$ is inscribed in a circle with center at O; $\overset{\frown}{AB} = 120$ and $\overset{\frown}{BC} = 72$. A point E is taken in minor arc AC such that OE is perpendicular to AC. Then the ratio of the magnitudes of $\angle OBE$ and $\angle BAC$ is:

$\textbf{(A)}\ \frac{5}{18} \qquad \textbf{(B)}\ \frac{2}{9} \qquad \textbf{(C)}\ \frac{1}{4} \qquad \textbf{(D)}\ \frac{1}{3} \qquad \textbf{(E)}\ \frac{4}{9}$

Problem 23

A gives B as many cents as B has and C as many cents as C has. Similarly, B then gives A and C as many cents as each then has. C, similarly, then gives A and B as many cents as each then has. If each finally has 16 cents, with how many cents does A start?

$\textbf{(A)}\ 24 \qquad \textbf{(B)}\ 26 \qquad \textbf{(C)}\ 28 \qquad \textbf{(D)}\ 30 \qquad \textbf{(E)}\ 32$

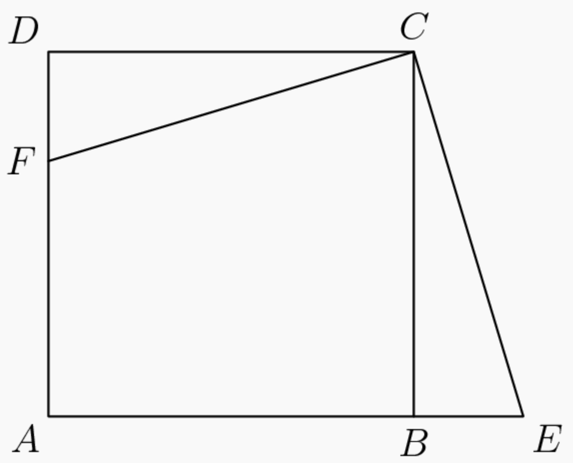
Problem 24

Consider equations of the form $x^2 + bx + c = 0$. How many such equations have real roots and have coefficients b and c selected from the set of integers $\{1,2,3,4,5,6\}$?

$\textbf{(A)}\ 20 \qquad \textbf{(B)}\ 19 \qquad \textbf{(C)}\ 18 \qquad \textbf{(D)}\ 17 \qquad \textbf{(E)}\ 16$

Problem 25

Point F is taken in side AD of square $ABCD$. At C a perpendicular is drawn to CF, meeting AB extended at E. The area of $ABCD$is 256 square inches and the area of $\triangle CEF$ is 200 square inches. Then the number of inches in BE is:



$\textbf{(A)}\ 12 \qquad \textbf{(B)}\ 14 \qquad \textbf{(C)}\ 15 \qquad \textbf{(D)}\ 16 \qquad \textbf{(E)}\ 20$

Problem 26

Version 1 Consider the statements:

$\textbf{(1)}\ p\wedge \sim q\wedge r \qquad \textbf{(2)}\ \sim p\wedge \sim q\wedge r \qquad \textbf{(3)}\ p\wedge \sim q\wedge \sim r \qquad \textbf{(4)}\ \sim p \wedge q \wedge r$

where p,q, and r are propositions. How many of these imply the truth of $(p\rightarrow q)\rightarrow r$?

$\textbf{(A)}\ 0 \qquad \textbf{(B)}\ 1 \qquad \textbf{(C)}\ 2 \qquad \textbf{(D)}\ 3 \qquad \textbf{(E)}\ 4$

Version 2 Consider the statements (1) p and r are true and q is false (2) r is true and p and q are false (3) p is true and q and r are false (4) q and r are true and p is false. How many of these imply the truth of the statement "r is implied by the statement that p implies q"?

