1991 AHSME Problems

Problem 1

If for any three distinct numbers a, b, and c we define $f(a,b,c)=\frac{c+a}{c-b}$, then $f(1,-2,-3)$ is

$\textbf{(A) } -2 \qquad \textbf{(B) } -\frac{2}{5} \qquad \textbf{(C) } -\frac{1}{4} \qquad \textbf{(D) } \frac{2}{5} \qquad \textbf{(E) } 2$

Problem 2

$|3-\pi|=$

$\textbf{(A) } \frac{1}{7} \qquad \textbf{(B) } 0.14 \qquad \textbf{(C) } 3-\pi \qquad \textbf{(D) } 3+\pi \qquad \textbf{(E) } \pi-3$

Problem 3

$(4^{-1}-3^{-1})^{-1}=$

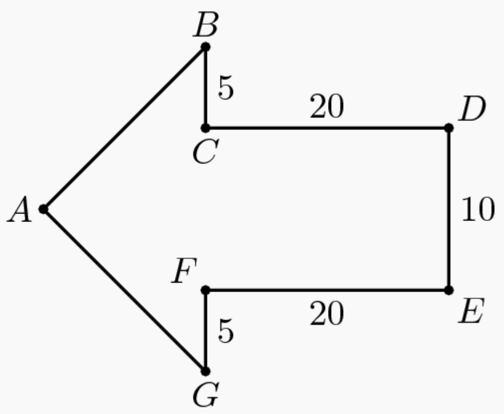
(A) -12 (B) -1 (C) $\frac{1}{12}$ (D) 1 (E) 12

Problem 4

Which of the following triangles cannot exist?

(A) An acute isosceles triangle (B) An isosceles right triangle (C) An obtuse right triangle (D) A scalene right triangle (E) A scalene obtuse tr iangle

Problem 5



In the arrow-shaped polygon [see figure], the angles at vertices A, C, D, E and F are right angles, $BC=FG=5, CD=FE=20, DE=10$, and $AB=AG$. The area of the polygon is closest to $\text{(A) } 288 \quad \text{(B) } 291 \quad \text{(C) } 294 \quad \text{(D) } 297 \quad \text{(E) } 300$

Problem 6

If $x\ge 0$, then $\sqrt{x\sqrt{x\sqrt{x}}}=$

(A) $x\sqrt{x}$ (B) $x\sqrt[4]{x}$ (C) $\sqrt[8]{x}$ (D) $\sqrt[8]{x^3}$ (E) $\sqrt[8]{x^7}$

Problem 7

If $x = \frac{a}{b}$, $a \neq b$ and $b \neq 0$, then $\frac{a+b}{a-b} =$

(A) $\frac{x}{x+1}$ (B) $\frac{x+1}{x-1}$ (C) 1 (D) $x - \frac{1}{x}$ (E) $x + \frac{1}{x}$

Problem 8

Liquid X does not mix with water. Unless obstructed, it spreads out on the surface of water to form a circular film 0.1cm thick. A rectangular box measuring 6cm by 3cm by 12cm is filled with liquid X. Its contents are poured onto a large body of water. What will be the radius, in centimeters, of the resulting circular film?

(A) $\frac{\sqrt{216}}{\pi}$ (B) $\sqrt{\frac{216}{\pi}}$ (C) $\sqrt{\frac{2160}{\pi}}$ (D) $\frac{216}{\pi}$ (E) $\frac{2160}{\pi}$

Problem 9

From time $t = 0$ to time $t = 1$ a population increased by $i\%$, and from time $t = 1$ to time $t = 2$ the population increased by $j\%$. Therefore, from time $t = 0$ to time $t = 2$ the population increased by

$\text{(A) (i+j)\%} \quad \text{(B) } ij\% \quad \text{(C) } (i+ij)\% \quad \text{(D) } \left(i+j+\frac{ij}{100}\right)\% \quad \text{(E) } \left(i+j+\frac{i+j}{100}\right)\%$

Problem 10

Point P is 9 units from the center of a circle of radius 15. How many different chords of the circle contain P and have integer lengths?

(A) 11 (B) 12 (C) 13 (D) 14 (E) 29

Problem 11

Jack and Jill run 10 km. They start at the same point, run 5 km up a hill, and reurn to the starting point by the same route. Jack has a 10 minute head start and runs at the rate of 15 km/hr uphill and 20 km/hr downhill. Jill runs 16 km/hr uphill and 22 km/hr downhill. How far from the top of the hill are they when they pass each other going in opposite directions (in km)?

$\text{(A) } \frac{5}{4} \quad \text{(B) } \frac{35}{27} \quad \text{(C) } \frac{27}{20} \quad \text{(D) } \frac{7}{3} \quad \text{(E) } \frac{28}{49}$

Problem 12

The measures (in degrees) of the interior angles of a convex hexagon form an arithmetic sequence of integers. Let m be the measure of the largest interior angle of the hexagon. The largest possible value of m, in degrees, is

(A) 165 (B) 167 (C) 170 (D) 175 (E) 179

Problem 13

Horses X, Y and Z are entered in a three-horse race in which ties are not possible. The odds against X winning are $3:1$ and the odds against Y winning are $2:3$, what are the odds against Z winning? (By "odds against H winning are $p:q$" we mean the probability of H winning the race is $\frac{q}{p+q}$.)

