1962 AHSME Problems

Problem 1

The expression $\frac{1^{4y-1}}{5^{-1}+3^{-1}}$ is equal to:

$\textbf{(A)}\ \frac{4y-1}{8} \qquad \textbf{(B)}\ 8 \qquad \textbf{(C)}\ \frac{15}{2} \qquad \textbf{(D)}\ \frac{15}{8} \qquad \textbf{(E)}\ \frac{1}{8}$

Problem 2

The expression $\sqrt{\frac{4}{3}}-\sqrt{\frac{3}{4}}$ is equal to:

$\textbf{(A)}\ \frac{\sqrt{3}}{6} \qquad \textbf{(B)}\ \frac{-\sqrt{3}}{6} \qquad \textbf{(C)}\ \frac{\sqrt{-3}}{6} \qquad \textbf{(D)}\ \frac{5\sqrt{3}}{6} \qquad \textbf{(E)}\ 1$

Problem 3

The first three terms of an arithmetic progression are $x-1, x+1, 2x+3$, in the order shown. The value of x is:

$\textbf{(A)}\ -2 \qquad \textbf{(B)}\ 0 \qquad \textbf{(C)}\ 2 \qquad \textbf{(D)}\ 4 \qquad \textbf{(E)}\ \text{undetermined}$

Problem 4

If $8^x = 32$, then x equals:

$\textbf{(A)}\ 4 \qquad \textbf{(B)}\ \frac{5}{3} \qquad \textbf{(C)}\ \frac{3}{2} \qquad \textbf{(D)}\ \frac{3}{5} \qquad \textbf{(E)}\ \frac{1}{4}$

Problem 5

If the radius of a circle is increased by 1 unit, the ratio of the new circumference to the new diameter is:

$\textbf{(A)}\ \pi+2 \qquad \textbf{(B)}\ \frac{2\pi+1}{2} \qquad \textbf{(C)}\ \pi \qquad \textbf{(D)}\ \frac{2\pi-1}{2} \qquad \textbf{(E)}\ \pi-2$

Problem 6

A square and an equilateral triangle have equal perimeters. The area of the triangle is $9\sqrt{3}$ square inches. Expressed in inches the diagonal of the square is:

$\textbf{(A)}\ \frac{9}{2} \qquad \textbf{(B)}\ 2\sqrt{5} \qquad \textbf{(C)}\ 4\sqrt{2} \qquad \textbf{(D)}\ \frac{9\sqrt{2}}{2} \qquad \textbf{(E)}\ \text{none of these}$

Problem 7

Let the bisectors of the exterior angles at B and C of $\triangle ABC$ meet at D. Then, if all measurements are in degrees, $\angle BDC$ equals:

$\textbf{(A)}\ \frac{1}{2}(90-A) \qquad \textbf{(B)}\ 90-A \qquad \textbf{(C)}\ \frac{1}{2}(180-A)$
$\textbf{(D)}\ 180-A \qquad \textbf{(E)}\ 180-2A$

Problem 8

Given the set of n numbers; $n > 1$, of which one is $1 - \frac{1}{n}$ and all the others are 1. The arithmetic mean of the n numbers is:

$\textbf{(A)}\ 1 \qquad \textbf{(B)}\ n-\frac{1}{n} \qquad \textbf{(C)}\ n-\frac{1}{n^2} \qquad \textbf{(D)}\ 1-\frac{1}{n^2} \qquad \textbf{(E)}\ 1-\frac{1}{n}-\frac{1}{n^2}$

Problem 9

When $x^9 - x$ is factored as completely as possible into polynomials and monomials with integral coefficients, the number of factors is:

$\textbf{(A)}\ \text{more than 5} \qquad \textbf{(B)}\ 5 \qquad \textbf{(C)}\ 4 \qquad \textbf{(D)}\ 3 \qquad \textbf{(E)}\ 2$

Problem 10

A man drives 150 miles to the seashore in 3 hours and 20 minutes. He returns from the shore to the starting point in 4 hours and 10minutes. Let r be the average rate for the entire trip. Then the average rate for the trip going exceeds r in miles per hour, by:

$\textbf{(A)}\ 5 \qquad \textbf{(B)}\ 4\frac{1}{2} \qquad \textbf{(C)}\ 4 \qquad \textbf{(D)}\ 2 \qquad \textbf{(E)}\ 1$

