1953 AHSME Problems

Problem 1

A boy buys oranges at 3 for 10 cents. He will sell them at 5 for 20 cents. In order to make a profit of \$ 1.00, he must sell:

$\textbf{(A)}\ 67 \text{ oranges} \qquad \textbf{(B)}\ 150 \text{ oranges} \qquad \textbf{(C)}\ 200\text{ oranges}$
$\textbf{(D)}\ \text{an infinite number of oranges} \qquad \textbf{(E)}\ \text{none of these}$

Problem 2

A refrigerator is offered at sale at \$250.00 less successive discounts of 20% and 15%. The sale price of the refrigerator is:

$\textbf{(A)}\ 35\% \text{ less than } 250.00 \qquad \textbf{(B)}\ 65\% \text{ of } 250.00 \qquad \textbf{(C)}\ 77\% \text{ of } 250.00 \qquad \textbf{(D)}\ 68\% \text{ of } 250.00 \qquad \textbf{(E)}\ \text{none of these}$

Problem 3

The factors of the expression x^2+y^2 are:

$\textbf{(A)}\ (x+y)(x-y) \qquad \textbf{(B)}\ (x+y)^2 \qquad \textbf{(C)}\ (x^{\frac{2}{3}}+y^{\frac{2}{3}})(x^{\frac{4}{3}}+y^{\frac{4}{3}})$
$\textbf{(D)}\ (x+iy)(x-iy) \qquad \textbf{(E)}\ \text{none of these}$

Problem 4

The roots of $x(x^2+8x+16)(4-x)=0$ are:

$\textbf{(A)}\ 0 \qquad \textbf{(B)}\ 0,4 \qquad \textbf{(C)}\ 0,4,-4 \qquad \textbf{(D)}\ 0,4,-4,-4 \qquad \textbf{(E)}\ \text{none of these}$

Problem 5

If $\log_6 x=2.5$, the value of x is:

$\textbf{(A)}\ 90 \qquad \textbf{(B)}\ 36 \qquad \textbf{(C)}\ 36\sqrt{6} \qquad \textbf{(D)}\ 0.5 \qquad \textbf{(E)}\ \text{none of these}$

Problem 6

Charles has $5q + 1$ quarters and Richard has $q + 5$ quarters. The difference in their money in dimes is:

$\textbf{(A)}\ 10(q - 1) \qquad \textbf{(B)}\ \frac {2}{5}(4q - 4) \qquad \textbf{(C)}\ \frac {2}{5}(q - 1)$
$\textbf{(D)}\ \frac {5}{2}(q - 1) \qquad \textbf{(E)}\ \text{none of these}$

Problem 7

The fraction $\frac{\sqrt{a^2+x^2}-\frac{x^2-a^2}{\sqrt{a^2+x^2}}}{a^2+x^2}$ reduces to:

$\textbf{(A)}\ 0 \qquad \textbf{(B)}\ \frac{2a^2}{a^2+x^2} \qquad \textbf{(C)}\ \frac{2x^2}{(a^2+x^2)^{\frac{3}{2}}} \qquad \textbf{(D)}\ \frac{2a^2}{(a^2+x^2)^{\frac{3}{2}}} \qquad \textbf{(E)}\ \frac{2x^2}{a^2+x^2}$

Problem 8

The value of x at the intersection of $y = \frac{8}{x^2+4}$ and $x + y = 2$ is:

$\textbf{(A)}\ -2+\sqrt{5} \qquad \textbf{(B)}\ -2-\sqrt{5} \qquad \textbf{(C)}\ 0 \qquad \textbf{(D)}\ 2 \qquad \textbf{(E)}\ \text{none of these}$

Problem 9

The number of ounces of water needed to reduce 9 ounces of shaving lotion containing 50 % alcohol to a lotion containing 30 % alcohol is:

$\textbf{(A)}\ 3 \qquad \textbf{(B)}\ 4 \qquad \textbf{(C)}\ 5 \qquad \textbf{(D)}\ 6 \qquad \textbf{(E)}\ 7$

Problem 10

The number of revolutions of a wheel, with fixed center and with an outside diameter of 6 feet, required to cause a point on the rim to go one mile is:

$\textbf{(A)}\ 880 \qquad \textbf{(B)}\ \frac{440}{\pi} \qquad \textbf{(C)}\ \frac{880}{\pi} \qquad \textbf{(D)}\ 440\pi \qquad \textbf{(E)}\ \text{none of these}$

