Problem: Emergency Medical Response

The Emergency Service Coordinator (ESC) for a county is interested in locating the county's three ambulances to best maximize the number of residents that can be reached within 8 minutes of an emergency call. The county is divided into 6 zones and the average time required to travel from one zone to the next under semi-perfect conditions is summarized in the following Table 1.

	Average Travel Times (min.)					
Zones	**1**	**2**	**3**	**4**	**5**	**6**
1	1	8	12	14	10	16
2	8	1	6	18	16	16
3	12	18	1.5	12	6	4
4	16	14	4	1	16	12
5	18	16	10	4	2	2
6	16	18	4	12	2	2

Table 1: Average travel times from Zone *i* to Zone *j* in semi-perfect conditions.

The population in zones 1, 2, 3, 4, 5 and 6 are given in Table 2 below:

Zones	Population
1	50,000
2	80,000
3	30,000
4	55,000
5	35,000
6	20,000
Total	270,000

Table 2: Population in each Zone

Goals of your model

1. Determine the locations for the three ambulances which would maximize the number of people who can be reached within 8 minutes of a 911 call. Can we cover everyone? If not, then how many people are left without coverage?
2. We now have only two ambulances since one has been set aside for an emergency call; where should we put them to maximize the number of people who can be reached within the 8 minute window? Can we cover everyone? If not, then how many people are left without coverage?
3. Two ambulances are now no longer available; where should the remaining ambulance be posted? Can we cover everyone? If not, then how many people are left without coverage?
4. If a catastrophic event occurs in one location with many people from all zones involved, could the ESC cover the situation? How do counties or cities design for those rare but catastrophic events?
5. In addition to the contest's format, prepare a short 1-2 page non-technical memo outlining your recommendations from your model and analysis finding for the ESC.

Problem A

Emergency Medical Response

Team#4170

High School Mathematical Contest in Modeling

For office use only
T1 ________________
T2 ________________
T3 ________________
T4 ________________

For office use only
F1 ________________
F2 ________________
F3 ________________
F4 ________________

2013

Summary Sheet

Team Control Number: 4170
Problem Chosen: A

Problem A: Emergency Medical Response

Summary

In order to best maximize the number of residents that can be reached as soon as possible of an emergency call, our team strives to develop models to work out the best solutions when simulating the real situation in the county. To simulate the realest situation, we analyze the average travel time table and find the shortest time from one zone to another.

Firstly, to determine the locations for 3 ambulances, we develop the first model not only regardless of the cost times in a zone but also considering the cost of travel times in a zone. When the cost times in the zone are ignored, ambulances located in Zone 2, 5 and 6 can cover the most residents, which is 300,000 people. In order to reduce the cost, we locate the 3 ambulances in Zone 1, 2, and 5 or Zone1, 2, and 6, and all the zones can be covered within 6 minutes. When consider travel times in a zone, 3 ambulances are located in Zone 2, 4 and 5 or Zone 2, 5, and 6, the zones covered maximize. And there are 275,000 people covered.

Secondly, after drawing the conclusion and giving out the answer of the first question, our team further discuss the model how we determine the location of *m* ambulances in an area which is divided into *n* zones. And we build a clear and detailed model which can be used to almost every situation. It is also with 'regardless the travel time in a zone' as well as 'considering the travel time in a zone'.

Then we substitute *m*, *n* into our new model. And we gain the best solutions of the second and the third question using the second model. The simulation results validate that our model is correct.

After that, we test our model in Shanghai. We choose 8 famous locations and analyze the average time from one location to another. We also choose 3 locations as the starting point. The result is that all the locations can be reached. This result proves that our model is feasible in the real life.

At last, to solve the forth question, we turn the problem into a realistic example to analyze it. At the time we verify our model, we use a real event as an example. We choose the case of earthquake which happened in Wenchuan, China. We search a lot of information on the Internet and get useful pictures and texts. After analyzing them, we make matrixes and use our models to solve the problem. The simulation result is that most of the stricken areas can be covered, but some roads are damaged so that several places cannot be covered.

Contents

1. Problem Restatement

What an ESC of a certain county does is to locate ambulances and dispatch them to a particular place in the county which is divided into 6 zones.

According to the Average Travel Times Table, the time which it takes to travel from Zone *i* to Zone *j* and the time that it takes from Zone *j* to Zone *i* are different. Maybe it is because there are one-way roads or one-way traffic congestion. Besides, the time given by the table which it takes to travel from one zone to another may be not the shortest. For example, the time it takes from Zone 4 to Zone 5 is 16 minutes. But if the ambulance goes to Zone 6 and then leave for Zone 5, it just takes 12 minutes. This is shorter than 16 minutes. The ESC has to maximize the number of people who can be reached within 8 minutes.

So as to maximum the people or zones to be covered, we make efforts to make and improve our model so that we can simulate the situation more real.

2. Assumptions and Justification

Assumptions of Model 1, 2, 3:

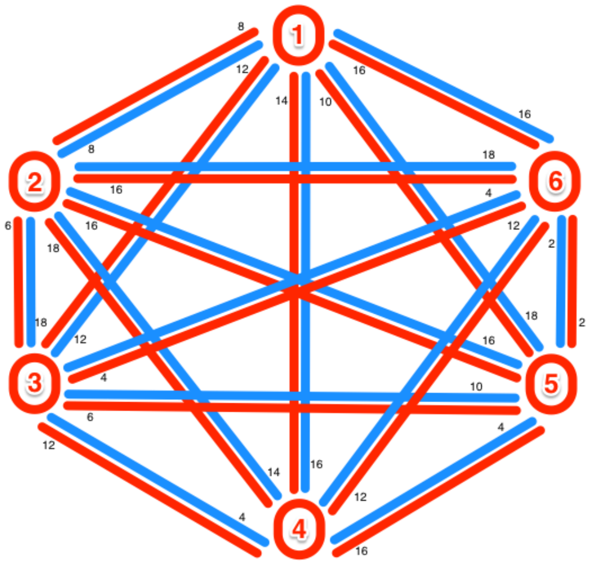
1. There's no accident on the way such as traffic jams, storms, etc.
2. The time that an ambulance travels from one spot to another is certain.
3. The population of each zone won't change.
4. Things such as machines of the ambulances run well.
5. There are hospitals in each zone.
6. Full fuel.

3. Analysis of the Average Time

It is easy to find a lot of ways from one zone to another in the picture, and their lengths are different.

Tab. 1: The Original Average Time

Zone	**1**	**2**	**3**	**4**	**5**	**7**
1	1	8	12	14	10	16
2	8	1	6	18	16	16
3	12	18	1.5	12	6	4
4	16	14	4	1	16	12
5	18	16	10	4	2	2
6	16	18	4	12	2	2



The Original Picture of Routes with the Average Times

(When **i<j**, every red row shows the route from Zone *i* to Zone *j*, and every blue row shows the route from Zone *j* to Zone *i*.)

