1976 AHSME Problems

Problem 1

If one minus the reciprocal of $(1-x)$ equals the reciprocal of $(1-x)$, then x equals

$\textbf{(A) }-2\qquad \textbf{(B) }-1\qquad \textbf{(C) }1/2\qquad \textbf{(D) }2\qquad \textbf{(E) }3$

Problem 2

For how many real numbers x is $\sqrt{-(x+1)^2}$ a real number?

$\textbf{(A) }\text{none}\qquad \textbf{(B) }\text{one}\qquad \textbf{(C) }\text{two}$
$\textbf{(D) }\text{a finite number greater than two}\qquad \textbf{(E) }\infty$

Problem 3

The sum of the distances from one vertex of a square with sides of length 2 to the midpoints of each of the sides of the square is

$\textbf{(A) }2\sqrt{5}\qquad \textbf{(B) }2+\sqrt{3}\qquad \textbf{(C) }2+2\sqrt{3}\qquad \textbf{(D) }2+\sqrt{5}\qquad \textbf{(E) }2+2\sqrt{5}$

Problem 4

Let a geometric progression with n terms have first term one, common ratio r and sum s, where r and s are not zero. The sum of the geometric progression formed by replacing each term of the original progression by its reciprocal is

$\textbf{(A) }\frac{1}{s}\qquad \textbf{(B) }\frac{1}{r^ns}\qquad \textbf{(C) }\frac{s}{r^{n-1}}\qquad \textbf{(D) }\frac{r^n}{s}\qquad \textbf{(E) }\frac{r^{n-1}}{s}$

Problem 5

How many integers greater than 10 and less than 100, written in base-10 notation, are increased by 9 when their digits are reversed?

$\textbf{(A) }0\qquad \textbf{(B) }1\qquad \textbf{(C) }8\qquad \textbf{(D) }9\qquad \textbf{(E) }10$

Problem 6

If c is a real number and the negative of one of the solutions of $x^2-3x+c=0$ is a solution of $x^2+3x-c=0$, then the solutions of $x^2-3x+c=0$ are

$\textbf{(A) }1,~2\qquad \textbf{(B) }-1,~-2\qquad \textbf{(C) }0,~3\qquad \textbf{(D) }0,~-3\qquad \textbf{(E) }\frac{3}{2},~\frac{3}{2}$

Problem 7

If x is a real number, then the quantity $(1-|x|)(1+x)$ is positive if and only if

$\textbf{(A) }|x|<1\qquad \textbf{(B) }|x|>1\qquad \textbf{(C) }x<-1\text{ or }-1<x<1$
$\textbf{(D) }x<1\qquad \textbf{(E) }x<-1$

Problem 8

A point in the plane, both of whose rectangular coordinates are integers with absolute values less than or equal to four, is chosen at random, with all such points having an equal probability of being chosen. What is the probability that the distance from the point to the origin is at most two units?

$\textbf{(A)}\ \frac{13}{81} \qquad \textbf{(B)}\ \frac{15}{81} \qquad \textbf{(C)}\ \frac{13}{64} \qquad \textbf{(D)}\ \frac{\pi}{16} \qquad \textbf{(E)}\ \text{the square of a rational number}$

Problem 9

In triangle ABC, D is the midpoint of AB; E is the midpoint of DB; and F is the midpoint of BC. If the area of $\triangle ABC$ is 96, then the area of $\triangle AEF$ is

$\textbf{(A)}\ 16 \qquad \textbf{(B)}\ 24 \qquad \textbf{(C)}\ 32 \qquad \textbf{(D)}\ 36 \qquad \textbf{(E)}\ 48$

Problem 10

If $m,~ n,~ p$, and q are real numbers and $f(x)=mx+n$ and $g(x)=px+q$, then the equation $f(g(x))=g(f(x))$ has a solution

$\textbf{(A)}\ \text{for all choices of }m,~ n,~ p, \text{ and } q$
$\textbf{(B)}\ \text{if and only if }m=p\text{ and }n=q$
$\textbf{(C)}\ \text{if and only if }mq-np=0$
$\textbf{(D)}\ \text{if and only if }n(1-p)-q(1-m)=0$
$\textbf{(E)}\ \text{if and only if }(1-n)(1-p)-(1-q)(1-m)=0$

Problem 11

Which of the following statements is (are) equivalent to the statement "If the pink elephant on planet alpha has purple eyes, then the wild pig on planet beta does not have a long nose"?