Problem 11

A running track is the ring formed by two concentric circles. It is 10 feet wide. The circumference of the two circles differ by about:

$\textbf{(A)}\ 10\text{ feet} \qquad \textbf{(B)}\ 30\text{ feet} \qquad \textbf{(C)}\ 60\text{ feet} \qquad \textbf{(D)}\ 100\text{ feet}$
$\textbf{(E)}\ \text{none of these}$

Problem 12

The diameters of two circles are 8 inches and 12 inches respectively. The ratio of the area of the smaller to the area of the larger circle is:

$\textbf{(A)}\ \frac{2}{3} \qquad \textbf{(B)}\ \frac{4}{9} \qquad \textbf{(C)}\ \frac{9}{4} \qquad \textbf{(D)}\ \frac{1}{2} \qquad \textbf{(E)}\ \text{none of these}$

Problem 13

A triangle and a trapezoid are equal in area. They also have the same altitude. If the base of the triangle is 18 inches, the median of the trapezoid is:

$\textbf{(A)}\ 36\text{ inches} \qquad \textbf{(B)}\ 9\text{ inches} \qquad \textbf{(C)}\ 18\text{ inches}$
$\textbf{(D)}\ \text{not obtainable from these data} \qquad \textbf{(E)}\ \text{none of these}$

Problem 14

Given the larger of two circles with center P and radius p and the smaller with center Q and radius q. Draw PQ. Which of the following statements is false?

$\textbf{(A)}\ p-q\text{ can be equal to }\overline{PQ}$
$\textbf{(B)}\ p+q\text{ can be equal to }\overline{PQ}$
$\textbf{(C)}\ p+q\text{ can be less than }\overline{PQ}$
$\textbf{(D)}\ p-q\text{ can be less than }\overline{PQ}$
$\textbf{(E)}\ \text{none of these}$

Problem 15

A circular piece of metal of maximum size is cut out of a square piece and then a square piece of maximum size is cut out of the circular piece. The total amount of metal wasted is:

(A) $\frac{1}{4}$ the area of the original square
(B) $\frac{1}{2}$ the area of the original square
(C) $\frac{1}{2}$ the area of the circular piece
(D) $\frac{1}{4}$ the area of the circular piece
(E) none of these

Problem 16

Adams plans a profit of 10 % on the selling price of an article and his expenses are 15 % of sales. The rate of markup on an article that sells for \$ 5.00 is:

(A) 20% **(B)** 25% **(C)** 30% **(D)** $33\frac{1}{3}$% **(E)** 35%

Problem 17

A man has part of \$ 4500 invested at 4 % and the rest at 6 %. If his annual return on each investment is the same, the average rate of interest which he realizes of the \$4500 is:

(A) 5% **(B)** 4.8% **(C)** 5.2% **(D)** 4.6% **(E)** none of these

Problem 18

One of the factors of x^4+4 is:

(A) x^2+2 **(B)** $x+1$ **(C)** x^2-2x+2 **(D)** x^2-4
(E) none of these

Problem 19

In the expression xy^2, the values of x and y are each decreased 25 %; the value of the expression is:

(A) decreased 50% **(B)** decreased 75%
(C) decreased $\frac{37}{64}$ of its value **(D)** decreased $\frac{27}{64}$ of its value
(E) none of these

Problem 20

If $y=x+\frac{1}{x}$, then $x^4+x^3-4x^2+x+1=0$ becomes:

(A) $x^2(y^2+y-2)=0$ **(B)** $x^2(y^2+y-3)=0$
(C) $x^2(y^2+y-4)=0$ **(D)** $x^2(y^2+y-6)=0$
(E) none of these

Problem 21

If $\log_{10}(x^2-3x+6)=1$, the value of x is:

(A) 10 or 2 **(B)** 4 or -2 **(C)** 3 or -1 **(D)** 4 or -1
(E) none of these

Problem 22

The logarithm of $27\sqrt[4]{9}\sqrt[3]{9}$ to the base 3 is:

$\textbf{(A)}\ 8\frac{1}{2} \qquad \textbf{(B)}\ 4\frac{1}{6} \qquad \textbf{(C)}\ 5 \qquad \textbf{(D)}\ 3 \qquad \textbf{(E)}\ \text{none of these}$