3.1 Variables Definition

Variable	Description
A_1	A matrix of average travel times from Zone *i* to Zone *j* in semi-perfect conditions
A_2	A matrix of the shortest travel times, regardless of the travel time in one zone, from Zone *i* to Zone *j* in semi-perfect conditions
A_3	A matrix with simplified information based on A_2 which marks the number larger than '8' as '0' and the rest as '1' so that '1' means that the ambulance can reach the place within 8 minutes
A_4	A matrix of the shortest travel times, considering the travel time in one zone, from Zone *i* to Zone *j* in semi-perfect conditions
A_5	A matrix with simplified information based on A_4 which marks the number larger than '8' as '0' and the rest as '1' so that '1' means that the ambulance can reach the place within 8 minutes
A_6	A matrix with simplified information based on A_2 which marks the number larger than '6' as '0' and the rest as '1' so that '1' means that the ambulance can reach the place within 6 minutes
A_7	A matrix with simplified information based on A_2 which marks the number larger than '4' as '0' and the rest as '1' so that '1' means that the ambulance can reach the place within 4 minutes
Pop	A matrix which shows the population of each zone

$$A_1 = \begin{pmatrix} 1 & 8 & 12 & 14 & 10 & 16 \\ 8 & 1 & 6 & 18 & 16 & 16 \\ 12 & 18 & 1.5 & 12 & 6 & 4 \\ 16 & 14 & 4 & 1 & 16 & 12 \\ 18 & 16 & 10 & 4 & 2 & 2 \\ 16 & 18 & 4 & 12 & 2 & 2 \end{pmatrix} A_2 = \begin{pmatrix} 1 & 8 & 12 & 14 & 10 & 12 \\ 8 & 1 & 6 & 18 & 12 & 10 \\ 12 & 18 & 1.5 & 10 & 6 & 4 \\ 16 & 14 & 4 & 1 & 10 & 8 \\ 18 & 16 & 6 & 4 & 2 & 2 \\ 16 & 18 & 4 & 6 & 2 & 2 \end{pmatrix} A_3 = \begin{pmatrix} 1 & 1 & 0 & 0 & 0 & 0 \\ 1 & 1 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 1 & 1 \\ 0 & 0 & 1 & 1 & 0 & 1 \\ 0 & 0 & 1 & 1 & 1 & 1 \\ 0 & 0 & 1 & 1 & 1 & 1 \end{pmatrix}$$

$$A_4 = \begin{pmatrix} 1 & 10 & 14.5 & 16 & 13 & 17 \\ 10 & 1 & 8.5 & 20 & 16.5 & 14.5 \\ 14.5 & 20.5 & 1.5 & 14.5 & 9.5 & 7.5 \\ 18 & 16 & 6.5 & 1 & 14.5 & 12.5 \\ 21 & 19 & 11.5 & 7 & 2 & 6 \\ 19 & 21 & 7.5 & 11 & 6 & 2 \end{pmatrix} A_5 = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 1 \\ 0 & 0 & 1 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 1 & 1 \\ 0 & 0 & 1 & 0 & 1 & 1 \end{pmatrix} A_6 = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 1 \\ 0 & 0 & 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 & 1 & 1 \\ 0 & 0 & 1 & 1 & 1 & 1 \end{pmatrix}$$

$$A_7 = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 1 \\ 0 & 0 & 1 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 1 & 1 \\ 0 & 0 & 1 & 0 & 1 & 1 \end{pmatrix}$$

$$Pop = \begin{pmatrix} 50 & 80 & 30 & 55 & 35 & 20 \end{pmatrix}$$

3.2 Process for the Shortest Path

Critical Path Method:

In A_1, we shorten the average travel time from one zone to another. For example, we use Zone 2 to Zone 6 as series of data to get the shortest time. The number underrowd in the table is the shortest time.

	1	2	3	4	5	6
i = 1		<u>1</u>	∞	∞	∞	∞
i = 2	8		<u>6</u>	18	16	16
i = 3	<u>8</u>			16	12	10
i = 4				16	12	<u>10</u>

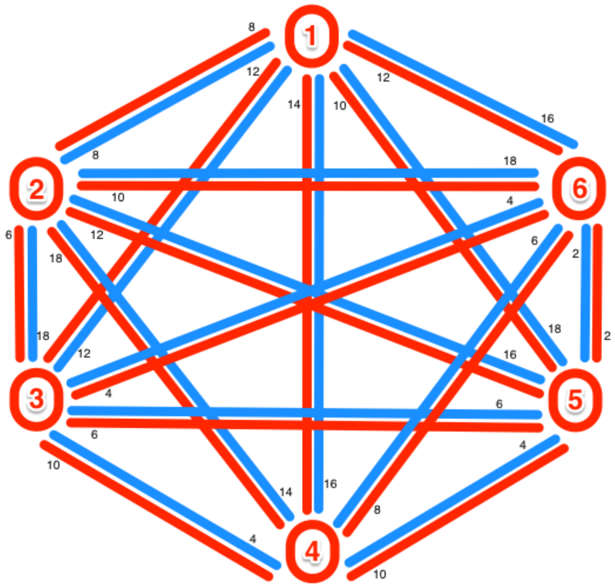
3.3 Conclusion

The shortest average time, <u>regardless of the time in one zone</u>, under semi-perfect conditions from Zone *i* to *j* is showed in the following table.

Tab. 2:Shortest Average Travel Times Regardless the Travel Time in a Zone

Zone	**1**	**2**	**3**	**4**	**5**	**6**
1	1	8	12	14	10	12
2	8	1	6	18	12	10
3	12	18	1.5	10	6	4
4	16	14	4	1	10	8

5	18	16	6	4	2	2
6	16	18	4	6	2	2



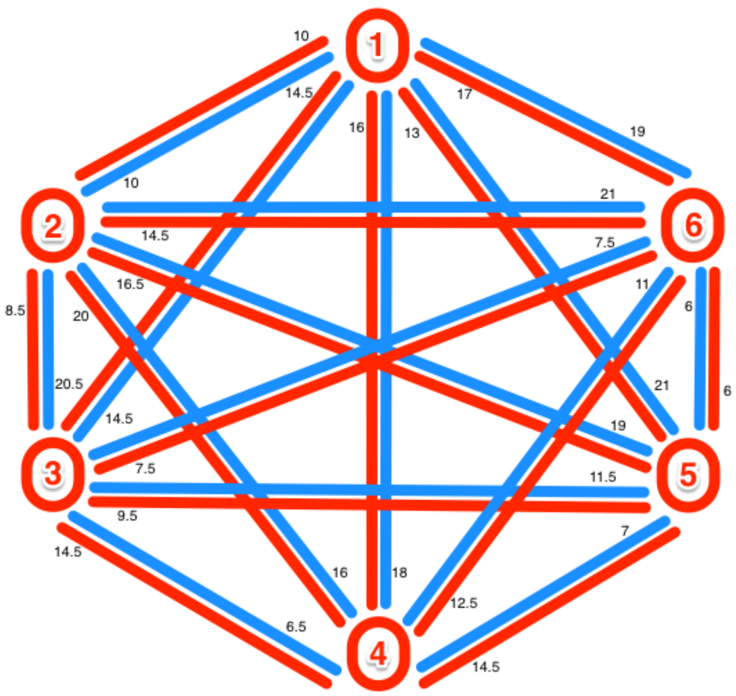
The Picture of Routes with the Shortest Average Times Regardless of Travel Times in a Zone

(When **i<j**, every red row shows the route from Zone *i* to Zone *j*, and every blue row shows the route from Zone *j* to Zone *i*.)

The shortest time, <u>considering the time in one zone</u>, under semi-perfect conditions from Zone *i* to *j* is showed in the following table.

Tab. 3: Shortest Average Travel Times Considering the Travel Time in a Zone

Zone	1	2	3	4	5	6
1	1	10	14.5	16	13	17
2	10	1	8.5	20	16.5	14.5
3	14.5	20.5	1.5	14.5	9.5	7.5
4	18	16	6.5	1	14.5	12.5
5	21	19	11.5	7	2	6
6	19	21	7.5	11	6	2



The Picture of Routes with Shortest Average Times Considering the Travel Time in a Zone

(When **i<j**, every red row shows the route from Zone *i* to Zone *j*, and every blue row shows the route from Zone *j* to Zone *i*.)

4. Model Design

4.1 Variables

In the optimal emergency medical response model, we define some variables as such:

Variable Name	Description
k	Index variable
n	The number of zones
m	The number of available ambulances
r_k	The number of people in Zone k
v_k	Zone k
v_x	The location of the first ambulance
v_y	The location of the second ambulance
v_z	The location of the third ambulance

v_{a_i}	The location where the i ambulance is located
$s(v_x, v_k)$	The value of row x, column k in A_3
$S(V_y,V_k)$	The value of row y, column k in A_3
$S(V_z,V_k)$	The value of row z, column k in A_3
S_k	The number of ambulances which can reach Zone k
y_k	Whether Zone k can be reached within 8 minutes
$\varphi(V_x,V_k)$	The value of row x, column k in A_5
$\varphi(V_y,V_k)$	The value of row y, column k in A_5
$\varphi(V_z,V_k)$	The value of row z, column k in A_5
z_k	Whether Zone k can be reached within 8 minutes

The variables in this table will be used in all of the following models.

