1982 AHSME Problems

Problem 1

When the polynomial x^3-2 is divided by the polynomial x^2-2, the remainder is

$\text{(A)}\ 2 \qquad \text{(B)}\ -2 \qquad \text{(C)}\ -2x-2 \qquad \text{(D)}\ 2x+2 \qquad \text{(E)}\ 2x-2$

Problem 2

If a number eight times as large as x is increased by two, then one fourth of the result equals

$\text{(A)}\ 2x+\frac{1}{2} \qquad \text{(B)}\ x+\frac{1}{2} \qquad \text{(C)}\ 2x+2 \qquad \text{(D)}\ 4 \qquad \text{(E)}\ 8$

Problem 3

Evaluate $(x^x)^{(x^x)}$ at $x=2$.

$\text{(A)}\ 16 \qquad \text{(B)}\ 64 \qquad \text{(C)}\ 256 \qquad \text{(D)}\ 1024 \qquad \text{(E)}\ 65,536$

Problem 4

The perimeter of a semicircular region, measured in centimeters, is numerically equal to its area, measured in square centimeters. The radius of the semicircle, measured in centimeters, is

$\text{(A)} \pi \qquad \text{(B)} \frac{2}{\pi} \qquad \text{(C)} 1 \qquad \text{(D)} \frac{1}{2} \qquad \text{(E)} \frac{4}{\pi}+2$

Problem 5

Two positive numbers x and y are in the ratio $a:b$ where $0<a<b$. If $x+y=c$, then the smaller of x and y is

$\text{(A)}\ \frac{ac}{b} \qquad \text{(B)}\ \frac{bc-ac}{b} \qquad \text{(C)}\ \frac{ac}{a+b} \qquad \text{(D)}\ \frac{bc}{a+b} \qquad \text{(E)}\ \frac{ac}{b-a}$

Problem 6

The sum of all but one of the interior angles of a convex polygon equals 2570°. The remaining angle is

$\text{(A)}\ 90^{\circ} \qquad \text{(B)}\ 105^{\circ} \qquad \text{(C)}\ 120^{\circ} \qquad \text{(D)}\ 130^{\circ} \qquad \text{(E)}\ 144^{\circ}$

Problem 7

If the operation $x \star y$ is defined by $x \star y = (x+1)(y+1)-1$, then which one of the following is FALSE?

$\text{(A)}\ x \star y = y \star x \text{ for all real } x,y.$
$\text{(B)}\ x \star (y+z) = (x \star y) + (x \star z) \text{ for all real } x,y, \text{ and } z.$
$\text{(C)}\ (x-1) \star (x+1) = (x \star x) - 1 \text{ for all real } x.$
$\text{(D)}\ x \star 0 = x \text{ for all real } x.$
$\text{(E)}\ x \star (y \star z) = (x \star y) \star z \text{ for all real } x,y, \text{ and } z.$

Problem 8

By definition, $r! = r(r-1)\cdots 1$ and $\binom{j}{k} = \dfrac{j!}{k!(j-k)!}$, where r, j, k are positive integers and $k < j$.
If $\binom{n}{1}, \binom{n}{2}, \binom{n}{3}$ form an arithmetic progression with $n > 3$, then n equals

$\textbf{(A)}\ 5 \qquad \textbf{(B)}\ 7 \qquad \textbf{(C)}\ 9 \qquad \textbf{(D)}\ 11 \qquad \textbf{(E)}\ 12$

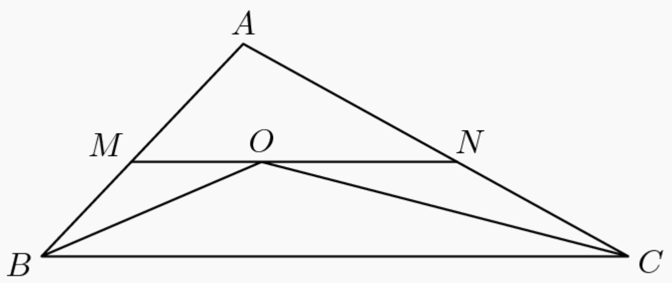
Problem 9

A vertical line divides the triangle with vertices $(0,0)$, $(1,1)$, and $(9,1)$ in the $xy\text{-plane}$ into two regions of equal area. The equation of the line is $x =$