Problem 23

The equation $\sqrt {x + 10} - \frac {6}{\sqrt {x + 10}} = 5$ has:

$\textbf{(A)}\ \text{an extraneous root between } - 5\text{ and } - 1$
$\textbf{(B)}\ \text{an extraneous root between } - 10\text{ and } - 6$
$\textbf{(C)}\ \text{a true root between }20\text{ and }25 \qquad \textbf{(D)}\ \text{two true roots}$
$\textbf{(E)}\ \text{two extraneous roots}$

Problem 24

If a,b,c are positive integers less than 10, then $(10a + b)(10a + c) = 100a(a + 1) + bc$ if:

$\textbf{(A)}\ b + c = 10 \qquad \textbf{(B)}\ b = c \qquad \textbf{(C)}\ a + b = 10 \qquad \textbf{(D)}\ a = b$
$\textbf{(E)}\ a + b + c = 10$

Problem 25

In a geometric progression whose terms are positive, any term is equal to the sum of the next two following terms. then the common ratio is:

$\textbf{(A)}\ 1 \qquad \textbf{(B)}\ \text{about }\frac {\sqrt {5}}{2} \qquad \textbf{(C)}\ \frac {\sqrt {5} - 1}{2} \qquad \textbf{(D)}\ \frac {1 - \sqrt {5}}{2} \qquad \textbf{(E)}\ \frac {2}{\sqrt {5}}$

Problem 26

The base of a triangle is 15 inches. Two lines are drawn parallel to the base, terminating in the other two sides, and dividing the triangle into three equal areas. The length of the parallel closer to the base is:

$\textbf{(A)}\ 5\sqrt{6}\text{ inches} \qquad \textbf{(B)}\ 10\text{ inches} \qquad \textbf{(C)}\ 4\sqrt{3}\text{ inches}\qquad \textbf{(D)}\ 7.5\text{ inches}$
$\textbf{(E)}\ \text{none of these}$

Problem 27

The radius of the first circle is 1 inch, that of the second $\frac{1}{2}$ inch, that of the third $\frac{1}{4}$ inch and so on indefinitely. The sum of the areas of the circles is:

$\textbf{(A)}\ \frac{3\pi}{4} \qquad \textbf{(B)}\ 1.3\pi \qquad \textbf{(C)}\ 2\pi \qquad \textbf{(D)}\ \frac{4\pi}{3}\qquad \textbf{(E)}\ \text{none of these}$

Problem 28

In $\triangle ABC$, sides a,b and c are opposite $\angle{A},\angle{B}$ and $\angle{C}$ respectively. AD bisects $\angle{A}$ and meets BC at D. Then if $x = \overline{CD}$ and $y = \overline{BD}$ the correct proportion is:

$\textbf{(A)}\ \frac {x}{a} = \frac {a}{b + c} \qquad \textbf{(B)}\ \frac {x}{b} = \frac {a}{a + c} \qquad \textbf{(C)}\ \frac{y}{c}=\frac{c}{b+c}$
$\textbf{(D)}\ \frac{y}{c}=\frac{a}{b+c}\qquad \textbf{(E)}\ \frac{x}{y}=\frac{c}{b}$

Problem 29

The number of significant digits in the measurement of the side of a square whose computed area is 1.1025 square inches to the nearest ten-thousandth of a square inch is:

$\textbf{(A) }2\qquad\textbf{(B) }3\qquad\textbf{(C) }4\qquad\textbf{(D) }5\qquad\textbf{(E) }1$

Problem 30

A house worth $ 9000 is sold by Mr. A to Mr. B at a 10 % loss. Mr. B sells the house back to Mr. A at a 10 % gain. The result of the two transactions is:

$\textbf{(A)}\ \text{Mr. A breaks even}\qquad\textbf{(B)}\ \text{Mr. B gains }\$900\qquad\textbf{(C)}\ \text{Mr. A loses }\$900$
$\textbf{(D)}\ \text{Mr. A loses }\$810\qquad\textbf{(E)}\ \text{Mr. B gains }\$1710$

Problem 31

The rails on a railroad are 30 feet long. As the train passes over the point where the rails are joined, there is an audible click. The speed of the train in miles per hour is approximately the number of clicks heard in:

$\textbf{(A)}\ 20\text{ seconds}\qquad\textbf{(B)}\ 2\text{ minutes}\qquad\textbf{(C)}\ 1\frac{1}{2}\text{ minutes}\qquad\textbf{(D)}\ 5\text{ minutes}$
$\textbf{(E)}\ \text{none of these}$