4.2 Model 1

We have designed an optimal emergency medical response model for the determinations of the locations of three ambulances.

4.2.1 Modeling

Under semi-perfect conditions and regardless of the travel time in one zone, we strive for

$$\max \sum_{k=1}^{6} y_k r_k$$

$$s.t.\begin{cases} y_k = \begin{cases} 0, & s_k = 0 \\ 1, & s_k \geq 1 \end{cases} \\ s_k = s(\mathrm{v}_x, \mathrm{v}_k) + \mathrm{s}(\mathrm{v}_y, \mathrm{v}_k) + \mathrm{s}(\mathrm{v}_z, \mathrm{v}_k) \\ v_x \in \{1,2,3,4,5,6\} \\ v_y \in \{1,2,3,4,5,6\} \\ v_z \in \{1,2,3,4,5,6\} \\ s(\mathrm{v}_x, \mathrm{v}_k) \in A_3 \\ s(\mathrm{v}_y, \mathrm{v}_k) \in A_3 \\ s(\mathrm{v}_z, \mathrm{v}_k) \in A_3 \\ v_x \neq v_y \neq v_z \\ r_k \in Pop, k \in \{1,2,3,4,5,6\} \end{cases}$$

A. Locations regardless the travel time in a zone

Following the model, we substitute for every possible x, y, z. And we programme and gain the following conclusion. ('Program solution_initial' is in the appendix; input $A_k = A_3$)

Table 4

	The numbers of the ambulances which cover the zone						The location of the ambulances
No.	**1**	**2**	**3**	**4**	**5**	**6**	

1	2	2	2	1	1	1	1	2	5
2	2	2	2	1	1	1	1	2	6
3	1	1	2	1	1	2	1	3	4
4	1	1	2	1	2	2	1	3	5
5	1	1	2	1	2	2	1	3	6
6	1	1	2	2	1	2	1	4	5
7	1	1	2	2	1	2	1	4	6
8	1	1	2	2	2	2	1	5	6
9	1	1	3	1	1	2	2	3	4
10	1	1	3	1	2	2	2	3	5
11	1	1	3	1	2	2	2	3	6
12	1	1	3	2	1	2	2	4	5
13	1	1	3	2	1	2	2	4	6
14	1	1	3	2	2	2	2	5	6

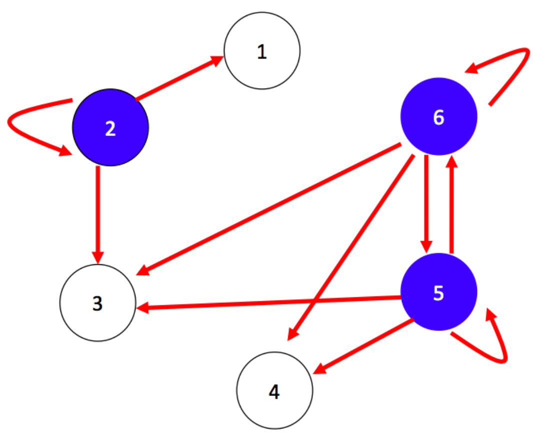
(The first six columns of *Table 4* mean the numbers of ambulances that cover this area. The last three columns of *D* mean the location of 3 ambulances. There are 14 ways.)

a. The places where the ambulances locate that can cover the maximum people regardless of the travel time in a zone

If an ambulance covers a whole zone, that ambulance covers everybody in this zone. The solutions that have the maximum sum of the people who can be covered by the three ambulances is the best solution.

We gain the best solution through 'Program solution_ initial'. ('Program solution_ initial' is in the appendix; input A_k = A_3) So, the best solution is that when the 3 ambulances are placed in **Zone 2, Zone 5 and Zone 6**, the number of the people who can be covered maximize. And there are **300,000 people covered**.

When there's a call, the 3 ambulances should go like what the following picture shows.



b. The places where the ambulances locate that cost least money or fuel regardless of the travel time in a zone

If an ambulance costs least money and fuel, that ambulance spends least time

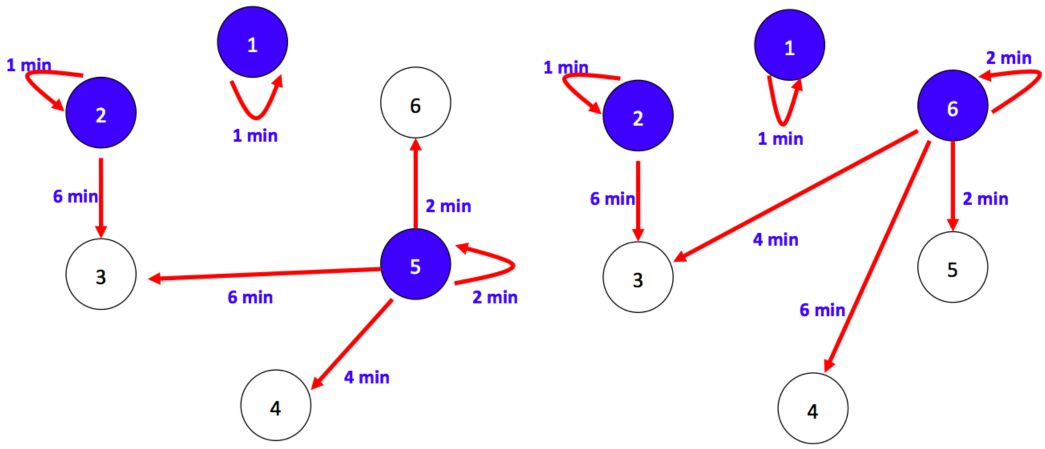
traveling on the way.

Firstly, we use 6 minutes as a stand to further discuss A_6 because the time of 6 minutes is the longest time besides that of 8 minutes on the way. We programme through A_6 and gain the following conclusion. ('Program solution_ initial' is in the appendix; input $A_k = A_6$) It is that when we locate the 3 ambulances in **Zone 1, Zone 2, Zone 5** or **Zone1, Zone 2, Zone 6**, all the zones can be covered within **6 minutes**. And the numbers of the people covered are the same. There are 300,000 people covered.

Secondly, we use 4 minutes as a stand to further discuss A_7 because the time of 4 minutes is the longest time besides that of 6 minutes on the way. We programme through A_7 and gain the following conclusion. ('Program solution_ initial' is in the appendix; input $A_k = A_7$) It is that ambulances **cannot** cover each zone within 4 minutes.

So, using this method, the best solution is to locate the 3 ambulances in **Zone 1, Zone 2, Zone 5 or Zone1, Zone 2, Zone 6.** And there are **300,000 people covered.**

When there's a call, the 3 ambulances should go like what the following pictures shows.