$\text{(A)} 2.5 \qquad \text{(B)} 3.0 \qquad \text{(C)} 3.5 \qquad \text{(D)} 4.0 \qquad \text{(E)} 4.5$

Problem 10

In the adjoining diagram, BO bisects $\angle CBA$, CO bisects $\angle ACB$, and MN is parallel to BC. If $AB = 12, BC = 24$, and $AC = 18$, then the perimeter of $\triangle AMN$ is



$\text{(A)} 30 \qquad \text{(B)} 33 \qquad \text{(C)} 36 \qquad \text{(D)} 39 \qquad \text{(E)} 42$

Problem 11

How many integers with four different digits are there between $1,000$ and $9,999$ such that the absolute value of the difference between the first digit and the last digit is 2?

$\text{(A)} 672 \qquad \text{(B)} 784 \qquad \text{(C)} 840 \qquad \text{(D)} 896 \qquad \text{(E)} 1008$

Problem 12

Let $f(x) = ax^7 + bx^3 + cx - 5$, where a, b and c are constants. If $f(-7) = 7$, the $f(7)$ equals

$\text{(A)} -17 \qquad \text{(B)} -7 \qquad \text{(C)} 14 \qquad \text{(D)} 21 \qquad \text{(E)} \text{not uniquely determined}$

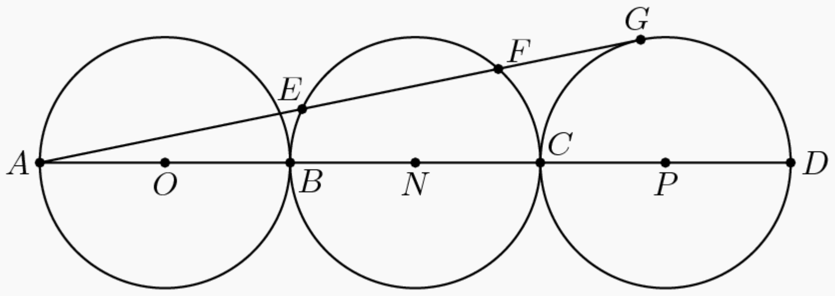
Problem 13

If $a > 1, b > 1$, and $p = \dfrac{\log_b(\log_b a)}{\log_b a}$, then a^p equals

$\text{(A)} 1 \qquad \text{(B)} b \qquad \text{(C)} \log_a b \qquad \text{(D)} \log_b a \qquad \text{(E)} a^{\log_b a}$

Problem 14

In the adjoining figure, points B and C lie on line segment AD, and AB, BC, and CD are diameters of circle O, N, and P, respectively. Circles O, N, and P all have radius 15 and the line AG is tangent to circle P at G. If AG intersects circle N at points $E$and F, then chord EF has length



(A)20 (B)$15\sqrt{2}$ (C)24 (D)25 (E)none of these

Problem 15

Let $[z]$ denote the greatest integer not exceeding z. Let x and y satisfy the simultaneous equations

$y = 2[x] + 3$
$y = 3[x-2] + 5.$

If x is not an integer, then $x + y$ is

(A) an integer (B) between 4 and 5 (C) between -4 and 4
(D) between 15 and 16 (E) 16.5

Problem 16

A wooden cube has edges of length 3 meters. Square holes, of side one meter, centered in each face are cut through to the opposite face. The edges of the holes are parallel to the edges of the cube. The entire surface area including the inside, in square meters, is

