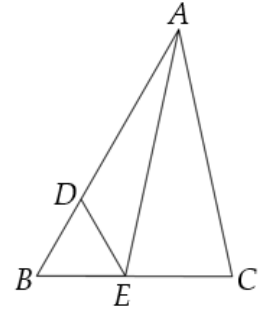# July, 2016 – Grades 8 & 9

Individual Questions

Each question is worth 10 points. Calculators are PROHIBITED.

#1. (*Time Limit: 7 minutes*) Reduce $\dfrac{(2+1)(2^2+1)(2^4+1)(2^8+1)(2^{16}+1)(2^{32}+1)}{2^{64}-1}$ to lowest terms.

#2. (*Time Limit: 7 minutes*) In the diagram at the right, $\triangle AEC$ is isosceles with $AE = AC$. If $\triangle DBE$ is equilateral, $AD = 10$, and $DB = 2$, what is EC?

#3. (*Time Limit: 7 minutes*) If $a^2+b^2+c^2+d^2+e^2$ is a perfect square, how many ordered 5-tuples of consecutive integers (a,b,c,d,e) satisfy $a<b<c<d<e< 2016$?

#4. (*Time Limit: 7 minutes*) If a and b are integers and $a < b$, how many ordered pairs (a,b) satisfy $\dfrac{2016}{a}-\dfrac{2016}{b} = b - a$?
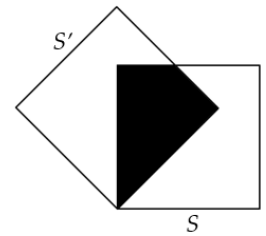
#5. (*Time Limit: 7 minutes*) Square S has area 16. Rotate square S 45° about one of its vertices, as shown, to form a second square, S'. What is the area of the shaded region common to these two squares?

#6. (*Time Limit: 7 minutes*) Of all points in the plane whose distance from (0,0) is twice their distance from (3,0), point (a,b) is furthest from (0,10). What is the value of $\dfrac{b-10}{a}$?

#7. (*Time Limit: 7 minutes*) What are all real numbers x for which x, $x + 2$, and 10 could be the lengths of the sides of an acute triangle?

#8. (*Time Limit: 7 minutes*) If $[x]$ denotes the greatest integer $\leq x$, what are all real values of x that satisfy $[x]^2 - 2x + 1 = 0$?

#9. (*Time Limit: 7 minutes*) A circle is inscribed in $\triangle ABC$, so that $\overline{AB}$ is tangent to the circle at D and $\overline{AC}$ is tangent to the circle at E. If $m\angle A = 60$, $AD = 6$, and $AB = 10$, what is BC?

#10. (*Time Limit: 7 minutes*) In how many different ways can one choose five of the first 20 positive integers such that the difference between any two of the five chosen integers is greater than 2?