B. Locations considering the travel time in a zone

We get A_4 in a situation that we consider the ambulance traveling in each zone. For example, if an ambulance travels from Zone 1 to Zone 6, we should consider then sum of time from Zone 1 to Zone 1, Zone 1 to Zone 6 and Zone 6 to Zone 6. Here is the new expression.

$$\max \sum_{k=1}^{6} z_k r_k$$

$$s.t.\begin{cases} z_k = \begin{cases} 0 & \phi_k = 0 \\ 1 & \phi_k \geq 1 \end{cases} \\ \phi_k = \phi(\mathrm{v}_x, \mathrm{v}_k) + \phi(\mathrm{v}_y, \mathrm{v}_k) + \phi(\mathrm{v}_z, \mathrm{v}_k) \\ v_x \in \{1,2,3,4,5,6\} \\ v_y \in \{1,2,3,4,5,6\} \\ v_z \in \{1,2,3,4,5,6\} \\ \phi(\mathrm{v}_x, \mathrm{v}_k) \in A_5 \\ \phi(\mathrm{v}_y, \mathrm{v}_k) \in A_5 \\ \phi(\mathrm{v}_z, \mathrm{v}_k) \in A_5 \\ v_x \neq v_y \neq v_z \\ r_k \in Pop, k \in \{1,2,3,4,5,6\} \end{cases}$$

We use 6 minutes as a stand and mark the number larger than '8' as '0' and the rest as '1' so that '1' means that the ambulance can reach the place within 8 minutes. After we simplify the matrix, we get A_5. We programme through A_5 and gain the following conclusion. ('Program solution_ initial' is in the appendix; input $A_k = A_5$) It is that we cannot cover every zone by 3 ambulances.

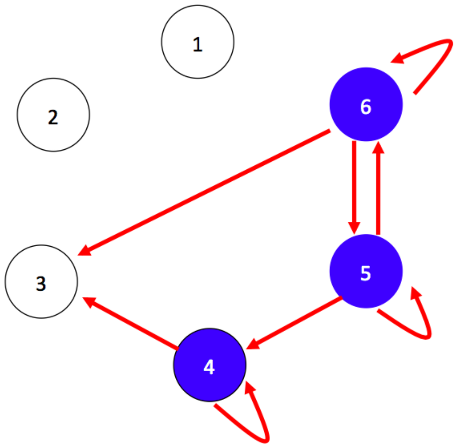
a. The places where the ambulances locate that can cover the maximum people considering the travel time in a zone

We programme and gain the following conclusion. ('Program solution_ initial' is in the appendix; input $A_k = A_5$)

	The numbers of the ambulances which cover the zone						The location of the ambulances			The largest no. of people covered (thousand)
No.	**1**	**2**	**3**	**4**	**5**	**6**				
1	1	1	1	0	0	1	1	2	3	180
2	1	1	1	1	0	0	1	2	4	215
3	1	1	0	1	1	1	1	2	5	240
4	1	1	1	0	1	1	1	2	6	215
5	1	0	2	1	0	1	1	3	4	185
6	1	0	1	1	1	2	1	3	5	210
7	1	0	2	0	1	2	1	3	6	185
8	1	0	1	2	1	1	1	4	5	245
9	1	0	2	1	1	1	1	4	6	220
10	1	0	1	1	2	2	1	5	6	245
11	0	1	2	1	0	1	2	3	4	215
12	0	1	1	1	1	2	2	3	5	240
13	0	1	2	0	1	2	2	3	6	215

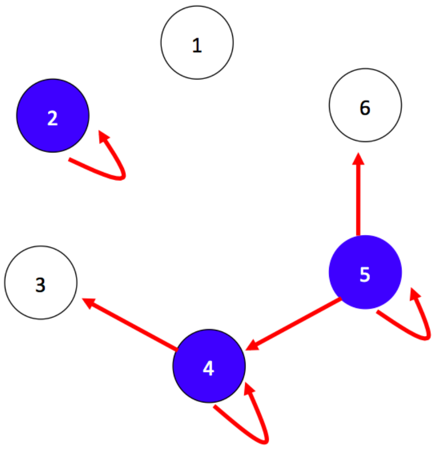
14	0	1	1	2	1	1	2	4	5	275
15	0	1	2	1	1	1	2	4	6	250
16	0	1	1	1	2	2	2	5	6	275
17	0	0	2	2	1	2	3	4	5	245
18	0	0	3	1	1	2	3	4	6	220
19	0	0	2	1	2	3	3	5	6	245
20	0	0	2	2	2	2	4	5	6	280

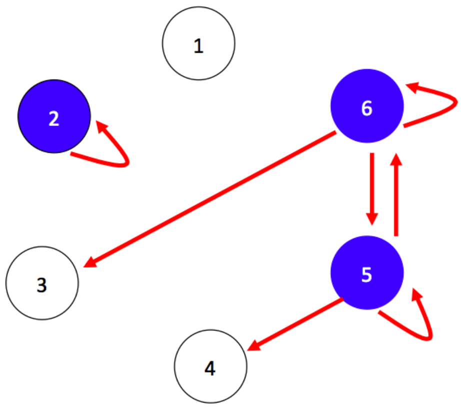
We can get the best solution that when the 3 ambulances are located in **Zone 4, Zone 5 and Zone 6**, the people covered maximize. And there are **280,000 people covered.**
When there's a call, the 3 ambulances should go like what the following picture shows.



b. The places where the ambulances locate that cover most zones

We programme and gain the following conclusion. ('Program solution_ initial' is in the appendix; input $A_k = A_5$) The best solution is that when the 3 ambulances are located in **Zone 2, Zone 4, Zone 5** or **Zone 2, Zone 5, Zone 6**, the zones covered maximize. And there are **275,000 people covered.**
When there's a call, the 3 ambulances should go like what the following pictures shows.





4.2.2 Conclusion

We have turned a complex mathematical problem into a series of expression through a model. And the problem has been worked out. At the same time, we have worked out 2 situations: regardless of the travel time in a zone or considering the travel time in a zone. In the situations of ‘Regardless of the travel time in a zone’, we have further discussed the two solutions to cost least money and fuel or to cover maximum people. Also, in the situation of ‘Considering the travel time in a zone’, we have explored the two solutions to cover most zones or people and give out different results.

4.3 Model 2

We have designed an optimal emergency medical response model for the determinations of the locations of two ambulances.

4.3.1 Modeling

Under semi-perfect conditions and regardless of the travel time in a zone, we strive for

$$\max \sum_{k=1}^{n} y_k r_k$$

$$s.t.\begin{cases} y_k = \begin{cases} 0, & s_k = 0 \\ 1, & s_k \geq 1 \end{cases} \\ s_k = \sum_{i=1}^{m} s(v_{a_i} v_k) \\ v_{a_i} \in \{1,2,3,\cdots,n\} \ , i \in \{1,2,3,\cdots,m\} \\ s(v_{a_i}, v_k) \in A_3 \\ v_{a_1} \neq v_{a_2} \neq \cdots \neq v_{a_m} \\ r_k \in Pop, k \in \{1,2,3\cdots,n\} \end{cases}$$

It’s obvious that m equals 2 and n equals 6.

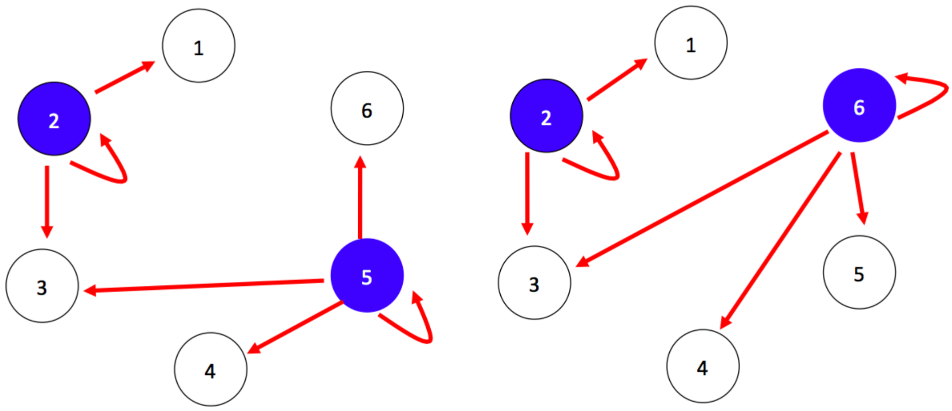
A. Locations regardless of the travel time in a zone

Following the model, we substitute for m, n and every possible x, y. And we programme and gain the following conclusion. (‘Program solution_cover’ is in the appendix; input A_k = A_3; input Pop = Pop)

a. The places where the ambulances locate that can cover the maximum people regardless of the travel time in a zone

We gain the best solution through ‘Program solution_cover’. (‘Program solution_cover’ is in the appendix; input A_k = A_3; input Pop = Pop) It is that when we locate the two ambulances in **Zone 2, Zone 5** or **Zone 2, Zone 6**, the number of people covered maximize. There are **300,000 people covered**.

