2016 Stanford International Math Tournament (High School)
August, 2016
Stanford University

Individual Questions
Each Question is worth 15 points. Time Limit 90 minutes.
Calculators are **PROHIBITED.**

Name (Print):_____ Grade: _____

1. Let $(x^{2015}+x^{2017}+2)^{2016}=a_0+a_1x+\ldots+a_nx^n$.

   What is the value of $a_0-\frac{a_1}{2}-\frac{a_2}{2}+a_3-\frac{a_4}{2}-\frac{a_5}{2}+a_6-\ldots$ ?

   Answer: _____

2. What are all the ordered pairs of real numbers ($x$, $y$) that satisfy the system of equations below?

$$\begin{cases} x-y=2016 \\ \frac{x+y}{2}-\sqrt{xy}=72 \end{cases}$$

   Answer: _____

3. What are all the real values for $x$ that satisfy $\frac{2x}{7x^2-5x-17}+\frac{13x}{7x^2+x-17}=6$?

   Answer: _____

4. Let $f(x)=\frac{1}{1+2016^{2x-1}}$. What is the value of $f(\frac{1}{2017})+f(\frac{2}{2017})+f(\frac{3}{2017})+\ldots+f(\frac{2016}{2017})$?

   Answer: _____

5. What are all possible $x$-coordinates of points on curve $r = 2(1 + \sin\theta)$ (interval $[0, 2\pi]$) whose $y$-coordinates is $\frac{\sqrt{5}+1}{4}$?

Answer: _____

6. Write the fully reduced form of the expression below.

$$\frac{\sum_{j=1}^{224} \sin\left(j * \frac{\pi}{900}\right)}{\sum_{i=1}^{224} \cos\left(i * \frac{\pi}{900}\right)}$$

Answer: _____

7. The side-lengths of $\triangle ABC$ are 3, 3, and 2. What is the positive difference between the diameter of its circumcircle (circumscribed circle) and the diameter of its incircle (inscribed circle)?

Answer: _____

8. $a$, $b$, $c$, $d$, and $e$, not necessary distinct, are all single digit numbers from 0 to 9, inclusive, such that $a + b + c + d + e = 37$.
How many different ways can the values of $a$, $b$, $c$, $d$, and $e$ be? For example, one possible way is $(a, b, c, d, e) = (9, 8, 7, 6, 7)$.

Answer: _____

9. Let $a$, $b$, and $c$ be non-zero complex numbers, such that

$$a - \frac{1}{b} = n - 2$$
$$b - \frac{1}{c} = n$$
$$c - \frac{1}{a} = n + 2$$

What is the value of $abc - \frac{1}{abc}$ in terms of $n$?

Answer: _____

10. What are all the real values for $x$ that satisfy $x^2 + \frac{9x^2}{(x+3)^2} = 16$?

Answer: _____