# 2016 Stanford International Math Tournament (High School)
## August, 2016
## Stanford University

## *"Speed Test 60" Questions*

## Time Limit 45 minutes (2 points each)—Calculators are PROHIBITED

## Directions

- This is a test of speed and accuracy.

- The questions are **NOT** arranged in order of difficulty.

- Answers exactly equivalent to the official answers will be given credit.

- No one is expected to solve all 60 questions in the time given.

- Do not spend too much time on any one question.

- Write your answer to each question in the answer box to the right of each question.

- Each question is worth 2 points.

- You will have 45 minutes in which to answer 60 questions.

Write your answer in the answer box to the right of each question. **(2 points each)**

Name:_____ Number Correct:_____

| | |
|---|---|
| 1. What is the value of $(1 + \dfrac{1}{0.1})^2$? | 1. |
| 2. $74 + 7 + 9 + 1926 = 2000 + \underline{?}$ | 2. |
| 3. When written in standard form, how many zeros does $(10^{10})^{10}$ have? | 3. |
| 4. A pen and a pencil together cost \$1.30. If the pen costs \$1 more than the pencil, how much does the pencil cost in dollars? | 4. |
| 5. What is the least value of $x$ that satisfies $x^2 + x = 2^2 + 2$. | 5. |
| 6. What is the reciprocal of $(\dfrac{1}{5} + \dfrac{1}{12})$? | 6. |
| 7. How many of the first 100 positive integers have exactly 3 positive integral divisors? | 7. |
| 8. For what real number $c$ are both roots of $10x^2 + 20x + c = 0$ equal? | 8. |
| 9. If the circumference of circle $C$ is the same as the perimeter of a square of area $\pi$, what is the area of circle $C$? | 9. |
| 10. Anna's brother has 1 more brother than he has sisters. How many more brothers than sisters does Anna have? | 10. |
| 11. Two integers between 10 and 99 have the same digits but in different order. What is the greatest possible value of the difference between these integers? | 11. |
| 12. True/False: Lines $4x + y = 7$ and $y = 4x - 9$ are parallel. | 12. |
| 13. How many real numbers have their reciprocals equal to their opposites? | 13. |
| 14. What is the greatest integer that is less than or equal to $-\pi$? | 14. |
| 15. True/False: It is possible for a trapezoid to have exactly 3 congruent angles. | 15. |
| 16. What value of $x$ satisfies $3^{336} \times 9^{336} \times 27^{336} = 3^x$? | 16. |
| 17. I bought a painting for \$1 and then sold it for a profit of 10000% on my cost. For what price in dollars did I sell this painting? | 17. |
| 18. What is the length of a side of a rhombus whose diagonals have lengths 3 and 4? | 18. |
| 19. A computer can download 2% of a file in 10 seconds. How many seconds does it take to download 100% of the file? | 19. |
| 20. What is the length of the altitude to the shortest side of a triangle with side-lengths of 3, 4, and 5? | 20. |

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### Time Limit 45 minutes—Calculators are PROHIBITED—PAGE 2 of 3
Write your answer in the answer box to the right of each question. **(2 points each)**