When there’s a call, the 2 ambulances should go like what the following pictures shows.



b. The places where the ambulances locate that cost least money or fuel regardless of the travel time in a zone

If an ambulance costs least money and fuel, that ambulance spends least time traveling on the way.

Firstly, we use 6 minutes as a stand because the time of 6 minutes is the longest time besides that of 8 minutes on the way. We programme and gain the following conclusion. ('Program solution_cover' is in the appendix; input $A_k = A_6$; input $Pop = Pop$) It is that there exists **no** solution that cost less than that of 8 minutes.

B. Locations considering the travel time in a zone

We get A_4 in a situation that we consider the ambulance traveling in each zone. For example, if an ambulance travels from Zone 1 to Zone 6, we should consider then sum of time from Zone 1 to Zone 1, Zone 1 to Zone 6 and Zone 6 to Zone 6. Here is the new expression.

$$\max \sum_{k=1}^{n} z_k r_k$$

$$s.t.\begin{cases} z_k = \begin{cases} 0, \phi_k = 0 \\ 1, \phi_k \ge 1 \end{cases} \\ \phi_k = \sum_{i=1}^{\mathrm{m}} \phi(v_{a_i} v_k) \\ v_{a_i} \in \{1,2,3,\cdots,\mathrm{n}\} \quad , \quad i \in \{1,2,3,\cdots,\mathrm{m}\} \\ \phi(\mathrm{v}_{a_i}, \mathrm{v}_k) \in A_5 \\ v_{a_1} \ne v_{a_2} \ne \cdots \ne v_{a_\mathrm{m}} \\ r_k \in Pop, \quad k \in \{1,2,3,\cdots,\mathrm{n}\} \end{cases}$$

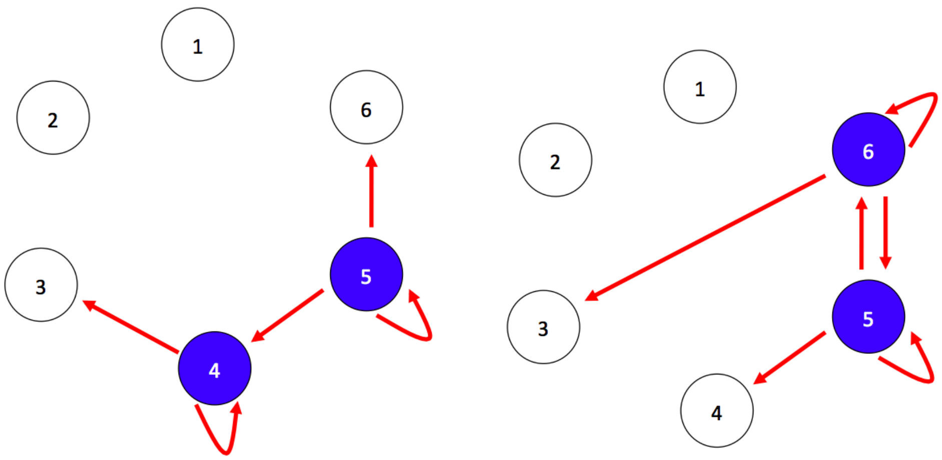
It's obvious that m equals 2 and n equals 6. We use 6 minutes as a stand and mark the number larger than '8' as '0' and the rest as '1' so that '1' means that the ambulance can reach the place within 8 minutes. After we simplify the matrix, we get A_5. We programme through A_5 and gain the following conclusion. ('Program solution_cover' is in the appendix; input $A_k = A_5$; input $Pop = Pop$) It is that we **cannot** cover every zone by 2 ambulances.

a. The places where the two ambulances locate that can cover the maximum people considering the travel time in a zone

We programme and gain the following conclusion. ('Program solution_cover' is in the appendix; input $A_k = A_5$; input $Pop = Pop$)

No.	The numbers of the ambulances which cover the zone: 1	2	3	4	5	6	The location of the ambulances		The largest no. of people covered (thousand)
1	1	1	0	0	0	0	1	2	130
2	1	0	1	0	0	1	1	3	100
3	1	0	1	1	0	0	1	4	135
4	1	0	0	1	1	1	1	5	160
5	1	0	1	0	1	1	1	6	135
6	0	1	1	0	0	1	2	3	130
7	0	1	1	1	0	0	2	4	165
8	0	1	0	1	1	1	2	5	190
9	0	1	1	0	1	1	2	6	165
10	0	0	2	1	0	1	3	4	135
11	0	0	1	1	1	2	3	5	160
12	0	0	2	0	1	2	3	6	135
13	0	0	1	2	1	1	4	5	195
14	0	0	2	1	1	1	4	6	170
15	0	0	1	1	2	2	5	6	195

The best solution is that when the two ambulances are located in **Zone 4, Zone 5** or **Zone 5, Zone 6**, the number of people covered maximize. There are **195,000 people covered**. When there's a call, the 2 ambulances should go like what the following pictures shows.

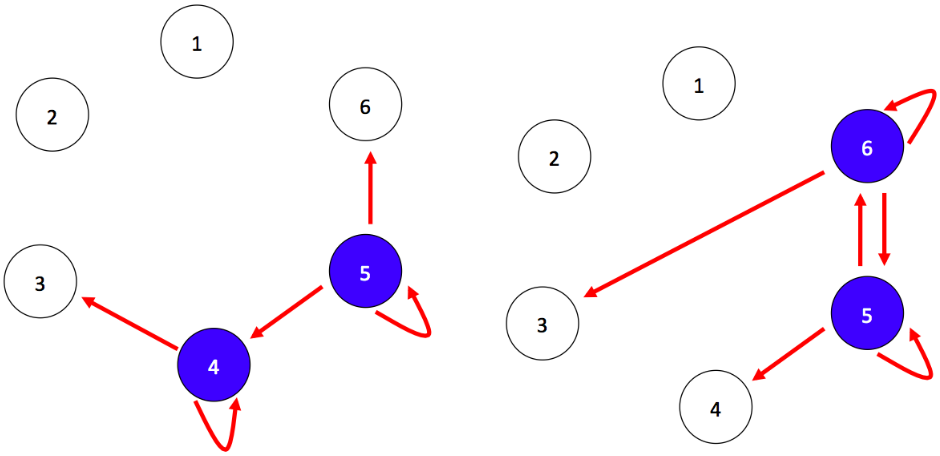


b. The places where the ambulances locate that cover most zones

We programme and gain the following conclusion. ('Program solution_cover' is in the

appendix; input A_k = A_5; input *Pop* = Pop)

The best solution is that when the two ambulances are located in **Zone 4, Zone 5** or **Zone 5, Zone 6**, the number of people covered maximize. There are **195,000 people covered**. When there's a call, the 2 ambulances should go like what the following pictures shows.



4.3.2 Conclusions

Our team further discussed the model how we determine the location of *m* ambulances in an area which is divided into *n* zones. At the same time, we have worked out 2 situations: regardless of the travel time in a zone or considering the travel time in a zone. In the situations of 'Regardless of the travel time in a zone', we have further discussed the two solutions to cost least money and fuel or to cover maximum people. Also, in the situation of 'Considering the travel time in a zone', we have explored the two solutions to cover most zones or people and give out different results.

4.4 Model 3

We have designed an optimal emergency medical response model for the determinations of the locations of an ambulance.

