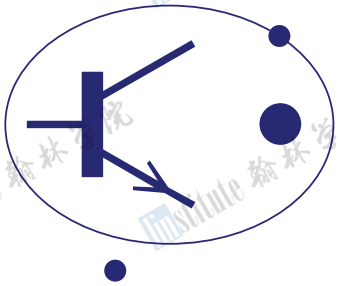
BRITISH PHYSICS OLYMPIAD



British Physics Olympiad 2010

Paper 2

13th November 2009

Section 2

Important Constants

Speed of light	c	3.00×10^{8}	ms^{-1}
Planck constant	h	6.63×10^{-34}	Js
Electronic charge	e	1.60×10^{-19}	C
Mass of electron	m_e	9.11×10^{-31}	kg
Permittivity of a vacuum	ε_0	8.85×10^{-12}	Fm^{-1}
Gravitational constant	G	6.67×10^{-11}	Nm^2kg^{-2}
Acceleration due to free fall	g	9.81	ms^{-2}
Mass of Earth	M_E	5.9700×10^{24}	kg
Mass of Moon	M_M	7.35×10^{22}	kg
Radius of Earth	R_E	6.38×10^{3}	km
Radius of the Moon	R_M	1.74×10^{3}	km
Earth – Moon distance	R_{EM}	3.84×10^{5}	km

Q2

(a) A glass block measures 5.0 cm x 5.0 cm x 8.0 cm. When the block stands on one of its smaller faces, and viewed directly from above, it appears to be a cube. Determine the refractive index of the block.

[2]

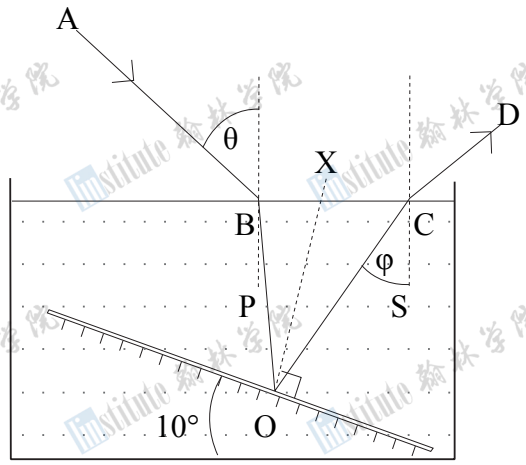


Figure 2.b

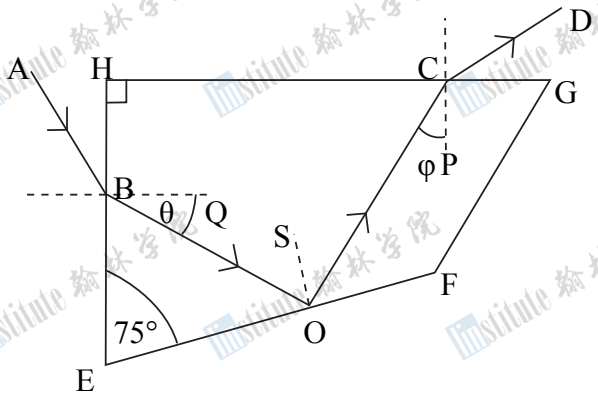


Figure 2.c

(b) A plane, inclined, mirror lies at the bottom of a long flat bottomed tank containing water. The mirror makes an angle of 10° with the horizontal, Figure 2.b. A narrow beam of monochromatic light falls on the surface of the water at an angle of incidence θ. If the refractive index of water is 4/3, determine the maximum value of θ for which light, after reflection from the mirror, would emerge from the upper surface of the water. The angle SCO is indicated by φ. OX is normal to the mirror.

[6]

(c) Figure 2.c shows the cross-section of a glass prism, refractive index *n*, with the angles indicated showing a light ray passing through it. OS is perpendicular to EF.

(i) Express angle BOC in terms of θ.
(ii) Determine the angle φ in terms of θ.
(iii) For what value of θ is θ = φ ? What condition does this impose on *n* if total internal reflection occurs at O?
(iv) What is the angle between the incident and emergent rays when θ = φ ?

[10]

(d) Why, at night, are the images of lights reflected from a river elongated?

[2]

Q3 The ends of a uniform wire of cross-sectional area 1.00×10^{-6} m^2 and negligible mass are attached to fixed points A and B which are 1.00 m apart, in the same horizontal line. The wire is initially straight and unstretched. A mass of 0.50 kg is attached to the mid-point of the wire and hangs in equilibrium with the mid-point at a distance of 10 mm below AB.
Calculate the Young's modulus for the wire.

[10]

(b) A man, mass 90 kg, begins to climb a 4.0 m ladder of mass 10 kg. The ladder rests against a wall at A. The foot of the ladder, B, is 2.0 m from the wall.

