

2002 AMC 10B Problems

Problem 1

The ratio $\frac{2^{2001} \cdot 3^{2003}}{6^{2002}}$ is:

- (A) $\frac{1}{6}$ (B) $\frac{1}{3}$ (C) $\frac{1}{2}$ (D) $\frac{2}{3}$ (E) $\frac{3}{2}$

Problem 2

For the nonzero numbers a , b , and c , define $(a, b, c) = \frac{abc}{a + b + c}$. Find $(2, 4, 6)$.

- (A) 1 (B) 2 (C) 4 (D) 6 (E) 24

Problem 3

The arithmetic mean of the nine numbers in the set $\{9, 99, 999, 9999, \dots, 999999999\}$ is a 9-digit number M , all of whose digits are distinct. The number M does not contain the digit

- (A) 0 (B) 2 (C) 4 (D) 6 (E) 8

Problem 4

What is the value of

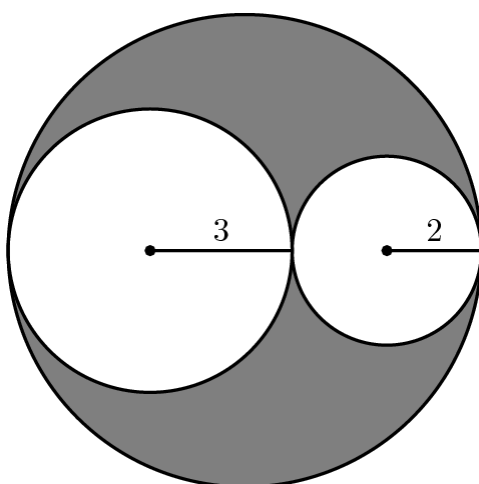
$$(3x - 2)(4x + 1) - (3x - 2)4x + 1$$

when $x = 4$?

- (A) 0 (B) 1 (C) 10 (D) 11 (E) 12

Problem 5

Circles of radius 2 and 3 are externally tangent and are circumscribed by a third circle, as shown in the figure. Find the area of the shaded region.



- (A) 3π (B) 4π (C) 6π (D) 9π (E) 12π

Problem 6

For how many positive integers n is $n^2 - 3n + 2$ a prime number?

- (A) none (B) one (C) two (D) more than two, but finitely many (E) infinitely many

Problem 7

Let n be a positive integer such that $\frac{1}{2} + \frac{1}{3} + \frac{1}{7} + \frac{1}{n}$ is an integer. Which of the following statements is **not** true?

- (A) 2 divides n (B) 3 divides n (C) 6 divides n (D) 7 divides n (E) $n > 84$

Problem 8

Suppose July of year N has five Mondays. Which of the following must occur five times in the August of year N ? (Note: Both months have 31 days.)

- (A) Monday (B) Tuesday (C) Wednesday (D) Thursday (E) Friday

Problem 9

Using the letters A, M, O, S , and U , we can form five-letter "words". If these "words" are arranged in alphabetical order, then the "word" $USAMO$ occupies position

- (A) 112 (B) 113 (C) 114 (D) 115 (E) 116

Problem 10

Suppose that a and b are nonzero real numbers, and that the equation $x^2 + ax + b = 0$ has solutions a and b . Then the pair (a, b) is

- (A) $(-2, 1)$ (B) $(-1, 2)$ (C) $(1, -2)$ (D) $(2, -1)$ (E) $(4, 4)$

Problem 11

The product of three consecutive positive integers is 8 times their sum. What is the sum of the squares?

- (A) 50 (B) 77 (C) 110 (D) 149 (E) 194

Problem 12

For which of the following values of k does the equation $\frac{x-1}{x-2} = \frac{x-k}{x-6}$ have no solution for x ?

- (A) 1 (B) 2 (C) 3 (D) 4 (E) 5

Problem 13

Find the value(s) of x such that $8xy - 12y + 2x - 3 = 0$ is true for all values of y .

- (A) $\frac{2}{3}$ (B) $\frac{3}{2}$ or $-\frac{1}{4}$ (C) $-\frac{2}{3}$ or $-\frac{1}{4}$ (D) $\frac{3}{2}$ (E) $-\frac{3}{2}$ or $-\frac{1}{4}$

Problem 14

The number $25^{64} \cdot 64^{25}$ is the square of a positive integer N . In decimal representation, the sum of the digits of N is

- (A) 7 (B) 14 (C) 21 (D) 28 (E) 35

Problem 15

The positive integers A , B , $A - B$, and $A + B$ are all prime numbers. The sum of these four primes is

- (A) even (B) divisible by 3 (C) divisible by 5 (D) divisible by 7 (E) prime

Problem 16

For how many integers n is $\frac{n}{20 - n}$ the square of an integer?

- (A) 1 (B) 2 (C) 3 (D) 4 (E) 10

Problem 17

A regular octagon $ABCDEFGH$ has sides of length two. Find the area of $\triangle ADG$.

- (A) $4 + 2\sqrt{2}$ (B) $6 + \sqrt{2}$ (C) $4 + 3\sqrt{2}$ (D) $3 + 4\sqrt{2}$ (E) $8 + \sqrt{2}$

Problem 18

Four distinct circles are drawn in a plane. What is the maximum number of points where at least two of the circles intersect?

- (A) 8 (B) 9 (C) 10 (D) 12 (E) 16

Problem 19

Suppose that $\{a_n\}$ is an arithmetic sequence with $a_1 + a_2 + \cdots + a_{100} = 100$ and $a_{101} + a_{102} + \cdots + a_{200} = 200$. What is the value of $a_2 - a_1$?

- (A) 0.0001 (B) 0.001 (C) 0.01 (D) 0.1 (E) 1

Problem 20

Let a , b , and c be real numbers such that $a - 7b + 8c = 4$ and $8a + 4b - c = 7$. Then $a^2 - b^2 + c^2$ is

- (A) 0 (B) 1 (C) 4 (D) 7 (E) 8

Problem 21

Andy's lawn has twice as much area as Beth's lawn and three times as much as Carlos' lawn. Carlos' lawn mower cuts half as fast as Beth's mower and one third as fast as Andy's mower. If they all start to mow their lawns at the same time, who will finish first?

- (A) Andy (B) Beth (C) Carlos (D) Andy and Carlos tie for first. (E) All three tie.

Problem 22

Let $\triangle XOY$ be a right-angled triangle with $m\angle XOY = 90^\circ$. Let M and N be the midpoints of the legs OX and OY , respectively. Given $XN = 19$ and $YM = 22$, find XY .

- (A) 24 (B) 26 (C) 28 (D) 30 (E) 32

Problem 23

Let $\{a_k\}$ be a sequence of integers such that $a_1 = 1$ and $a_{m+n} = a_m + a_n + mn$, for all positive integers m and n . Then a_{12} is

- (A) 45 (B) 56 (C) 67 (D) 78 (E) 89

Problem 24

Riders on a Ferris wheel travel in a circle in a vertical plane. A particular wheel has radius 20 feet and revolves at the constant rate of one revolution per minute. How many seconds does it take a rider to travel from the bottom of the wheel to a point 10 vertical feet above the bottom?

- (A) 5 (B) 6 (C) 7.5 (D) 10 (E) 15

Problem 25

When 15 is appended to a list of integers, the mean is increased by 2. When 1 is appended to the enlarged list, the mean of the enlarged list is decreased by 1. How many integers were in the original list?

- (A) 4 (B) 5 (C) 6 (D) 7 (E) 8