$\textbf{(A)}\ 0 \qquad \textbf{(B)}\ 1 \qquad \textbf{(C)}\ 2 \qquad \textbf{(D)}\ 3 \qquad \textbf{(E)}\ 4$

Problem 27

Six straight lines are drawn in a plane with no two parallel and no three concurrent. The number of regions into which they divide the plane is:

$\textbf{(A) } 16 \qquad \textbf{(B) } 20 \qquad \textbf{(C) } 22 \qquad \textbf{(D) } 24 \qquad \textbf{(E) } 26$

Problem 28

Given the equation $3x^2 - 4x + k = 0$ with real roots. The value of k for which the product of the roots of the equation is a maximum is:

$\textbf{(A) } \frac{16}{9} \qquad \textbf{(B) } \frac{16}{3} \qquad \textbf{(C) } \frac{4}{9} \qquad \textbf{(D) } \frac{4}{3} \qquad \textbf{(E) } -\frac{4}{3}$

Problem 29

A particle projected vertically upward reaches, at the end of t seconds, an elevation of s feet where $s = 160t - 16t^2$. The highest elevation is:

$\textbf{(A) } 800 \qquad \textbf{(B) } 640 \qquad \textbf{(C) } 400 \qquad \textbf{(D) } 320 \qquad \textbf{(E) } 160$

Problem 30

Let $F = \log \frac{1+x}{1-x}$. Find a new function G by replacing each x in F by $\frac{3x+x^3}{1+3x^2}$, and simplify. The simplified expression G is equal to:

$\textbf{(A) } -F \qquad \textbf{(B) } F \qquad \textbf{(C) } 3F \qquad \textbf{(D) } F^3 \qquad \textbf{(E) } F^3 - F$

Problem 31

The number of solutions in positive integers of $2x + 3y = 763$ is:

$\textbf{(A) } 255 \qquad \textbf{(B) } 254 \qquad \textbf{(C) } 128 \qquad \textbf{(D) } 127 \qquad \textbf{(E) } 0$

Problem 32

The dimensions of a rectangle R are a and b, $a < b$. It is required to obtain a rectangle with dimensions x and y, $x < a, y < a$, so that its perimeter is one-third that of R, and its area is one-third that of R. The number of such (different) rectangles is:

$\textbf{(A) } 0 \qquad \textbf{(B) } 1 \qquad \textbf{(C) } 2 \qquad \textbf{(D) } 4 \qquad \textbf{(E) } \infty$

Problem 33

Given the line $y = \frac{3}{4}x + 6$ and a line L parallel to the given line and 4 units from it. A possible equation for L is:

$\textbf{(A) } y = \frac{3}{4}x + 1 \qquad \textbf{(B) } y = \frac{3}{4}x \qquad \textbf{(C) } y = \frac{3}{4}x - \frac{2}{3}$

$\textbf{(D) } y = \frac{3}{4}x - 1 \qquad \textbf{(E) } y = \frac{3}{4}x + 2$

Problem 34

In $\triangle ABC$, side $a = \sqrt{3}$, side $b = \sqrt{3}$, and side $c > 3$. Let x be the largest number such that the magnitude, in degrees, of the angle opposite side c exceeds x. Then x equals:

$\textbf{(A) }150^\circ\qquad\textbf{(B) }120^\circ\qquad\textbf{(C) }105^\circ\qquad\textbf{(D) }90^\circ\qquad\textbf{(E) }60^\circ$

Problem 35

The lengths of the sides of a triangle are integers, and its area is also an integer. One side is 21 and the perimeter is 48. The shortest side is:

$\textbf{(A) }8\qquad\textbf{(B) }10\qquad\textbf{(C) }12\qquad\textbf{(D) }14\qquad\textbf{(E) }16$

Problem 36

A person starting with $\$64$ and making 6 bets, wins three times and loses three times, the wins and losses occurring in random order. The chance for a win is equal to the chance for a loss. If each wager is for half the money remaining at the time of the bet, then the final result is:

$\textbf{(A)}\text{ a loss of }\$27\qquad\textbf{(B)}\text{ a gain of }\$27\qquad\textbf{(C)}\text{ a loss of }\$37$
$\textbf{(D)}\text{ neither a gain nor a loss}$
$\textbf{(E)}\text{ a gain or a loss depending upon the order in which the wins and losses occur}$