4.4.1 Modeling

Under semi-perfect conditions and regardless of the travel time in a zone, we strive for

$$\max \sum_{k=1}^{n} y_k r_k$$

$$s.t.\begin{cases} y_k = \begin{cases} 0, & s_k = 0 \\ 1, & s_k \geq 1 \end{cases} \\ s_k = \sum_{i=1}^{m} s(v_{a_i} v_k) \\ v_{a_i} \in \{1, 2, 3, \cdots, n\} \ , i \in \{1, 2, 3, \cdots, m\} \\ s(v_{a_i}, v_k) \in A_3 \\ v_{a_1} \neq v_{a_2} \neq \cdots \neq v_{a_m} \\ r_k \in Pop, k \in \{1, 2, 3 \cdots, n\} \end{cases}$$

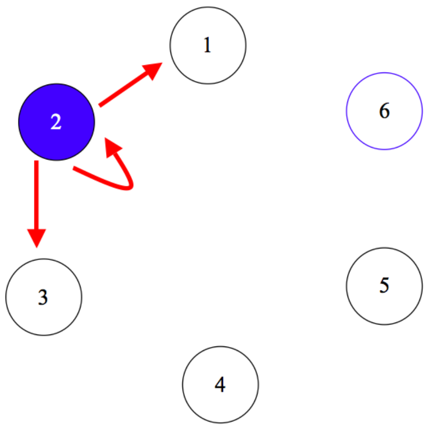
It's obvious that m equals 1 and n equals 6.

A. Locations regardless of the travel time in a zone

Following the model, we substitute for every possible x. And we programme and gain the following conclusion. ('Program solution_cover' is in the appendix; input A_k = A_3; input Pop = Pop)

a. The places where the ambulance locate that can cover the maximum people regardless of the travel time in a zone

We gain the best solution through 'Program solution_cover'. ('Program solution_cover' is in the appendix; input A_k = A_3; input Pop = Pop) It is that when we locate the ambulance in **Zone 2**, the number of people covered maximize. There are **160,000 people covered**. When there's a call, the ambulance should go like what the following picture shows.

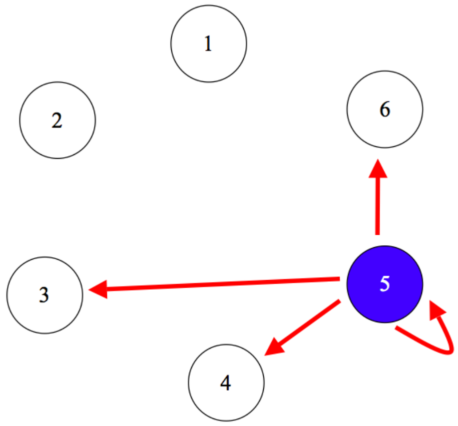


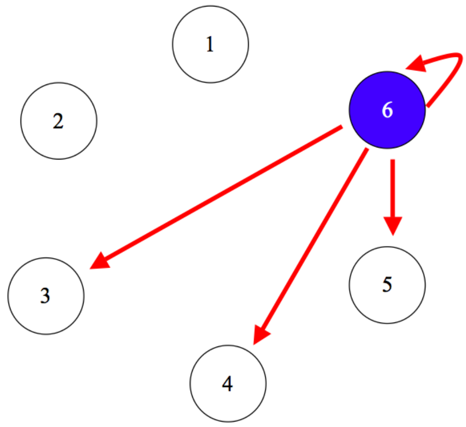
b. The places where the ambulances locate that cost least money or fuel regardless of the travel time in a zone

We programme and gain the following conclusion. ('Program solution_cover' is in the

appendix; input A_k = A_3; input *Pop* = *Pop*) It is that when we locate the ambulance in **Zone 5** or **Zone 6**, the number of people covered maximize. There are **140,000 people covered**.

When there's a call, the ambulance should go like what the following pictures shows.





B. Locations considering the travel time in a zone

We get A_4 in a situation that we consider the ambulance traveling in each zone. For example, if an ambulance travels from Zone 1 to Zone 6, we should consider then sum of time from Zone 1 to Zone 1, Zone 1 to Zone 6 and Zone 6 to Zone 6. It's obvious that m equals 1 and n equals 6. Here is the new expression.

$$\max \sum_{k=1}^{n} z_k r_k$$

$$s.t.\begin{cases} z_k = \begin{cases} 0, \phi_k = 0 \\ 1, \phi_k \geq 1 \end{cases} \\ \phi_k = \sum_{i=1}^{m} \phi(v_{a_i} v_k) \\ v_{a_i} \in \{1,2,3,\cdots,n\} \ , \ i \in \{1,2,3,\cdots,m\} \\ \phi(v_{a_i}, v_k) \in A_5 \\ v_{a_1} \neq v_{a_2} \neq \cdots \neq v_{a_m} \\ r_k \in Pop, \ \ k \in \{1,2,3,\cdots,n\} \end{cases}$$

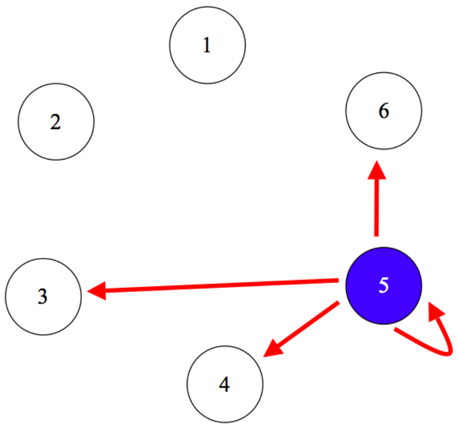
We use 6 minutes as a stand and mark the number larger than '8' as '0' and the rest as '1' so that '1' means that the ambulance can reach the place within 8 minutes. After we simplify the matrix, we get A_5. We programme through A_5 and gain the following conclusion. ('Program solution_cover' is in the appendix; input A_k = A_5; input *Pop* = *Pop*)

It is that we cannot cover every zone by 1 ambulance.

a. The places where the two ambulances locate that can cover the maximum people considering the travel time in a zone

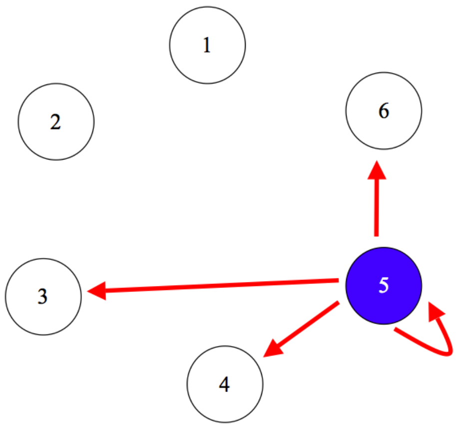
We programme and gain the following conclusion. ('Program solution_cover' is in the

appendix; input A_k = A_5; input *Pop* = Pop) It is that when we locate the ambulance in **Zone 5**, the number of people covered maximize. There are **110,000 people covered**. When there's a call, the ambulance should go like what the following picture shows.



b. The places where the ambulance locate that cover most zones

We programme and gain the following conclusion. ('Program solution_cover' is in the appendix; input A_k = A_5; input *Pop* = Pop) It is that when we locate the ambulance in **Zone 5**, the number of people covered maximize. There are **110,000 people covered**. When there's a call, the ambulance should go like what the following picture shows.



4.4.2 Conclusion

Just like model 2, we further discussed the model how we determine the location of *m* ambulances in an area which is divided into *n* zones. At the same time, we have worked out 2 situations: regardless of the travel time in a zone or considering the travel time in a zone. In the situations of 'Regardless of the travel time in a zone', we have further discussed the two solutions to cost least money and fuel or to cover maximum people. Also, in the situation of 'Considering the travel time in a zone', we have explored the two solutions to cover most zones or people and give out different results.

4.5 Testing for the model

We prove that our model is correct via some examples.