(i) Draw a diagram indicating all the forces present in terms of their vertical and horizontal components using the notation F_{AV}, F_{AH}, F_{BV} and F_{BH} for components at A and B; the vertical components have subscripts V and the horizontal components have subscripts H.

(ii) What conditions must be satisfied by the forces, and the moment of the forces about A, if the man is to climb a distance x up the ladder when it is against a smooth wall?

(iii) If the man is to climb to the top of the ladder, what is the minimum value of the coefficient of friction, μ, required on the ground?

[10]

Q4 A weight of mass M hangs from the end of a light vertical spring that is attached, at its upper end, to a rigid support . When in equilibrium the spring has an extension x_1. The natural angular frequency of the system is ω.

(i) Determine x_1 in terms of ω. [2]

(ii) The mass vibrates about its equilibrium position, the spring having an extension $(x_1 + x)$ at time t. Obtain an expression for the total potential energy, V, of the system. The zero of the gravitational potential energy of the mass is to be taken at the equilibrium extension, x_1. [3]

(iii) If the amplitude of the motion is A_1, determine the maximum kinetic energy, T_1 , of the mass and the total energy, E_1, of the system . [2]

(iv) The mass collides elastically with a stationary body of mass $M/2$ at $x = 0$. Determine the new velocity, v_2, after the collision and amplitude, A_2, of the motion of the mass M. [5]

(v) If in (iv) the masses coalesce, determine the new equilibrium position, x_2, and the new amplitude, A_2, in terms of A_1.

[8]

Q5

(a) Two capacitors, of capacitance 2.0 μF and 4.0 μF, are each given a charge of 120 μC. The positive plates are now connected together, as are the negative plates. Calculate:

(i) the new potential difference between the plates of the capacitors
(ii) the change in energy. Explain this energy change. [6]

(b)

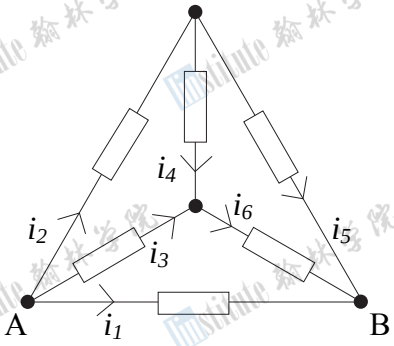


Figure 5.b

All the resistors in the circuit in Figure 5.b have resistance *R*. The potential difference across AB is *V* and the currents in the arms are $i_1, i_2, \ldots i_6$, as indicated.

(i) How are the currents altered if *V* is reversed? Give the relations between the currents by comparing the circuits in the two situations; *V* and -*V*.
(ii) Deduce the currents in terms of *V* and *R*.
(iii) Determine the resistance across AB. [7]

(c)

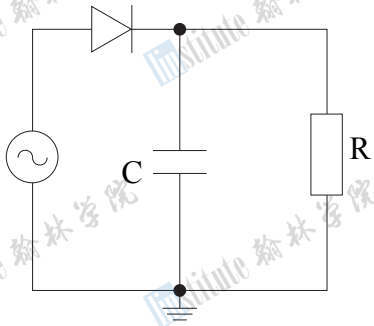


Figure 5.c

The circuit in Figure 5.c has a capacitor C, with capacitance 50 μF, a diode and a resistor R, resistance *R*. It is used to rectify an a.c. supply of frequency 50 Hz and peak to peak voltage of 20 V. R can have the values of 10 kΩ or 100 Ω . In each case:

(i) determine the time constant for the circuit
(ii) sketch the voltage waveforms across R

If the a.c. supply is replaced by a square wave supply of the same frequency and amplitude, determine the greatest fractional change in the voltage across R for the circuit with the 10 kΩ resistor.

[7]

Q6

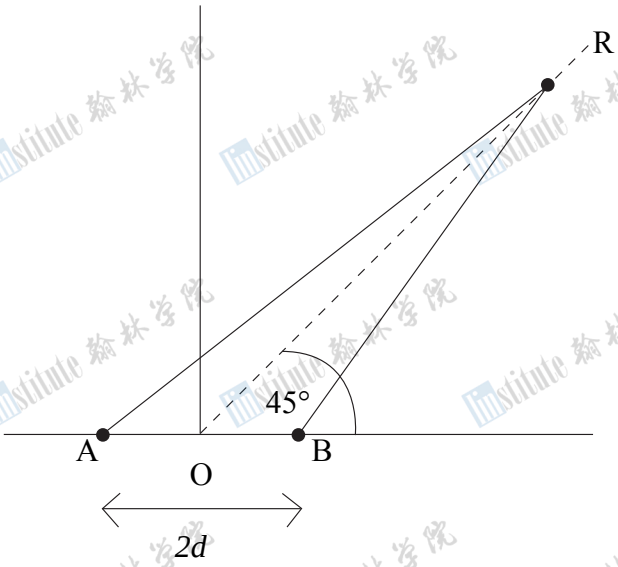


Figure 6.1

Two loud speakers, A and B, are mounted at ground level, a distance $2d$ = 50.0 cm apart, and are connected in parallel to the output of an amplifier, which is fed from a variable frequency oscillator. An observer, also at ground level, stands at a point R that is 1.20 km from the line joining the loud speakers and 1.20 km from the perpendicular bisector of AB at O. OR = D, Figure 6.1. The oscillator is adjusted so that its frequency, f, rises linearly with time, at 10 Hz per sec from $f = 0$ at time $t = 0$. The sound heard by the observer drops to a minimum for the first time when t = 52.2 s.