$\text{(A) } 3:20 \quad \text{(B) } 5:6 \quad \text{(C) } 8:5 \quad \text{(D) } 17:3 \quad \text{(E) } 20:3$

Problem 14

If x is the cube of a positive integer and d is the number of positive integers that are divisors of x, then d could be

(A) 200 (B) 201 (C) 202 (D) 203 (E) 204

Problem 15

A circular table has 60 chairs around it. There are N people seated at this table in such a way that the next person seated must sit next to someone. What is the smallest possible value for N?

$\text{(A) } 15\quad \text{(B) } 20\quad \text{(C) } 30\quad \text{(D) } 40\quad \text{(E) } 58$

Problem 16

One hundred students at Century High School participated in the AHSME last year, and their mean score was 100. The number of non-seniors taking the AHSME was 50% more than the number of seniors, and the mean score of the seniors was 50% higher than that of the non-seniors. What was the mean score of the seniors?

(A) 100 (B) 112.5 (C) 120 (D) 125 (E) 150

Problem 17

A positive integer N is a *palindrome* if the integer obtained by reversing the sequence of digits of N is equal to N. The year 1991 is the only year in the current century with the following 2 properties:

(a) It is a palindrome (b) It factors as a product of a 2-digit prime palindrome and a 3-digit prime palindrome.

How many years in the millenium between 1000 and 2000 have properties (a) and (b)?

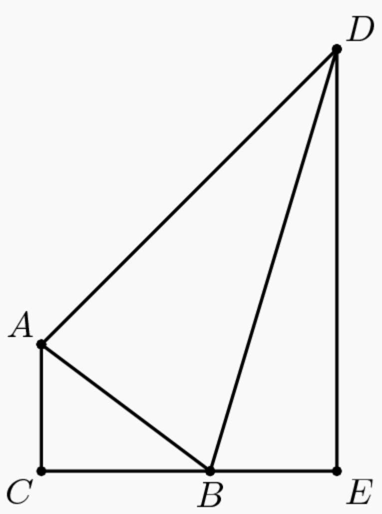
$\text{(A) } 1\quad \text{(B) } 2\quad \text{(C) } 3\quad \text{(D) } 4\quad \text{(E) } 5$

Problem 18

If S is the set of points z in the complex plane such that $(3+4i)z$ is a real number, then S is a

(A) right triangle (B) circle (C) hyperbola (D) line (E) parabola

Problem 19



Triangle ABC has a right angle at $C, AC=3$ and $BC=4$. Triangle ABD has a right angle at A and $AD=12$. Points C and D are on opposite sides of $\overline{AB}$. The line through D parallel to $\overline{AC}$ meets $\overline{CB}$ extended at E. If $\frac{DE}{DB}=\frac{m}{n}$, where m and n are relatively prime positive integers, then $m+n=$

$\text{(A) } 25\quad \text{(B) } 128\quad \text{(C) } 153\quad \text{(D) } 243\quad \text{(E) } 256$

Problem 20

The sum of all real x such that $(2^x-4)^3+(4^x-2)^3=(4^x+2^x-6)^3$ is

(A) 3/2 (B) 2 (C) 5/2 (D) 3 (E) 7/2

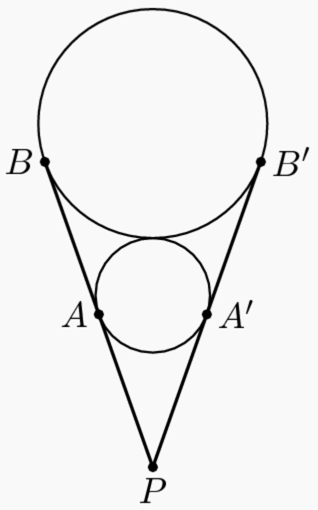
Problem 21

For all real numbers x except $x=0$ and $x=1$ the function $f(x)$ is defined by $f(x/(1-x))=1/x$. Suppose $0\leq t\leq \pi/2$.

What is the value of $f(\sec^2 t)$?

