1973 AHSME Problems

Problem 1

1 A chord which is the perpendicular bisector of a radius of length 12 in a circle, has length

$\textbf{(A)}\ 3\sqrt{3}\qquad\textbf{(B)}\ 27\qquad\textbf{(C)}\ 6\sqrt{3}\qquad\textbf{(D)}\ 12\sqrt{3}\qquad\textbf{(E)}\ \text{ none of these}$

Problem 2

2 One thousand unit cubes are fastened together to form a large cube with edge length 10 units; this is painted and then separated into the original cubes. The number of these unit cubes which have at least one face painted is

$\textbf{(A)}\ 600\qquad\textbf{(B)}\ 520\qquad\textbf{(C)}\ 488\qquad\textbf{(D)}\ 480\qquad\textbf{(E)}\ 400$

Problem 3

3 The stronger Goldbach conjecture states that any even integer greater than 7 can be written as the sum of two different prime numbers. For such representations of the even number 126, the largest possible difference between the two primes is

$\textbf{(A)}\ 112\qquad\textbf{(B)}\ 100\qquad\textbf{(C)}\ 92\qquad\textbf{(D)}\ 88\qquad\textbf{(E)}\ 80$

Problem 4

4 Two congruent -- are placed so that they overlap partly and their hypotenuses coincide. If the hypotenuse of each triangle is 12, the area common to both triangles is

$\textbf{(A)}\ 6\sqrt{3}\qquad\textbf{(B)}\ 8\sqrt{3}\qquad\textbf{(C)}\ 9\sqrt{3}\qquad\textbf{(D)}\ 12\sqrt{3}\qquad\textbf{(E)}\ 24$

Problem 5

5 Of the following five statements, I to V, about the binary operation of averaging (arithmetic mean),

I. Averaging is associative II. Averaging is commutative III. Averaging distributes over addition IV. Addition distributes over averaging V. Averaging has an identity element

those which are always true are

$\textbf{(A)}\ \text{All}\qquad\textbf{(B)}\ \text{I and II only}\qquad\textbf{(C)}\ \text{II and III only}\qquad\textbf{(D)}\ \text{II and IV only}\qquad\textbf{(E)}\ \text{II and V only}$

Problem 6

6 If 554 is the base representation of the square of the number whose base representation is 24, then , when written in base 10, equals

$\textbf{(A)}\ 6\qquad\textbf{(B)}\ 8\qquad\textbf{(C)}\ 12\qquad\textbf{(D)}\ 14\qquad\textbf{(E)}\ 16$

Problem 7

7 The sum of all integers between 50 and 350 which end in 1 is

$\textbf{(A)}\ 5880\qquad\textbf{(B)}\ 5539\qquad\textbf{(C)}\ 5208\qquad\textbf{(D)}\ 4877\qquad\textbf{(E)}\ 4566$

Problem 8

8 If 1 pint of paint is needed to paint a statue 6 ft. high, then the number of pints it will take to paint (to the same thickness) 540 statues similar to the original but only 1 ft. high is

$\textbf{(A)}\ 90\qquad\textbf{(B)}\ 72\qquad\textbf{(C)}\ 45\qquad\textbf{(D)}\ 30\qquad\textbf{(E)}\ 15$

Problem 9

9 In with right angle at , altitude and median trisect the right angle. If the area of is , then the area of is

(A) $6K$ **(B)** $4\sqrt{3}\ K$ **(C)** $3\sqrt{3}\ K$ **(D)** $3K$ **(E)** $4K$

Problem 10

10 If is a real number, then the simultaneous system

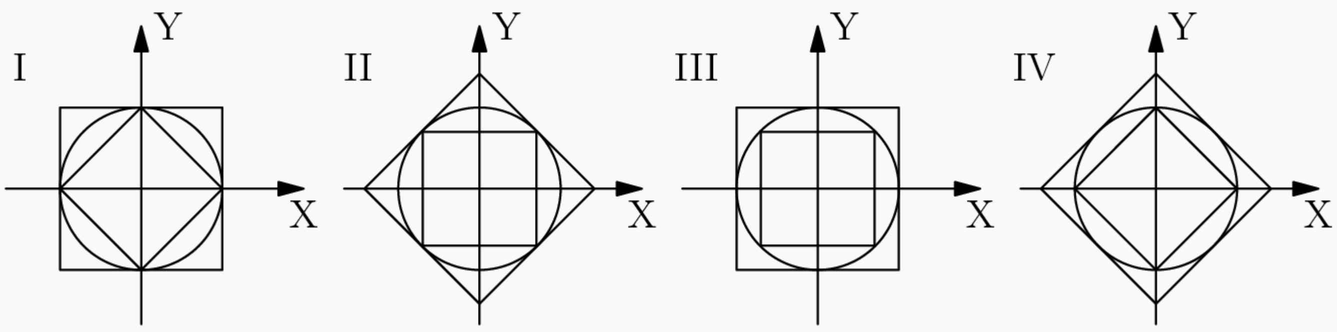
$$nx + y = 1$$

has no solution if and only if is equal to

(A) -1 **(B)** 0 **(C)** 1 **(D)** 0 or 1 **(E)** $\frac{1}{2}$

Problem 11

11 A circle with a circumscribed and an inscribed square centered at the origin of a rectangular coordinate system with positive and axes and is shown in each figure to below.