(A)54 (B)72 (C)76 (D)84 (E)86

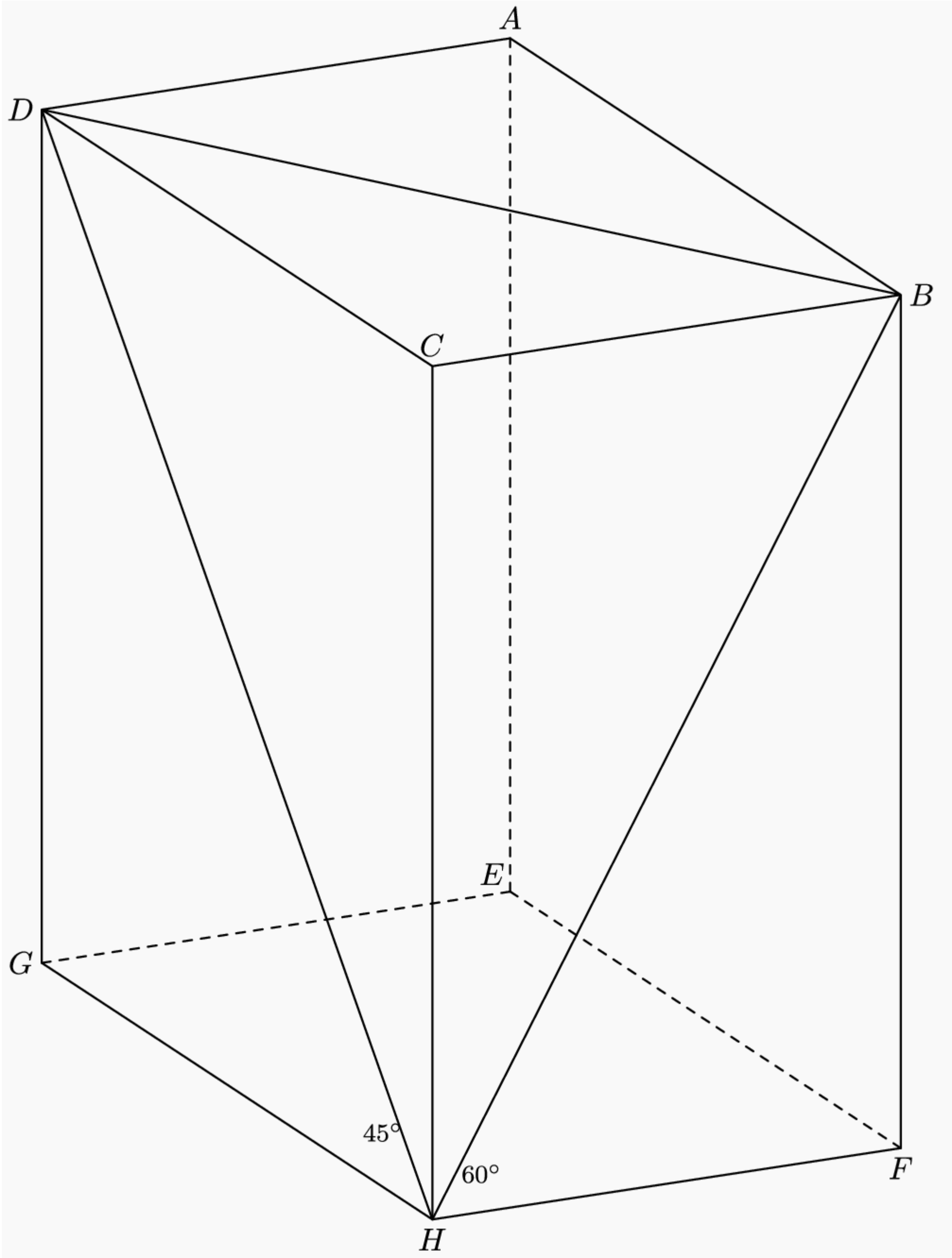
Problem 17

How many real numbers x satisfy the equation $3^{2x+2} - 3^{x+3} - 3^x + 3 = 0$?