(a) Obtain an expression for the time taken by the sound, velocity c_s, to reach R from:

(i) A (ii) B [4]

(b) Find an expression for the frequency of the sound reaching R at time t from:

(i) A (ii) B [4]

(c) As D is much greater than d, the frequencies in (b) are approximately equal. Give an expression for the value, and accuracy, of the 'common' frequency.

[4]

(d) Deduce an expression for the wavelength of the sound reaching R.

[1]

(e) Calculate the speed of sound, c_s.

[7]

Q7

(a) The abundances of U^{238} and U^{235} are in the ratio, respectively, of 140:1. Equal amounts of each isotope existed in the Earth's crust at its formation. Estimate the age of the Earth. The half-lives are: U^{238} 4.5×10^9 years, U^{235} 7.1×10^8 years.

[6]

(b) A steel compression ring for the piston of a car is irradiated with neutrons until it has a uniformly distributed activity of 4×10^5 Bq due to the formation of Fe^{59}. The ring is immediately installed in the engine. After the engine has been running for 30 days, a 100 cm^3 of the engine oil is taken out and 126 disintegrations are recorded from it during a 10 min counting period. If the total volume of the oil is 5.0×10^{-3} m^3, what fraction of the ring has worn away during the running period? Assume all the metal worn away is in suspension in the oil.

(1 Bq is one disintegration per sec., half-life Fe^{59} = 45 days)

[6]

(c) A radioactive detector is used to measure the count rate of a single radioactive source. It initially registered 82 counts s^{-1}, which dropped after 210 s to 19 counts s^{-1}. The half life of the substance was 70 s. Verify that the count rate does *not* satisfy an exponential decay law. A constant background radiation was present during the measurements. Determine its count rate.

[5]

(d) If in (c) the anomaly was due to the presence of a radioactive substance, with a much greater half life than the source, how would one deduce its half life from experimental measurements?

[3]

Q8

(a) Calculate the numerical value of the acceleration due to free fall, g, at the surface of the Earth by applying Newton's law of gravitation. Neglect rotational effects and assume a spherical Earth of mass M_E and radius R_E.

[3]

(b) If the Earth's crust, density 2.7×10^3 kgm^{-3} , is assumed to be 30 km deep, what is the fractional change in g would occur upon descending to a depth of 15 km ? Indicate if g increased or decreased .

[7]

(c) A spherical deposit of iron, density 7.9×10^3 kgm^{-3}, has a diameter of 5.0 km and lies just beneath the Earth's surface. Determine the fractional change in g, measured at the Earth's surface immediately above the deposit, due to this deposit.

[6]

(d) If the iron in (c) is mined, leaving a cavity, determine the fractional change in g.

[4]

Q9

A rocket carrying a 1,000 kg satellite is to be launched from Earth with a velocity such that the combined vehicle has *just sufficient* energy to reach the Moon. At a height of 20 km above the lunar surface, the satellite is detached. Its velocity is altered, and redirected by its internal motors, so as to place it in a stable orbit 10 km above the lunar surface. The rocket's speed is not affected by the freeing of the satellite and it is allowed to continue on its flight until it crashes into the Moon. It can be assumed that the rocket has constant mass and there is no relative motion between the Earth and the Moon.

(a) Determine the distances, R_{MF} and R_{MP} respectively, measured from the centre of the Moon, at which the magnitude of (i) the gravitational forces, F, and (ii) the gravitational potential energies, P , of the Moon and Earth on the combined vehicle are equal in magnitude.

[6]

(b) Derive an *algebraic* expression, using the symbols in the table below, for the velocity at which the rocket would impact on the lunar surface if it did not detach the satellite.

[5]

(c) Obtain the speed of the satellite in orbit at 10 km above the Moon's surface.

[4]

(d) Determine the energy, E_m, required to be extracted from the satellite, in orbit 10 km above the Moon's surface, in order to achieve a soft landing on the lunar surface.

[5]

Mass of the Earth	$M_E = 5.97 \times 10^{24}$	kg
Mass of the Moon	$M_M = 7.35 \times 10^{22}$	kg
Radius of the Earth	$R_E = 6.38 \times 10^{3}$	km
Radius of the Moon	$R_M = 1.74 \times 10^{3}$	km
Earth – Moon, centre to centre, distance	$R_{EM} = 3.84 \times 10^{5}$	km

End of Section 2
End of Paper