Name:_____ Number Correct:_____

| | |
|---|---|
| 21. What is the perimeter of a right triangle with one leg of length 51 and hypotenuse of length 85? | 21. |
| 22. True/False: The product of 2 different prime numbers is always odd. | 22. |
| 23. True/False: The sum of 2 different composite numbers can be a prime. | 23. |
| 24. True/False: The sum of 2 different primes can be a composite number. | 24. |
| 25. What is the smallest integer greater than 999 which uses only the digit 9 and is divisible by 999? | 25. |
| 26. If the last three digits of an integer $n$ are 625, then what is the largest integer that must be a divisor of $n$? | 26. |
| 27. Rotating an image 2016° clockwise is the same as rotating it $d°$ counterclockwise, where $0 < d < 360$. What is the value of $d$? | 27. |
| 28. A watch loses 1 second per day. How many days will it take for this watch to lose 24 hours? | 28. |
| 29. What is the largest integer less than 10 000 that is included in the arithmetic sequence, 1, 5, 9, . . . ? | 29. |
| 30. What is the smallest multiple of 4 between 100 and 1000 whose digits have a sum of 20? | 30. |
| 31. If the hypotenuse of a right triangle is 3 times as long as the altitude to the hypotenuse and the numerical area of the triangle is 12 times the numerical length of this altitude, what is the length of the hypotenuse? | |
| 32. What is the smallest positive integer $n$ that satisfies $2^n > 6n^2$? | 32. |
| 33. What is the smallest $n$ for which a regular $n$-gon has each interior angle $\geq 162°$? | 33. |
| 34. What is the sum of the remainders when each of the first 100 positive integers is divided by 6? | 34. |
| 35. What is the sum of the remainders when 20 is divided by each of the positive integers greater than 1 and less than 20? | 35. |
| 36. If $ab = 2$ and $a^2 + b^2 = 24$, what is $|a - b|$? | 36. |
| 37. If $ab = 2$ and $a^2 + b^2 = 24$, what is $\left| \dfrac{1}{a} + \dfrac{1}{b} \right|$? | 37. |
| 38. If $ab = 2$ and $a^2 + b^2 = 24$, what is $|a^3 + b^3|$? | 38. |
| 39. If for all $x$, $2f(x) + 3g(x) = 12x - 1$ and $4f(x) + 8g(x) = 28x - 8$, what is $g(0)$? | 39. |
| 40. True or False: $\dfrac{1}{\sqrt{21} - \sqrt{20}} > 9$? | 40. |

Write your answer in the answer box to the right of each question. **(2 points each)**

Name:_____ Number Correct:_____

| Question | |
|---|---|
| 41. If the surface area of a cube is 150 cm$^2$, what is the cube's volume in cm$^3$? | 41. |
| 42. True or False: If $a$, $b$, and $c$ are the lengths of the sides of a triangle, then there is always a triangle with sides of length $a^2$, $b^2$, and $c^2$? | 42. |
| 43. Plant 1 is now 5 cm tall and grows uniformly at a rate of 1 cm/week. Plant 2 is now 7 cm tall and grows uniformly at a rate of 2 cm every 3 weeks. In how many weeks will the two plants be the same height? | 43. |
| 44. When my spinner stops spinning, it lands on 1 of 3 numbers. It lands on each number with equal probability. If my spinner is spun 3 times, what is the probability that it does **not** land on a different number each time? | 44. |
| 45. What is the area of a circle whose circumference is $2\pi^2$? | 45. |
| 46. I traveled 1 km at 2 km/hr and 2 km at 1 km/hr. What is my average speed in km/hr for the entire 3 km trip? | 46. |
| 47. If the area between a square and its inscribed circle is $16 - \pi^2$, what is the area of the square? | 47. |
| 48. True/False: The sum of 2017 odd integers can never equal 2016. | 48. |
| 49. What is the least integer $x$ that satisfies $x^5 > x$? | 49. |
| 50. How many real numbers satisfy $x + x^3 + x^5 = 0$? | 50. |
| 51. A rectangular box has dimensions 1, 4, and 5. What is the greatest possible distance between any 2 points in or on the box? | 51. |
| 52. True/False: $100^{\frac{1}{500}} > 1000^{\frac{1}{1000}}$ . | 52. |
| 53. What is the greatest prime factor of the sum of the first 200 positive integers? | 53. |
| 54. If $a + b = 2$, $b + c = 4$, $a + c = 2016$, what is the value of $a + b + c$? | 54. |
| 55. If the length of the median to the hypotenuse of a right triangle is 5 and one leg of the triangle has length 8, what is the area of the triangle? | 55. |
| 56. True/False: If $abc = 1. 2016$, then $a^2 + b^2 + c^2 > 1$. | 56. |
| 57. True/False: Given any 4 integers, there is always a nonempty subset of the integers whose sum is divisible by 4. | 57. |
| 58. If $P(x)$ is a 3rd degree polynomial with roots 1, 2, and 3, what is the sum of the roots of $P(2x + 1)$? | 58. |
| 59. True/False: If $a$, $b$, and $c$ are positive numbers and $a^2 + b^2 + c^2 > 3$, $abc > 1$. | 59. |
| 60. What is the largest even integer less than 200 with exactly 4 positive divisors? | 60. |