Problem 32

Each angle of a rectangle is trisected. The intersections of the pairs of trisectors adjacent to the same side always form:

$\textbf{(A)}\ \text{a square}\qquad\textbf{(B)}\ \text{a rectangle}\qquad\textbf{(C)}\ \text{a parallelogram with unequal sides}$
$\textbf{(D)}\ \text{a rhombus}\qquad\textbf{(E)}\ \text{a quadrilateral with no special properties}$

Problem 33

The perimeter of an isosceles right triangle is $2p$. Its area is:

$\textbf{(A)}\ (2+\sqrt{2})p\qquad\textbf{(B)}\ (2-\sqrt{2})p\qquad\textbf{(C)}\ (3-2\sqrt{2})p^2$
$\textbf{(D)}\ (1-2\sqrt{2})p^2\qquad\textbf{(E)}\ (3+2\sqrt{2})p^2$

Problem 34

If one side of a triangle is 12 inches and the opposite angle is 30°, then the diameter of the circumscribed circle is:

$\textbf{(A)}\ 18\text{ inches}\qquad\textbf{(B)}\ 30\text{ inches}\qquad\textbf{(C)}\ 24\text{ inches}\qquad\textbf{(D)}\ 20\text{ inches}$
$\textbf{(E)}\ \text{none of these}$

Problem 35

If $f(x)=\frac{x(x-1)}{2}$, then $f(x+2)$ equals:

$\textbf{(A)}\ f(x)+f(2)\qquad\textbf{(B)}\ (x+2)f(x)\qquad\textbf{(C)}\ x(x+2)f(x)\qquad\textbf{(D)}\ \frac{xf(x)}{x+2}$
$\textbf{(E)}\ \frac{(x+2)f(x+1)}{x}$

Problem 36

Determine m so that $4x^2-6x+m$ is divisible by $x-3$. The obtained value, m, is an exact divisor of:

$\textbf{(A)}\ 12\qquad\textbf{(B)}\ 20\qquad\textbf{(C)}\ 36\qquad\textbf{(D)}\ 48\qquad\textbf{(E)}\ 64$

Problem 37

The base of an isosceles triangle is 6 inches and one of the equal sides is 12 inches. The radius of the circle through the vertices of the triangle is:

$\textbf{(A)}\ \frac{7\sqrt{15}}{5} \qquad \textbf{(B)}\ 4\sqrt{3} \qquad \textbf{(C)}\ 3\sqrt{5} \qquad \textbf{(D)}\ 6\sqrt{3} \qquad \textbf{(E)}\ \text{none of these}$

Problem 38

If $f(a)=a-2$ and $F(a,b)=b^2+a$, then $F(3,f(4))$ is:

$\textbf{(A)}\ a^2-4a+7 \qquad \textbf{(B)}\ 28 \qquad \textbf{(C)}\ 7 \qquad \textbf{(D)}\ 8 \qquad \textbf{(E)}\ 11$

Problem 39

The product, $\log_a b \cdot \log_b a$ is equal to:

$\textbf{(A)}\ 1 \qquad \textbf{(B)}\ a \qquad \textbf{(C)}\ b \qquad \textbf{(D)}\ ab \qquad \textbf{(E)}\ \text{none of these}$

Problem 40

The negation of the statement "all men are honest," is:

$\textbf{(A)}\ \text{no men are honest} \qquad \textbf{(B)}\ \text{all men are dishonest}$
$\textbf{(C)}\ \text{some men are dishonest} \qquad \textbf{(D)}\ \text{no men are dishonest}$
$\textbf{(E)}\ \text{some men are honest}$

Problem 41

A girls' camp is located 300 rods from a straight road. On this road, a boys' camp is located 500 rods from the girls' camp. It is desired to build a canteen on the road which shall be exactly the same distance from each camp. The distance of the canteen from each of the camps is:

$\textbf{(A)}\ 400\text{ rods} \qquad \textbf{(B)}\ 250\text{ rods} \qquad \textbf{(C)}\ 87.5\text{ rods} \qquad \textbf{(D)}\ 200\text{ rods}$
$\textbf{(E)}\ \text{none of these}$