Problem 11

The difference between the larger root and the smaller root of $x^2 - px + (p^2 - 1)/4 = 0$ is:

$\textbf{(A)}\ 0 \qquad \textbf{(B)}\ 1 \qquad \textbf{(C)}\ 2 \qquad \textbf{(D)}\ p \qquad \textbf{(E)}\ p+1$

Problem 12

When $\left(1 - \frac{1}{a}\right)^6$ is expanded the sum of the last three coefficients is:

$\textbf{(A)}\ 22 \qquad \textbf{(B)}\ 11 \qquad \textbf{(C)}\ 10 \qquad \textbf{(D)}\ -10 \qquad \textbf{(E)}\ -11$

Problem 13

R varies directly as S and inverse as T. When $R = \frac{4}{3}$ and $T = \frac{9}{14}, S = \frac{3}{7}$. Find S when $R = \sqrt{48}$ and $T = \sqrt{75}$.

$\textbf{(A)}\ 28 \qquad \textbf{(B)}\ 30 \qquad \textbf{(C)}\ 40 \qquad \textbf{(D)}\ 42 \qquad \textbf{(E)}\ 60$

Problem 14

Let s be the limiting sum of the geometric series $4 - \frac{8}{3} + \frac{16}{9} - \dots$, as the number of terms increases without bound. Then s equals:

$\textbf{(A)}\ \text{a number between 0 and 1} \qquad \textbf{(B)}\ 2.4 \qquad \textbf{(C)}\ 2.5 \qquad \textbf{(D)}\ 3.6 \qquad \textbf{(E)}\ 12$

Problem 15

Given $\triangle ABC$ with base AB fixed in length and position. As the vertex C moves on a straight line, the intersection point of the three medians moves on:

$\textbf{(A)}\ \text{a circle} \qquad \textbf{(B)}\ \text{a parabola} \qquad \textbf{(C)}\ \text{an ellipse} \qquad \textbf{(D)}\ \text{a straight line} \qquad \textbf{(E)}\ \text{a curve here not listed}$

Problem 16

Given rectangle R_1 with one side 2 inches and area 12 square inches. Rectangle R_2 with diagonal 15 inches is similar to R_1. Expressed in square inches the area of R_2 is:

$\textbf{(A)}\ \frac{9}{2} \qquad \textbf{(B)}\ 36 \qquad \textbf{(C)}\ \frac{135}{2} \qquad \textbf{(D)}\ 9\sqrt{10} \qquad \textbf{(E)}\ \frac{27\sqrt{10}}{4}$

Problem 17

If $a = \log_8 225$ and $b = \log_2 15$, then a, in terms of b, is:

$\textbf{(A)}\ \frac{b}{2} \qquad \textbf{(B)}\ \frac{2b}{3} \qquad \textbf{(C)}\ b \qquad \textbf{(D)}\ \frac{3b}{2} \qquad \textbf{(E)}\ 2b$

Problem 18

A regular dodecagon (12 sides) is inscribed in a circle with radius r inches. The area of the dodecagon, in square inches, is:

$\textbf{(A)}\ 3r^2 \qquad \textbf{(B)}\ 2r^2 \qquad \textbf{(C)}\ \frac{3r^2\sqrt{3}}{4} \qquad \textbf{(D)}\ r^2\sqrt{3} \qquad \textbf{(E)}\ 3r^2\sqrt{3}$

Problem 19

If the parabola $y = ax^2 + bx + c$ passes through the points $(-1, 12)$, $(0, 5)$, and $(2, -3)$, the value of $a + b + c$ is:

$\textbf{(A)}\ -4 \qquad \textbf{(B)}\ -2 \qquad \textbf{(C)}\ 0 \qquad \textbf{(D)}\ 1 \qquad \textbf{(E)}\ 2$

Problem 20

The angles of a pentagon are in arithmetic progression. One of the angles in degrees, must be:

$\textbf{(A)}\ 108 \qquad \textbf{(B)}\ 90 \qquad \textbf{(C)}\ 72 \qquad \textbf{(D)}\ 54 \qquad \textbf{(E)}\ 36$

Problem 21

It is given that one root of $2x^2 + rx + s = 0$, with r and s real numbers, is $3+2i (i = \sqrt{-1})$. The value of s is:

$\textbf{(A)}\ \text{undetermined} \qquad \textbf{(B)}\ 5 \qquad \textbf{(C)}\ 6 \qquad \textbf{(D)}\ -13 \qquad \textbf{(E)}\ 26$