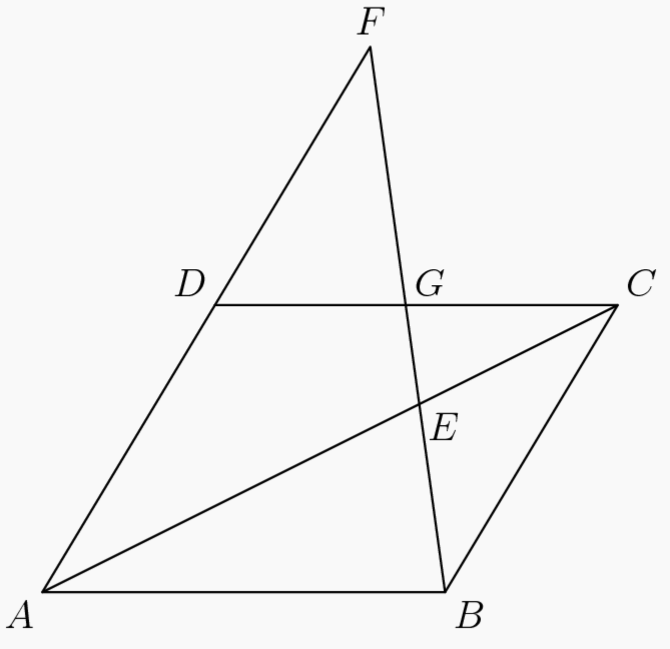
Problem 37

Given points $P_1, P_2, \cdots, P_7$ on a straight line, in the order stated (not necessarily evenly spaced). Let P be an arbitrarily selected point on the line and let s be the sum of the undirected lengths $PP_1, PP_2, \cdots, PP_7$. Then s is smallest if and only if the point P is:

$\textbf{(A)}\text{ midway between }P_1\text{ and }P_7$
$\textbf{(B)}\text{ midway between }P_2\text{ and }P_6$
$\textbf{(C)}\text{ midway between }P_3\text{ and }P_5$
$\textbf{(D)}\text{ at }P_4\qquad\textbf{(E)}\text{ at }P_1$

Problem 38

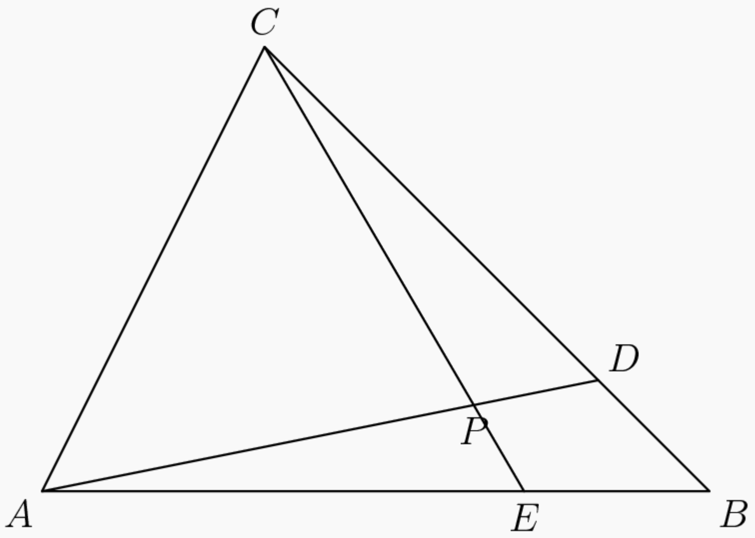
Point F is taken on the extension of side AD of parallelogram $ABCD$. BF intersects diagonal AC at E and side DC at G. If $EF = 32$ and $GF = 24$, then BE equals:



$\textbf{(A) }4\qquad\textbf{(B) }8\qquad\textbf{(C) }10\qquad\textbf{(D) }12\qquad\textbf{(E) }16$

Problem 39

In $\triangle ABC$ lines CE and AD are drawn so that $\dfrac{CD}{DB}=\dfrac{3}{1}$ and $\dfrac{AE}{EB}=\dfrac{3}{2}$. Let $r=\dfrac{CP}{PE}$ where P is the intersection point of CE and AD. Then r equals:



(A) 3 **(B)** $\frac{3}{2}$ **(C)** 4 **(D)** 5 **(E)** $\frac{5}{2}$

Problem 40

If x is a number satisfying the equation $\sqrt[3]{x+9} - \sqrt[3]{x-9} = 3$, then x^2 is between:

(A) 55 and 65 **(B)** 65 and 75 **(C)** 75 and 85 **(D)** 85 and 95 **(E)** 95 and 105