I. "If the wild pig on planet beta has a long nose, then the pink elephant on planet alpha has purple eyes."
II. "If the pink elephant on planet alpha does not have purple eyes, then the wild pig on planet beta does not have a long nose."
III. "If the wild pig on planet beta has a long nose, then the pink elephant on planet alpha does not have purple eyes."
IV. "The pink elephant on planet alpha does not have purple eyes, or the wild pig on planet beta does not have a long nose."

$\textbf{(A)}\ \textbf{I.} \text{ and } \textbf{II.} \text{ only} \qquad \textbf{(B)}\ \textbf{III.} \text{ and } \textbf{IV.} \text{ only} \qquad \textbf{(C)}\ \textbf{II.} \text{ and } \textbf{IV.} \text{ only} \qquad \textbf{(D)}\ \textbf{II.} \text{ and } \textbf{III.} \text{ only} \qquad \textbf{(E)}\ \textbf{III.} \text{ only}$

Problem 12

A supermarket has 128 crates of apples. Each crate contains at least 120 apples and at most 144 apples. What is the largest integer $n$such that there must be at least n crates containing the same number of apples?

$\textbf{(A)}\ 4 \qquad \textbf{(B)}\ 5 \qquad \textbf{(C)}\ 6 \qquad \textbf{(D)}\ 24 \qquad \textbf{(E)}\ 25$

Problem 13

If x cows give $x+1$ cans of milk in $x+2$ days, how many days will it take $x+3$ cows to give $x+5$ cans of milk?

$\textbf{(A)}\ \frac{x(x+2)(x+5)}{(x+1)(x+3)} \qquad \textbf{(B)}\ \frac{x(x+1)(x+5)}{(x+2)(x+3)}$
$\textbf{(C)}\ \frac{(x+1)(x+3)(x+5)}{x(x+2)} \qquad \textbf{(D)}\ \frac{(x+1)(x+3)}{x(x+2)(x+5)}$
$\textbf{(E)}\ \text{none of these}$

Problem 14

The measures of the interior angles of a convex polygon are in arithmetic progression. If the smallest angle is 100°, and the largest is 140°, then the number of sides the polygon has is

$\textbf{(A)}\ 6 \qquad \textbf{(B)}\ 8 \qquad \textbf{(C)}\ 10 \qquad \textbf{(D)}\ 11 \qquad \textbf{(E)}\ 12$

Problem 15

If r is the remainder when each of the numbers $1059,\ 1417$, and 2312 is divided by d, where d is an integer greater than 1, then $d-r$equals

$\textbf{(A) }1\qquad\textbf{(B) }15\qquad\textbf{(C) }179\qquad\textbf{(D) }d-15\qquad\textbf{(E) }d-1$

Problem 16

In triangles ABC and DEF, lengths $AC,\ BC,\ DF$, and EF are all equal. Length AB is twice the length of the altitude of $\triangle DEF$from F to DE. Which of the following statements is (are) true?

$\textbf{I.}$ $\angle ACB$ and $\angle DFE$ must be complementary.
$\textbf{II.}$ $\angle ACB$ and $\angle DFE$ must be supplementary.
$\textbf{III.}$ The area of $\triangle ABC$ must equal the area of $\triangle DEF$.
$\textbf{IV.}$ The area of $\triangle ABC$ must equal twice the area of $\triangle DEF$.

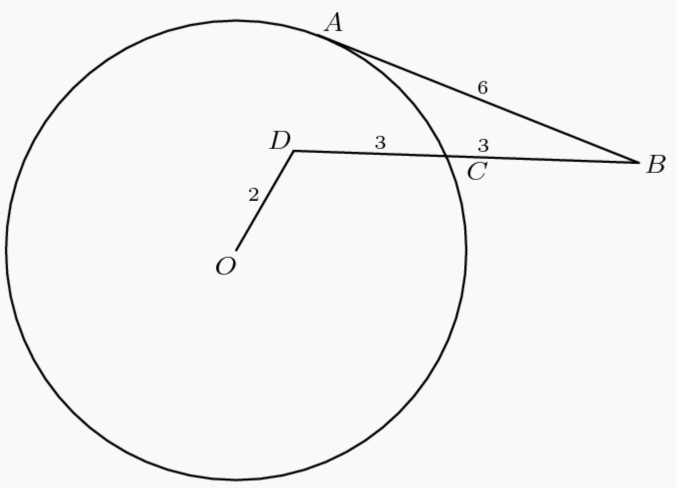
$\textbf{(A) }\textbf{II.}\text{ only}\qquad\textbf{(B) }\textbf{III.}\text{ only}\qquad\textbf{(C) }\textbf{IV.}\text{ only}\qquad\textbf{(D) }\text{I. and }\textbf{III.}\text{ only}\qquad\textbf{(E) }\textbf{II.}\text{ and }\textbf{III.}\text{ only}$