(A)0 (B)1 (C)2 (D)3 (E)4

Problem 18

In the adjoining figure of a rectangular solid, $\angle DHG = 45^\circ$ and $\angle FHB = 60^\circ$. Find the cosine of $\angle BHD$.



(A) $\dfrac{\sqrt{3}}{6}$ (B) $\dfrac{\sqrt{2}}{6}$ (C) $\dfrac{\sqrt{6}}{3}$ (D) $\dfrac{\sqrt{6}}{4}$ (E) $\dfrac{\sqrt{6}-\sqrt{2}}{4}$

Problem 19

Let $f(x) = |x-2| + |x-4| - |2x-6|$ for $2 \leq x \leq 8$. The sum of the largest and smallest values of $f(x)$ is

$(\text{A})1 \qquad (\text{B})2 \qquad (\text{C})4 \qquad (\text{D})6 \qquad (\text{E})\text{none of these}$

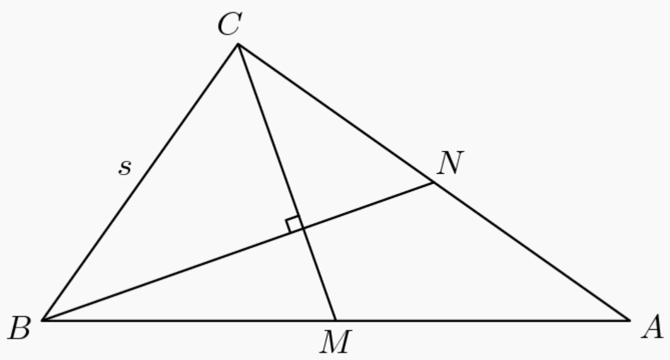
Problem 20

The number of pairs of positive integers (x, y) which satisfy the equation $x^2 + y^2 = x^3$ is

$(\text{A})0 \qquad (\text{B})1 \qquad (\text{C})2 \qquad (\text{D})\text{not finite} \qquad (\text{E})\text{none of these}$

Problem 21

In the adjoining figure, the triangle ABC is a right triangle with $\angle BCA = 90^\circ$. Median CM is perpendicular to median BN, and side $BC = s$. The length of BN is



(A)$s\sqrt{2}$ (B)$\frac{3}{2}s\sqrt{2}$ (C)$2s\sqrt{2}$ (D)$\frac{1}{2}s\sqrt{5}$ (E)$\frac{1}{2}s\sqrt{6}$

Problem 22

In a narrow alley of width w a ladder of length a is placed with its foot at point P between the walls. Resting against one wall at Q, the distance k above the ground makes a 45° angle with the ground. Resting against the other wall at R, a distance h above the ground, the ladder makes a 75° angle with the ground. The width w is equal to