4.5.1 Variables

Variables	Description
A_{10}	A matrix which describes the average times among the 8 famous location
A_{11}	A matrix with simplified information based on A_{11} which marks the number larger than '10' as '0' and the rest as '1' so that '1' means that the ambulances can reach the place within 10 minutes

4.5.2 Example

Shanghai is an international metropolis. We choose 8 famous location from the center of Shanghai and estimate the average time from one location to another. And we pick 3 suitable starting point where the 3 ambulances are located.

Dim Zone 1 as Xu Jia Hui

Dim Zone 2 as The Bund

Dim Zone 3 as West Shanghai Station

Dim Zone 4 as Jing'an Temple

Dim Zone 5 as Hongkou stadium

Dim Zone 6 as Shanghai Circus Stage

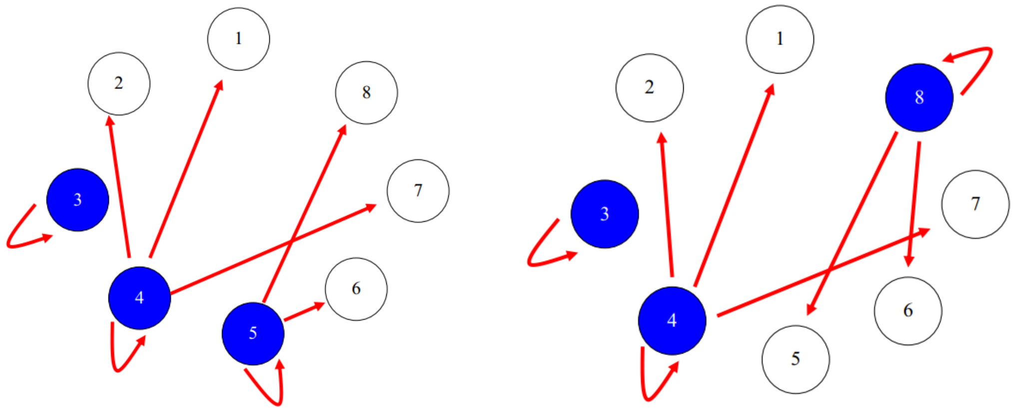
Dim Zone 7 as Zhongshan Park

Dim Zone 8 as Wujiao Court

$$A_{10}=\begin{pmatrix} 1 & 40 & 17 & 9 & 22 & 20 & 11 & 26 \\ 40 & 4 & 18 & 8 & 13 & 13 & 13 & 16 \\ 17 & 18 & 0.5 & 12 & 18 & 12 & 11 & 19 \\ 9 & 8 & 12 & 1 & 15 & 12 & 8 & 18 \\ 22 & 13 & 18 & 15 & 2 & 9 & 21 & 9 \\ 20 & 13 & 12 & 12 & 9 & 1 & 16 & 10 \\ 11 & 13 & 11 & 8 & 21 & 16 & 2 & 21 \\ 26 & 16 & 19 & 18 & 9 & 10 & 21 & 4 \end{pmatrix} A_{11}=\begin{pmatrix} 1 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 1 & 1 & 0 & 1 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 & 1 & 0 & 1 \\ 0 & 0 & 0 & 0 & 1 & 1 & 0 & 1 \\ 0 & 0 & 0 & 1 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 & 1 & 0 & 1 \end{pmatrix}$$

Through programming we can draw the conclusion. ('Program solution_cover' is in the appendix; input A_k = A_{11}, input $Pop=(1085 \quad 300 \quad 1130 \quad 349 \quad 792 \quad 707 \quad 690 \quad 1313)$)

We get the result that when the 3 ambulances are located at Zone 3, Zone 4, Zone 5 or Zone 3, Zone 4, Zone 8, all the zones can be covered. So, our model can be used in the real situation and has a good feasibility.



5. The Solutions under Catastrophic Event

Considering that a serious disaster happen, a lot of unexpected situations appear. The roads may be destroyed so that ambulances would be delayed. The numbers of population of each region are different, so there will be weights among regions in terms of the regions' importance. So there are a lot of things we cannot predict. Considering these things, we simulate the situation and improve our model.

5.1 Variables

Variables	Description
M	A matrix with each weight of each region in terms of the regions' importance
A_8	The Average travel time from one city/county to another
A_9	A matrix with simplified information based on A_8 which marks the number larger than '150' as '0' and the rest as '1' so that '1' means that the ambulances can reach the place within 150 minutes

5.2 Model

$$\max \sum_{k=1}^{n} m_k y_\mathrm{k} r_\mathrm{k}$$

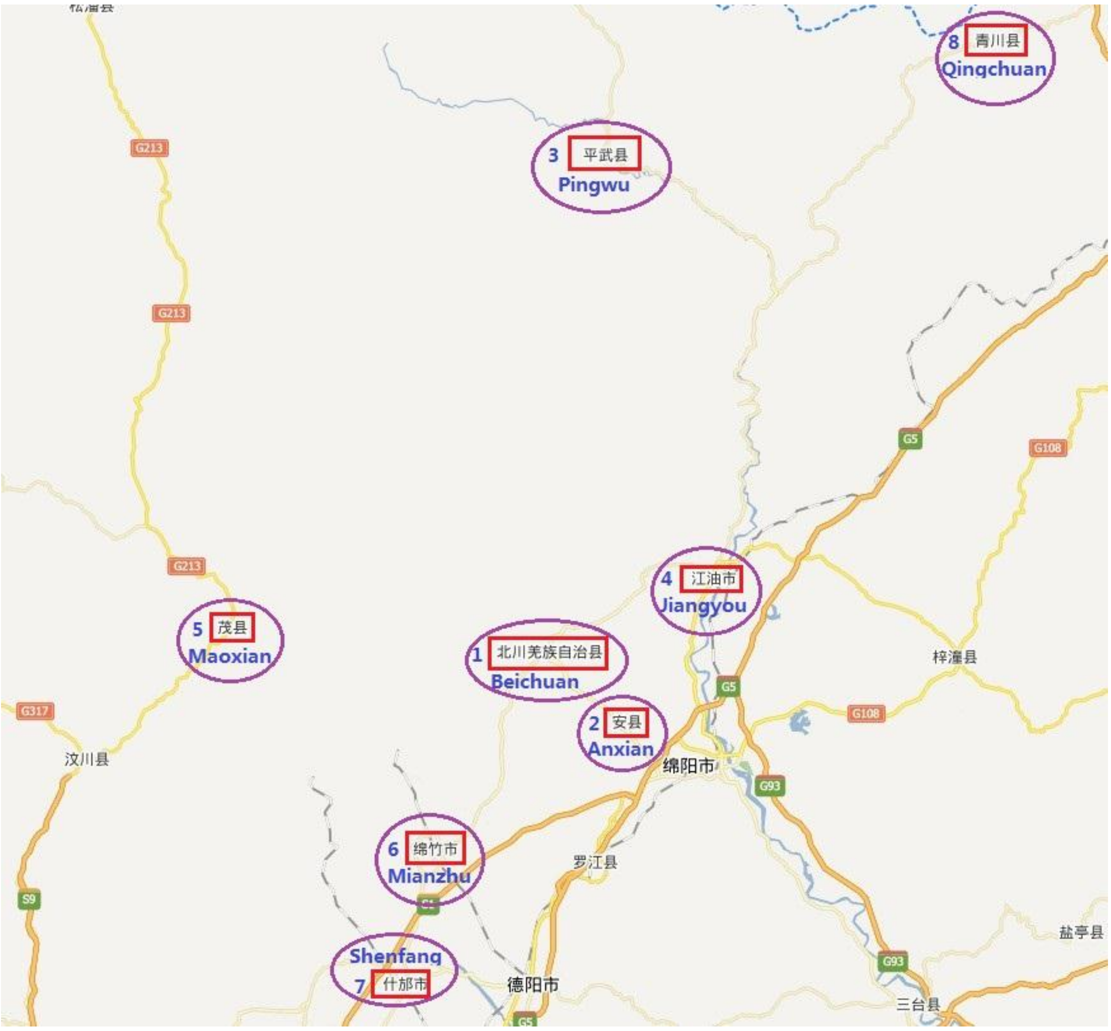
$$s.t.\begin{cases} y_k = \begin{cases} 0, & s_k = 0 \\ 1, & s_k \geq 1 \end{cases} \\ s_k = \sum_{i=1}^{\mathrm{m}} s(v_{a_\mathrm{i}} v_k) \\ v_{a_i} \in \{1,2,3,\cdots,\mathrm{n}\} \ \ , i \in \{1,2,3,\cdots,\mathrm{m}\} \\ s(\mathrm{v}_{a_i}, \mathrm{v}_k) \in A_8 \\ v_{a_1} \neq v_{a_2} \neq \cdots \neq v_{a_\mathrm{m}} \\ r_k \in B, \mathrm{m}_k \in \{\underline{\mathrm{m}}_k, \bar{\mathrm{m}}_k\}, k \in \{1,2,3\cdots,\mathrm{n}\} \end{cases}$$

$\underline{\mathrm{m}}_k$ means the min weight which is given out. $\bar{\mathrm{m}}_k$ means the max weight which is given out.