Problem 42

The centers of two circles are 41 inches apart. The smaller circle has a radius of 4 inches and the larger one has a radius of 5 inches. The length of the common internal tangent is:

$\textbf{(A)}\ 41\text{ inches} \qquad \textbf{(B)}\ 39\text{ inches} \qquad \textbf{(C)}\ 39.8\text{ inches} \qquad \textbf{(D)}\ 40.1\text{ inches}$
$\textbf{(E)}\ 40\text{ inches}$

Problem 43

If the price of an article is increased by percent p, then the decrease in percent of sales must not exceed d in order to yield the same income. The value of d is:

$\textbf{(A)}\ \frac{1}{1+p} \qquad \textbf{(B)}\ \frac{1}{1-p} \qquad \textbf{(C)}\ \frac{p}{1+p} \qquad \textbf{(D)}\ \frac{p}{p-1} \qquad \textbf{(E)}\ \frac{1-p}{1+p}$

Problem 44

In solving a problem that reduces to a quadratic equation one student makes a mistake only in the constant term of the equation and obtains 8 and 2 for the roots. Another student makes a mistake only in the coefficient of the first degree term and find -9 and -1 for the roots. The correct equation was:

$\textbf{(A)}\ x^2-10x+9=0 \qquad \textbf{(B)}\ x^2+10x+9=0 \qquad \textbf{(C)}\ x^2-10x+16=0$
$\textbf{(D)}\ x^2-8x-9=0 \qquad \textbf{(E)}\ \text{none of these}$

Problem 45

The lengths of two line segments are a units and b units respectively. Then the correct relation between them is:

$\textbf{(A)}\ \frac{a+b}{2} > \sqrt{ab} \qquad \textbf{(B)}\ \frac{a+b}{2} < \sqrt{ab} \qquad \textbf{(C)}\ \frac{a+b}{2} = \sqrt{ab}$
$\textbf{(D)}\ \frac{a+b}{2} \leq \sqrt{ab} \qquad \textbf{(E)}\ \frac{a+b}{2} \geq \sqrt{ab}$

Problem 46

Instead of walking along two adjacent sides of a rectangular field, a boy took a shortcut along the diagonal of the field and saved a distance equal to $\frac{1}{2}$ the longer side. The ratio of the shorter side of the rectangle to the longer side was:

$\textbf{(A)}\ \frac{1}{2} \qquad \textbf{(B)}\ \frac{2}{3} \qquad \textbf{(C)}\ \frac{1}{4} \qquad \textbf{(D)}\ \frac{3}{4} \qquad \textbf{(E)}\ \frac{2}{5}$

Problem 47

If $x > 0$, then the correct relationship is:

$\textbf{(A)}\ \log(1+x) = \frac{x}{1+x} \qquad \textbf{(B)}\ \log(1+x) < \frac{x}{1+x}$
$\textbf{(C)}\ \log(1+x) > x \qquad \textbf{(D)}\ \log(1+x) < x \qquad \textbf{(E)}\ \text{none of these}$

Problem 48

If the larger base of an isosceles trapezoid equals a diagonal and the smaller base equals the altitude, then the ratio of the smaller base to the larger base is:

$\textbf{(A)}\ \frac{1}{2} \qquad \textbf{(B)}\ \frac{2}{3} \qquad \textbf{(C)}\ \frac{3}{4} \qquad \textbf{(D)}\ \frac{3}{5} \qquad \textbf{(E)}\ \frac{2}{5}$

Problem 49

The coordinates of A, B and C are $(5,5), (2,1)$ and $(0,k)$ respectively. The value of k that makes $\overline{AC}+\overline{BC}$ as small as possible is:

$\textbf{(A)}\ 3 \qquad \textbf{(B)}\ 4\frac{1}{2} \qquad \textbf{(C)}\ 3\frac{6}{7} \qquad \textbf{(D)}\ 4\frac{5}{6} \qquad \textbf{(E)}\ 2\frac{1}{7}$

Problem 50

One of the sides of a triangle is divided into segments of 6 and 8 units by the point of tangency of the inscribed circle. If the radius of the circle is 4, then the length of the shortest side of the triangle is:

$\textbf{(A)}\ 12\text{ units} \qquad \textbf{(B)}\ 13\text{ units} \qquad \textbf{(C)}\ 14\text{ units} \qquad \textbf{(D)}\ 15\text{ units} \qquad \textbf{(E)}\ 16\text{ units}$