(A)a (B)RQ (C)k (D)$\frac{h+k}{2}$ (E)h

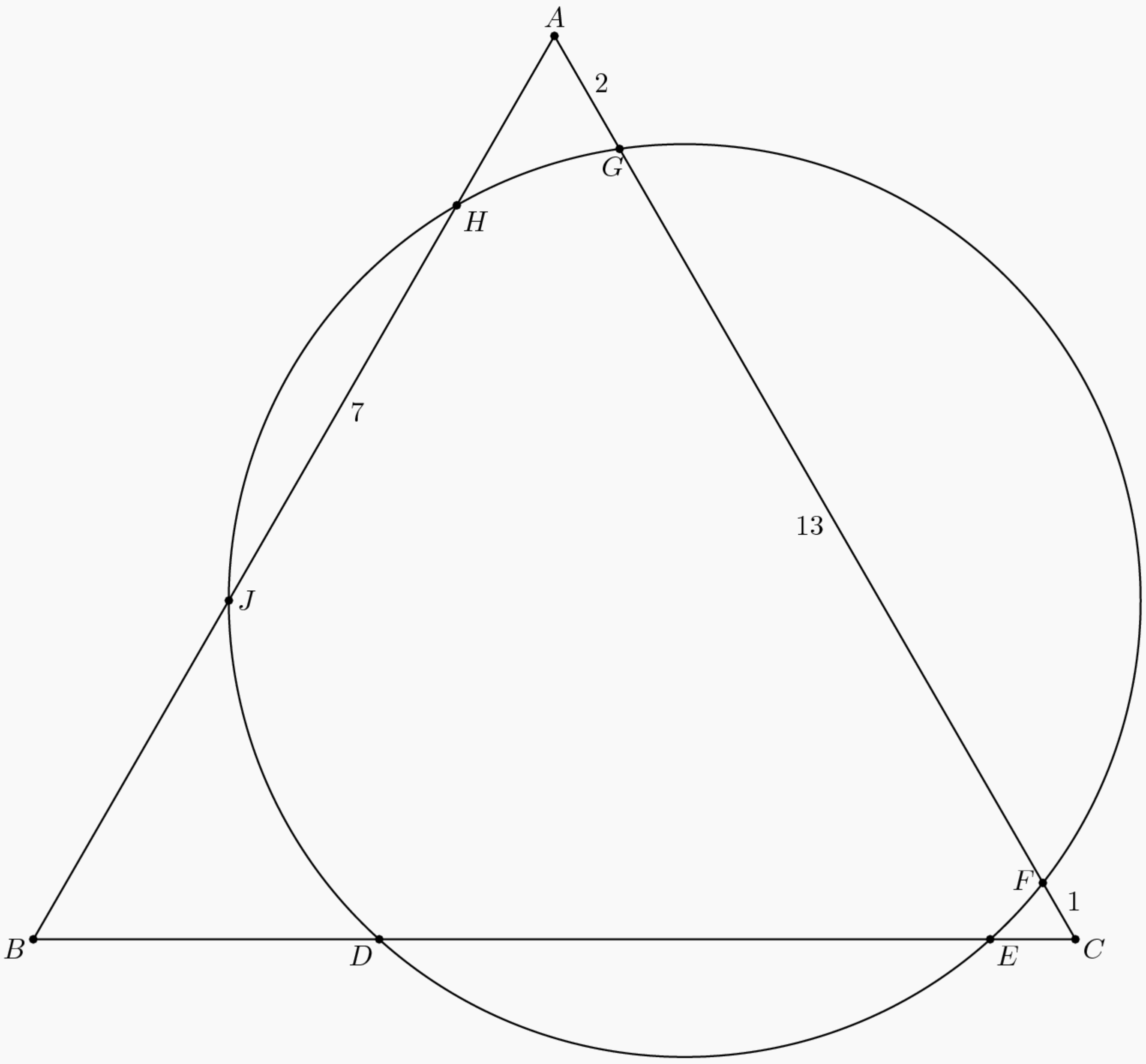
Problem 23

The lengths of the sides of a triangle are consecutive integers, and the largest angle is twice the smallest angle. The cosine of the smallest angle is

(A)$\frac{3}{4}$ (B)$\frac{7}{10}$ (C)$\frac{2}{3}$ (D)$\frac{9}{14}$ (E)none of these