Problem 17

If θ is an acute angle, and $\sin 2\theta=a$, then $\sin\theta+\cos\theta$ equals

$\textbf{(A) }\sqrt{a+1}\qquad\textbf{(B) }(\sqrt{2}-1)a+1\qquad\textbf{(C) }\sqrt{a+1}-\sqrt{a^2-a}$
$\textbf{(D) }\sqrt{a+1}+\sqrt{a^2-a}\qquad\textbf{(E) }\sqrt{a+1}+a^2-a$

Problem 18



In the adjoining figure, AB is tangent at A to the circle with center O; point D is interior to the circle; and DB intersects the circle at C. If $BC=DC=3$, $OD=2$, and $AB=6$, then the radius of the circle is

$\textbf{(A) }3+\sqrt{3}\qquad\textbf{(B) }15/\pi\qquad\textbf{(C) }9/2\qquad\textbf{(D) }2\sqrt{6}\qquad\textbf{(E) }\sqrt{22}$

Problem 19

A polynomial $p(x)$ has remainder three when divided by $x-1$ and remainder five when divided by $x-3$. The remainder when $p(x)$ is divided by $(x-1)(x-3)$ is

$\textbf{(A) }x-2\qquad\textbf{(B) }x+2\qquad\textbf{(C) }2\qquad\textbf{(D) }8\qquad\textbf{(E) }15$

Problem 20

Let $a,\ b$, and x be positive real numbers distinct from one. Then $4(\log_a x)^2+3(\log_b x)^2=8(\log_a x)(\log_b x)$

$\textbf{(A)}$ for all values of $a,\ b,$ and x
$\textbf{(B)}$ if and only if $a=b^2$
$\textbf{(C)}$ if and only if $b=a^2$
$\textbf{(D)}$ if and only if $x=ab$
$\textbf{(E)}$ for none of these

Problem 21

What is the smallest positive odd integer n such that the product $2^{1/7}2^{3/7}\cdots 2^{(2n+1)/7}$ is greater than 1000? (In the product the denominators of the exponents are all sevens, and the numerators are the successive odd integers from 1 to $2n+1$.)

$\textbf{(A) }7\qquad \textbf{(B) }9\qquad \textbf{(C) }11\qquad \textbf{(D) }17\qquad \textbf{(E) }19$

Problem 22

Given an equilateral triangle with side of length s, consider the locus of all points P in the plane of the triangle such that the sum of the squares of the distances from P to the vertices of the triangle is a fixed number a. This locus

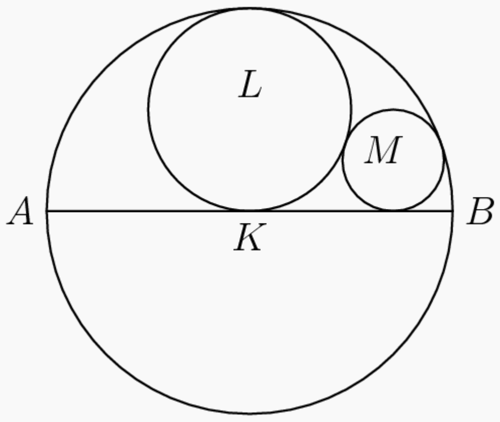
$\textbf{(A)}$ is a circle if $a>s^2$
$\textbf{(B)}$ contains only three points if $a=2s^2$ and is a circle if $a>2s^2$
$\textbf{(C)}$ is a circle with positive radius only if $s^2<a<2s^2$
$\textbf{(D)}$ contains only a finite number of points for any value of a
$\textbf{(E)}$ is none of these

Problem 23

For integers k and n such that $1\le k<n$, let $C_k^n=\frac{n!}{k!(n-k)!}$. Then $\left(\frac{n-2k-1}{k+1}\right)C_k^n$ is an integer

$\textbf{(A)}$ for all k and n
$\textbf{(B)}$ for all even values of k and $n,$ but not for all k and n
$\textbf{(C)}$ for all odd values of k and $n,$ but not for all k and n
$\textbf{(D)}$ if $k=1$ or $n-1,$ but not for all odd values k and n
$\textbf{(E)}$ if n is divisible by $k,$ but not for all even values k and n