The inequalities

$$|x| + |y| \leq \sqrt{2(x^2 + y^2)} \leq 2\text{Max}(|x|, |y|)$$

are represented geometrically* by the figure numbered

(A) I **(B)** II **(C)** III **(D)** IV **(E)** none of these

- An inequality of the form , for all and is represented geometrically by a figure showing the containment

for a typical real number .

Problem 12

12 The average (arithmetic mean) age of a group consisting of doctors and lawyers in 40. If the doctors average 35 and the lawyers 50 years old, then the ratio of the numbers of doctors to the number of lawyers is

(A) $3:2$ **(B)** $3:1$ **(C)** $2:3$ **(D)** $2:1$ **(E)** $1:2$

Problem 13

13 The fraction is equal to

$$\frac{2(\sqrt{2}+\sqrt{6})}{3\sqrt{2+\sqrt{3}}}$$

Problem 14

14 Each valve , , and , when open, releases water into a tank at its own constant rate. With all three valves open, the tank fills in 1 hour, with only valves and open it takes 1.5 hours, and with only valves and open it takes 2 hours. The number of hours required with only valves and open is

$(\mathrm{A})\ 1.1 \qquad (\mathrm{B})\ 1.15 \qquad (\mathrm{C})\ 1.2 \qquad (\mathrm{D})\ 1.25 \qquad (\mathrm{E})\ 1.75$

Problem 15

15 A sector with acute central angle is cut from a circle of radius 6. The radius of the circle circumscribed about the sector is

Problem 16

16 If the sum of all the angles except one of a convex polygon is , then the number of sides of the polygon must be

Problem 17

17 If is an acute angle and , then equals

Problem 18

18 If is a prime number, then divides without remainder

Problem 19

19 Define for and positive to be

where is the greatest integer for which . Then the quotient is equal to

Problem 20

20 A cowboy is 4 miles south of a stream which flows due east. He is also 8 miles west and 7 miles north of his cabin. He wishes to water his horse at the stream and return home. The shortest distance (in miles) he can travel and accomplish this is

Problem 21

21 The number of sets of two or more consecutive positive integers whose sum is 100 is

Problem 22

22 The set of all real solutions of the inequality

is

[Note: I updated the notation on this problem.]

Problem 23

23 There are two cards; one is red on both sides and the other is red on one side and blue on the other. The cards have the same probability (1/2) of being chosen, and one is chosen and placed on the table. If the upper side of the card on the table is red, then the probability that the under-side is also red is

Problem 24

24 The check for a luncheon of 3 sandwiches, 7 cups of coffee and one piece of pie came to . The check for a luncheon consisting of 4 sandwiches, 10 cups of coffee and one piece of pie came to at the same place. The cost of a luncheon consisting of one sandwich, one cup of coffee, and one piece of pie at the same place will come to

Problem 25

25 A circular grass plot 12 feet in diameter is cut by a straight gravel path 3 feet wide, one edge of which passes through the center of the plot. The number of square feet in the remaining grass area is

Problem 26

26 The number of terms in an A.P. (Arithmetic Progression) is even. The sum of the odd and even-numbered terms are 24 and 30, respectively. If the last term exceeds the first by 10.5, the number of terms in the A.P. is

Problem 27

27 Cars A and B travel the same distance. Care A travels half that distance at miles per hour and half at miles per hour. Car B travels half the time at miles per hour and half at miles per hour. The average speed of Car A is miles per hour and that of Car B is miles per hour. Then we always have

Problem 28

28 If , , and are in geometric progression (G.P.) with and is an integer, then , , form a sequence

Problem 29

29 Two boys start moving from the same point A on a circular track but in opposite directions. Their speeds are 5 ft. per second and 9 ft. per second. If they start at the same time and finish when they first me at the point A again, then the number of times they meet, excluding the start and finish, is

Problem 30

30 Let denote the greatest integer where and . Then we have

Problem 31

31 In the following equation, each of the letters represents uniquely a different digit in base ten:

The sum equals

Problem 32

32 The volume of a pyramid whose base is an equilateral triangle of side length 6 and whose other edges are each of length is

Problem 33

33 When one ounce of water is added to a mixture of acid and water, the new mixture is acid. When one ounce of acid is added to the new mixture, the result is acid. The percentage of acid in the original mixture is

Problem 34

34 A plane flew straight against a wind between two towns in 84 minutes and returned with that wind in 9 minutes less than it would take in still air. The number of minutes (2 answers) for the return trip was

Problem 35

35 In the unit circle shown in the figure, chords and are parallel to the unit radius of the circle with center at . Chords , , and are each units long and chord is units long.

Of the three equations those which are necessarily true are