$\text{(A)}\ \sin^2\theta \quad \text{(B)}\ \cos^2\theta \quad \text{(C)}\ \tan^2\theta \quad \text{(D)}\ \cot^2\theta \quad \text{(E)}\ \csc^2\theta$

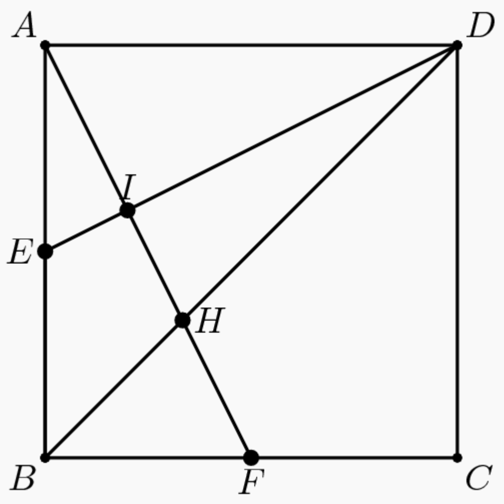
Problem 22



Two circles are externally tangent. Lines $\overline{PAB}$ and $\overline{PA'B'}$ are common tangents with A and A' on the smaller circle B and B' on the larger circle. If $PA=AB=4$, then the area of the smaller circle is

$\text{(A)}\ 1.44\pi \quad \text{(B)}\ 2\pi \quad \text{(C)}\ 2.56\pi \quad \text{(D)}\ \sqrt{8}\pi \quad \text{(E)}\ 4\pi$

Problem 23



If $ABCD$ is a $2X2$ square, E is the midpoint of $\overline{AB}$,F is the midpoint of $\overline{BC}$,$\overline{AF}$ and $\overline{DE}$ intersect at I, and $\overline{BD}$ and $\overline{AF}$ intersect at H, then the area of quadrilateral $BEIH$ is

$\text{(A)}\ \frac{1}{3} \quad \text{(B)}\ \frac{2}{5} \quad \text{(C)}\ \frac{7}{15} \quad \text{(D)}\ \frac{8}{15} \quad \text{(E)}\ \frac{3}{5}$

Problem 24

The graph, G of $y = \log_{10} x$ is rotated 90° counter-clockwise about the origin to obtain a new graph G'. Which of the following is an equation for G'?

(A) $y = \log_{10}\left(\frac{x+90}{9}\right)$ (B) $y = \log_x 10$ (C) $y = \frac{1}{x+1}$ (D) $y = 10^{-x}$ (E) $y = 10^x$

Problem 25

If $T_n = 1 + 2 + 3 + \cdots + n$ and $P_n = \frac{T_2}{T_2 - 1} \cdot \frac{T_3}{T_3 - 1} \cdot \frac{T_4}{T_4 - 1} \cdot \cdots \cdot \frac{T_n}{T_n - 1}$ for $n = 2, 3, 4, \cdots$, then P_{1991} is closest to which of the following numbers?

$\text{(A) } 2.0 \quad \text{(B) } 2.3 \quad \text{(C) } 2.6 \quad \text{(D) } 2.9 \quad \text{(E) } 3.2$

Problem 26

An n-digit positive integer is cute if its n digits are an arrangement of the set $\{1, 2, ..., n\}$ and its first k digits form an integer that is divisible by k , for $k = 1, 2, ..., n$. For example, 321 is a cute 3-digit integer because 1 divides 3, 2 divides 32, and 3 divides 321. Howmany cute 6-digit integers are there?

$\text{(A) } 0 \quad \text{(B) } 1 \quad \text{(C) } 2 \quad \text{(D) } 3 \quad \text{(E) } 4$

Problem 27

If $x + \sqrt{x^2 - 1} + \frac{1}{x - \sqrt{x^2 - 1}} = 20$ then $x^2 + \sqrt{x^4 - 1} + \frac{1}{x^2 + \sqrt{x^4 - 1}} =$

(A) 5.05 (B) 20 (C) 51.005 (D) 61.25 (E) 400

Problem 28

Initially an urn contains 100 white and 100 black marbles. Repeatedly 3 marbles are removed (at random) from the urn and replaced with some marbles from a pile outside the urn as follows: 3 blacks are replaced with 1 black, or 2 blacks and 1 white are replaced with a white and a black, or 1 black and 2 whites are replaced with 2 whites, or 3 whites are replaced with a black and a white. Which of the following could be the contents of the urn after repeated applications of this procedure?

(A) 2 black (B) 2 white (C) 1 black (D) 1 black and 1 white (E) 1 white

Problem 29

Equilateral triangle ABC has P on AB and Q on AC. The triangle is folded along PQ so that vertex A now rests at A' on side BC. If $BA' = 1$ and $A'C = 2$ then the length of the crease PQ is

(A) $\frac{8}{5}$ (B) $\frac{7}{20}\sqrt{21}$ (C) $\frac{1+\sqrt{5}}{2}$ (D) $\frac{13}{8}$ (E) $\sqrt{3}$

Problem 30

For any set S, let $|S|$ denote the number of elements in S, and let $n(S)$ be the number of subsets of S, including the empty set and the set S itself. If A, B, and C are sets for which $n(A) + n(B) + n(C) = n(A \cup B \cup C)$ and $|A| = |B| = 100$, then what is the minimum possible value of $|A \cap B \cap C|$?

(A) 96 (B) 97 (C) 98 (D) 99 (E) 100