Problem 22

The number 121_b, written in the integral base b, is the square of an integer, for

$\textbf{(A)}\ b = 10,\text{ only} \qquad \textbf{(B)}\ b = 10\text{ and }b = 5,\text{ only}$
$\textbf{(C)}\ 2\le b\le 10 \qquad \textbf{(D)}\ b > 2 \qquad \textbf{(E)}\ \text{no value of }b$

Problem 23

In $\triangle ABC$, CD is the altitude to AB and AE is the altitude to BC. If the lengths of AB, CD, and AE are known, the length of DB is:

$\textbf{(A)}\ \text{not determined by the information given}$
$\textbf{(B)}\ \text{determined only if A is an acute angle}$
$\textbf{(C)}\ \text{determined only if B is an acute angle}$
$\textbf{(D)}\ \text{determined only in ABC is an acute triangle}$
$\textbf{(E)}\ \text{none of these is correct}$

Problem 24

Three machines P, Q, and R, working together, can do a job in x hours. When working alone, P needs an additional 6 hours to do the job; Q, one additional hour; and R, x additional hours. The value of x is:

$\textbf{(A)}\ \frac{2}{3} \qquad \textbf{(B)}\ \frac{11}{12} \qquad \textbf{(C)}\ \frac{3}{2} \qquad \textbf{(D)}\ 2 \qquad \textbf{(E)}\ 3$

Problem 25

Given square $ABCD$ with side 8 feet. A circle is drawn through vertices A and D and tangent to side BC. The radius of the circle, in feet, is:

$\textbf{(A)}\ 4 \qquad \textbf{(B)}\ 4\sqrt{2} \qquad \textbf{(C)}\ 5 \qquad \textbf{(D)}\ 5\sqrt{2} \qquad \textbf{(E)}\ 6$

Problem 26

For any real value of x the maximum value of $8x - 3x^2$ is:

$\textbf{(A)}\ 0 \qquad \textbf{(B)}\ \frac{8}{3} \qquad \textbf{(C)}\ 4 \qquad \textbf{(D)}\ 5 \qquad \textbf{(E)}\ \frac{16}{3}$

Problem 27

Let $a@b$ represent the operation on two numbers, a and b, which selects the larger of the two numbers, with $a@a = a$. Let $a!b$ represent the operator which selects the smaller of the two numbers, with $a!a = a$. Which of the following three rules is (are) correct?

$\textbf{(1)}\ a@b = b@a$
$\textbf{(2)}\ a@(b@c) = (a@b)@c$
$\textbf{(3)}\ a!(b@c) = (a!b)@(a!c)$

$\textbf{(A)}\ (1)\text{ only} \qquad \textbf{(B)}\ (2)\text{ only} \qquad \textbf{(C)}\ (1)\text{ and }(2)\text{ only} \qquad \textbf{(D)}\ (1)\text{ and }(3)\text{ only} \qquad \textbf{(E)}\ \text{all three}$

Problem 28

The set of x-values satisfying the equation $$x^{\log_{10} x} = \frac{x^3}{100}$$ consists of:

$\textbf{(A)}\ \frac{1}{10} \qquad \textbf{(B)}\ 10\text{, only} \qquad \textbf{(C)}\ 100\text{, only} \qquad \textbf{(D)}\ 10\text{ or }100\text{, only} \qquad \textbf{(E)}\ \text{more than two real numbers.}$

Problem 29

Which of the following sets of x-values satisfy the inequality $2x^2 + x < 6$?

$\textbf{(A)}\ -2 < x < \frac{3}{2} \qquad \textbf{(B)}\ x > \frac{3}{2}\text{ or }x < -2 \qquad \textbf{(C)}\ x < \frac{3}{2}$
$\textbf{(D)}\ \frac{3}{2} < x < 2 \qquad \textbf{(E)}\ x < -2$

Problem 30

Consider the statements:

$\textbf{(1)}\ \text{p and q are both true}$
$\textbf{(2)}\ \text{p is true and q is false}$
$\textbf{(3)}\ \text{p is false and q is true}$
$\textbf{(4)}\ \text{p is false and q is false.}$

How many of these imply the negative of the statement "p and q are both true?"

$\textbf{(A) } 0 \qquad \textbf{(B) } 1 \qquad \textbf{(C) } 2 \qquad \textbf{(D) } 3 \qquad \textbf{(E) } 4$

Problem 31

The ratio of the interior angles of two regular polygons with sides of unit length is $3:2$. How many such pairs are there?