Problem 24



In the adjoining figure, circle K has diameter AB; circle L is tangent to circle K and to AB at the center of circle K; and circle Mtangent to circle K, to circle L and AB. The ratio of the area of circle K to the area of circle M is

$\textbf{(A) }12\qquad \textbf{(B) }14\qquad \textbf{(C) }16\qquad \textbf{(D) }18\qquad \textbf{(E) }\text{not an integer}$

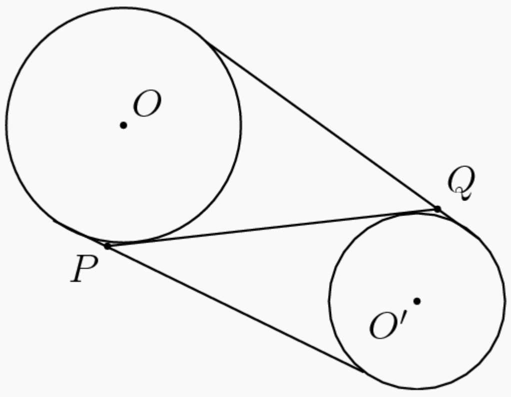
Problem 25

For a sequence $u_1,u_2\dots$, define $\Delta^1(u_n)=u_{n+1}-u_n$ and, for all integer $k>1, \Delta^k(u_n)=\Delta^1(\Delta^{k-1}(u_n))$.

If $u_n=n^3+n$, then $\Delta^k(u_n)=0$ for all n

$\textbf{(A)}\ \text{if}\ k=1$
$\textbf{(B)}\ \text{if}\ k=2,\ \text{but not if}\ k=1$
$\textbf{(C)}\ \text{if}\ k=3,\ \text{but not if}\ k=2$
$\textbf{(D)}\ \text{if}\ k=4,\ \text{but not if}\ k=3$
$\textbf{(E)}\ \text{for no value of}\ k$

Problem 26



In the adjoining figure, every point of circle O' is exterior to circle O. Let P and Q be the points of intersection of an internal common tangent with the two external common tangents. Then the length of PQ is

$\textbf{(A)}$ the average of the lengths of the internal and external common tangents
$\textbf{(B)}$ equal to the length of an external common tangent if and only if circles O and O' have equal radii
$\textbf{(C)}$ always equal to the length of an external common tangent
$\textbf{(D)}$ greater than the length of an external common tangent
$\textbf{(E)}$ the geometric mean of the lengths of the internal and external common tangents

Problem 27

If $N=\frac{\sqrt{\sqrt{5}+2}+\sqrt{\sqrt{5}-2}}{\sqrt{\sqrt{5}+1}}-\sqrt{3-2\sqrt{2}}$, then N equals

$\textbf{(A)}\ 1 \qquad \textbf{(B)}\ 2\sqrt{2}-1 \qquad \textbf{(C)}\ \frac{\sqrt{5}}{2} \qquad \textbf{(D)}\ \sqrt{\frac{5}{2}} \qquad \textbf{(E)}\ \text{none of these}$

Problem 28

Lines $L_1, L_2, \dots, L_{100}$ are distinct. All lines L_{4n}, n a positive integer, are parallel to each other. All lines L_{4n-3}, n a positive integer, pass through a given point A. The maximum number of points of intersection of pairs of lines from the complete set $\{L_1, L_2, \dots, L_{100}\}$ is

$\textbf{(A)}\ 4350 \qquad \textbf{(B)}\ 4351 \qquad \textbf{(C)}\ 4900 \qquad \textbf{(D)}\ 4901 \qquad \textbf{(E)}\ 9851$

Problem 29

Ann and Barbara were comparing their ages and found that Barbara is as old as Ann was when Barbara was as old as Ann had been when Barbara was half as old as Ann is. If the sum of their present ages is 44 years, then Ann's age is

$\textbf{(A)}\ 22 \qquad \textbf{(B)}\ 24 \qquad \textbf{(C)}\ 25 \qquad \textbf{(D)}\ 26 \qquad \textbf{(E)}\ 28$

Problem 30

How many distinct ordered triples (x,y,z) satisfy the equations $x+2y+4z=12xy+4yz+2xz=22xyz=6$

$\textbf{(A)}\ \text{none} \qquad \textbf{(B)}\ 1 \qquad \textbf{(C)}\ 2 \qquad \textbf{(D)}\ 4 \qquad \textbf{(E)}\ 6$