Problem 24

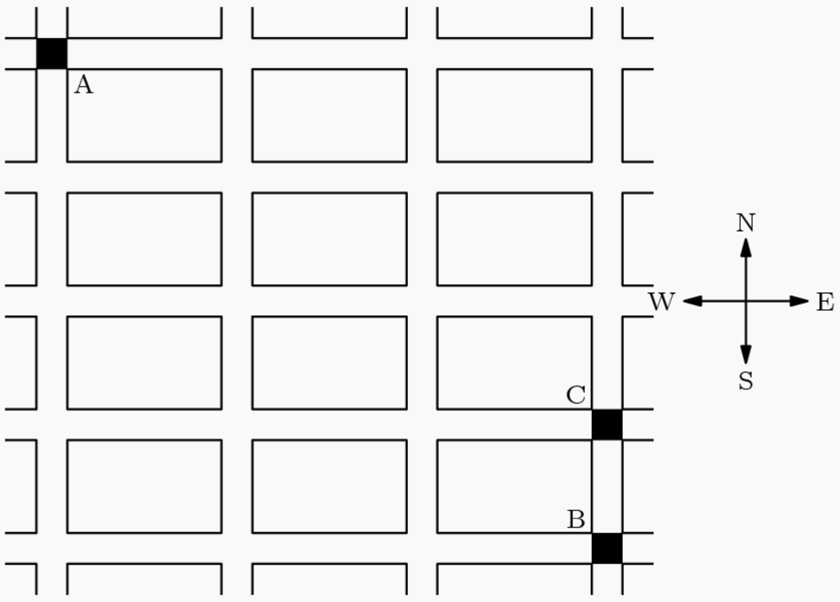
In the adjoining figure, the circle meets the sides of an equilateral triangle at six points. If $AG=2, GF=13, FC=1$, and $HJ=7$, then DE equals



(A)$2\sqrt{22}$ (B)$7\sqrt{3}$ (C)9 (D)10 (E)13

Problem 25

The adjacent map is part of a city: the small rectangles are rocks, and the paths in between are streets. Each morning, a student walks from intersection A to intersection B, always walking along streets shown, and always going east or south. For variety, at each intersection where he has a choice, he chooses with probability $\frac{1}{2}$ whether to go east or south. Find the probability that through any given morning, he goes through C.



$(\mathrm{A})\frac{11}{32}$ $\qquad (\mathrm{B})\frac{1}{2}$ $\qquad (\mathrm{C})\frac{4}{7}$ $\qquad (\mathrm{D})\frac{21}{32}$ $\qquad (\mathrm{E})\frac{3}{4}$

Problem 26

If the base 8 representation of a perfect square is $ab3c$, where $a \neq 0$, then c equals

$(\mathrm{A})0 \qquad (\mathrm{B})1 \qquad (\mathrm{C})3 \qquad (\mathrm{D})4 \qquad (\mathrm{E})\text{not uniquely determined}$

Problem 27

Suppose $z = a + bi$ is a solution of the polynomial equation $c_4z^4 + ic_3z^3 + c_2z^2 + ic_1z + c_0 = 0$, where c_0, c_1, c_2, c_3, a, and b are real constants and $i^2 = -1$. Which of the following must also be a solution?

$(\mathrm{A}) -a-bi \qquad (\mathrm{B})a-bi \qquad (\mathrm{C}) -a+bi \qquad (\mathrm{D})b+ai \qquad (\mathrm{E})\text{none of these}$

Problem 28

A set of consecutive positive integers beginning with 1 is written on a blackboard. One number is erased. The average (arithmetic mean) of the remaining numbers is $35\frac{7}{17}$. What number was erased?

$(\mathrm{A})6 \qquad (\mathrm{B})7 \qquad (\mathrm{C})8 \qquad (\mathrm{D})9 \qquad (\mathrm{E})\text{cannot be determined}$

Problem 29

Let x, y, and z be three positive real numbers whose sum is 1. If no one of these numbers is more than twice any other, then the minimum possible value of the product xyz is

$\textbf{(A)}\ \frac{1}{32} \qquad \textbf{(B)}\ \frac{1}{36} \qquad \textbf{(C)}\ \frac{4}{125} \qquad \textbf{(D)}\ \frac{1}{127} \qquad \textbf{(E)}\ \text{none of these}$

Problem 30

Find the units digit of the decimal expansion of

$(15 + \sqrt{220})^{19} + (15 + \sqrt{220})^{82}$.

$\textbf{(A)}\ 0 \qquad \textbf{(B)}\ 2 \qquad \textbf{(C)}\ 5 \qquad \textbf{(D)}\ 9 \qquad \textbf{(E)}\ \text{none of these}$