$\textbf{(A) } 1 \qquad \textbf{(B) } 2 \qquad \textbf{(C) } 3 \qquad \textbf{(D) } 4 \qquad \textbf{(E) } \infty$

Problem 32

If $x_{k+1} = x_k + \frac{1}{2}$ for $k = 1, 2, \dots, n-1$ and $x_1 = 1$, find $x_1 + x_2 + \dots + x_n$.

$\textbf{(A) } \frac{n+1}{2} \qquad \textbf{(B) } \frac{n+3}{2} \qquad \textbf{(C) } \frac{n^2-1}{2} \qquad \textbf{(D) } \frac{n^2+n}{4} \qquad \textbf{(E) } \frac{n^2+3n}{4}$

Problem 33

The set of x-values satisfying the inequality $2 \le |x-1| \le 5$ is:

$\textbf{(A) } -4 \le x \le -1 \text{ or } 3 \le x \le 6 \qquad \textbf{(B) } 3 \le x \le 6 \text{ or } -6 \le x \le -3$
$\textbf{(C) } x \le -1 \text{ or } x \ge 3 \qquad \textbf{(D) } -1 \le x \le 3 \qquad \textbf{(E) } -4 \le x \le 6$

Problem 34

For what real values of K does $x = K^2(x-1)(x-2)$ have real roots?

$\textbf{(A) } \text{none} \qquad \textbf{(B) } -2 < K < 1 \qquad \textbf{(C) } -2\sqrt{2} < K < 2\sqrt{2}$
$\textbf{(D) } K > 1 \text{ or } K < -2 \qquad \textbf{(E) } \text{all}$

Problem 35

A man on his way to dinner short after 6: 00 p.m. observes that the hands of his watch form an angle of 110°. Returning before 7: 00 p.m. he notices that again the hands of his watch form an angle of 110°. The number of minutes that he has been away is:

$\textbf{(A) } 36\frac{2}{3} \qquad \textbf{(B) } 40 \qquad \textbf{(C) } 42 \qquad \textbf{(D) } 42.4 \qquad \textbf{(E) } 45$

Problem 36

If both x and y are both integers, how many pairs of solutions are there of the equation $(x-8)(x-10) = 2^y$?

$\textbf{(A) } 0 \qquad \textbf{(B) } 1 \qquad \textbf{(C) } 2 \qquad \textbf{(D) } 3 \qquad \textbf{(E) } \text{more than 3}$

Problem 37

$ABCD$ is a square with side of unit length. Points E and F are taken respectively on sides AB and AD so that $AE = AF$ and the quadrilateral $CDFE$ has maximum area. In square units this maximum area is:

$\textbf{(A) } \frac{1}{2} \qquad \textbf{(B) } \frac{9}{16} \qquad \textbf{(C) } \frac{19}{32} \qquad \textbf{(D) } \frac{5}{8} \qquad \textbf{(E) } \frac{2}{3}$

Problem 38

The population of Nosuch Junction at one time was a perfect square. Later, with an increase of 100, the population was one more than a perfect square. Now, with an additional increase of 100, the population is again a perfect square.

The original population is a multiple of:

$\textbf{(A)}\ 3\qquad\textbf{(B)}\ 7\qquad\textbf{(C)}\ 9\qquad\textbf{(D)}\ 11\qquad\textbf{(E)}\ 17$

Problem 39

Two medians of a triangle with unequal sides are 3 inches and 6 inches. Its area is $3\sqrt{15}$ square inches. The length of the third median in inches, is:

$\textbf{(A)}\ 4\qquad\textbf{(B)}\ 3\sqrt{3}\qquad\textbf{(C)}\ 3\sqrt{6}\qquad\textbf{(D)}\ 6\sqrt{3}\qquad\textbf{(E)}\ 6\sqrt{6}$

Problem 40

The limiting sum of the infinite series, $\frac{1}{10}+\frac{2}{10^2}+\frac{3}{10^3}+\dots$ whose nth term is $\frac{n}{10^n}$ is:

$\textbf{(A)}\ \frac{1}{9}\qquad\textbf{(B)}\ \frac{10}{81}\qquad\textbf{(C)}\ \frac{1}{8}\qquad\textbf{(D)}\ \frac{17}{72}\qquad\textbf{(E)}\ \infty$