5.3 Example

To solve the forth question better, we turned the problem into a realistic example to analyze it. And we chose the earthquake which happened in Wenchuan, China 5 years ago.

Just like the process we solve Problem 1, 2, 3, we have to give out matrixes like A_2, Pop. So we chose 8 cities/counties randomly.



We got the population of each city/county and the average time from one place to another through the Internet. (The table of analyzing the data is in the appendix)We marked the routes where the time is over 4 hours as the route which had been damaged and we deleted these routes. And then we got the following matrix.

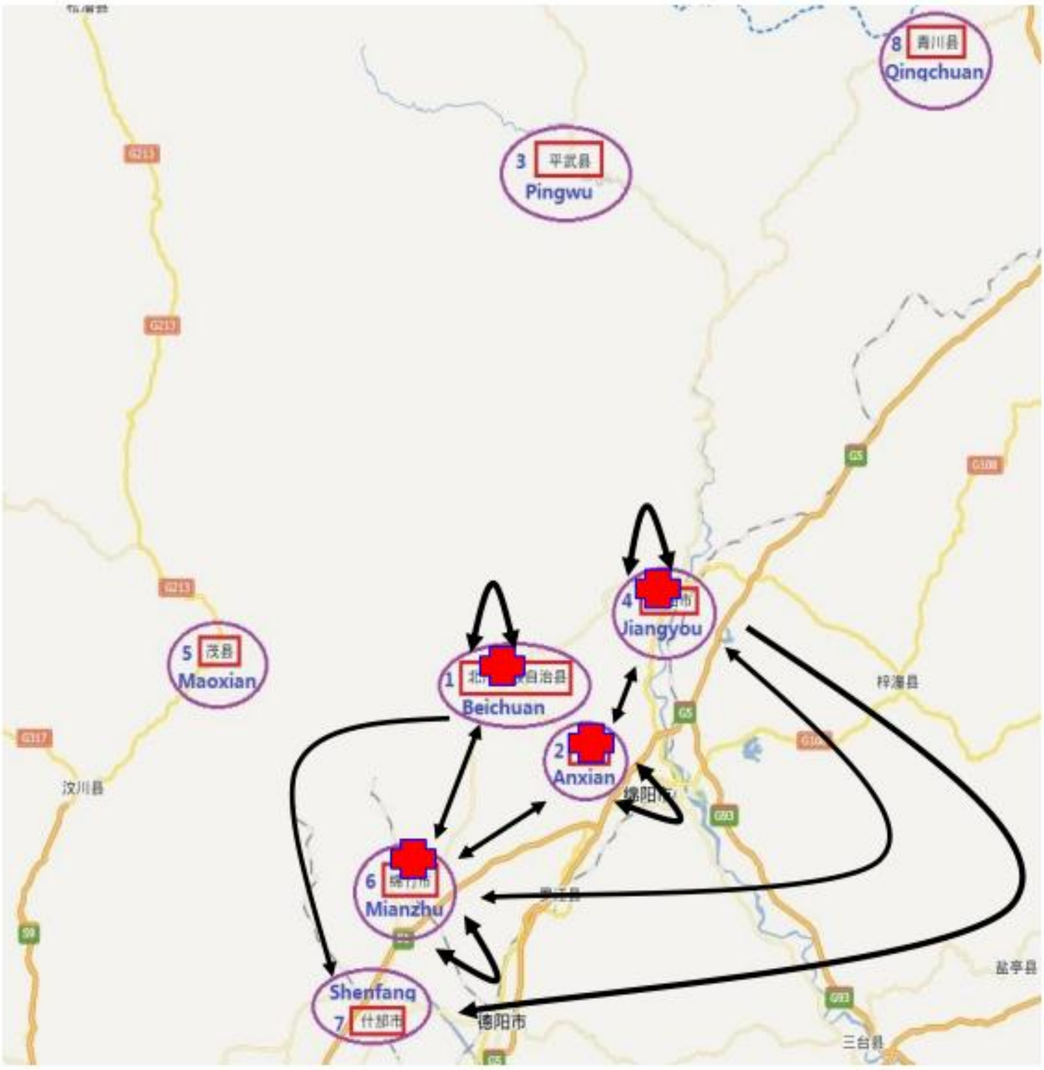
$$A_8 = \begin{pmatrix} 47 & \infty & \infty & \infty & 193 & 116 & 146 & \infty \\ \infty & 34 & \infty & 74 & \infty & 66 & 76 & 194 \\ \infty & \infty & 77 & \infty & \infty & \infty & \infty & 227 \\ \infty & 74 & \infty & 52 & \infty & 94 & 103 & 172 \\ 193 & \infty & \infty & \infty & 62 & \infty & 223 & \infty \\ 116 & 66 & \infty & 94 & \infty & 35 & \infty & 225 \\ 146 & 76 & \infty & 103 & 223 & \infty & 29 & \infty \\ \infty & 194 & 227 & 172 & \infty & 225 & \infty & 56 \end{pmatrix} \quad A_9 = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 1 & 1 & 0 \\ 0 & 1 & 0 & 1 & 0 & 1 & 1 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 1 & 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 1 & 1 & 0 & 1 & 0 & 1 & 0 & 0 \\ 1 & 1 & 0 & 1 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix}$$

To sum up, we got the two matrix and information just like those of question 1, 2, 3.

After that, we worked out the average injured people of the 8 regions. $The\ \ average\ \ injured\ \ people = \frac{All\ \ the\ \ injured\ \ people}{8}$. We made a program ('Program solution_disaster' is in the appendix; input $M_k = (1\ \ 1\ \ 2\ \ 1\ \ 1\ \ 2\ \ 2\ \ 1)$; input $A_k = A_9$; input $Pop = (161\ \ 500\ \ 186\ \ 310\ \ 90\ \ 520\ \ 430\ \ 250)$) Finally, we get the result that when the ambulances are mostly in Place 1, 2, 4, 6, the people covered maximize and the ambulances can reach the most serious stricken area.

But there exists disadvantages. First, there are less people in the most serious stricken area. Second, maybe some places cannot be reached because of the damage of the roads.

When there’s a call, the ambulances should go like what the following pictures shows.



6. Conclusion

We build a simple model to solve the first problem. In the process, we gradually develop the model and build our final model. Furthermore, we test our final model via the map of Shanghai and the results are perfect. To solve the forth problem, we test our model again via the map of Si Chuan Province and the simulation result is that most of the stricken areas can be covered, but some roads are damaged so that several places cannot be covered.

7. Appendix

a. solution_initial

In Matrix C, each row means each solution. The first six columns means the numbers of ambulances that cover this area. The last three columns of C means the location of the 3 ambulances. In Matrix D, each row means the sum number of the zones covered. In Matrix P, each row means the sum number of the people covered by each ambulance. And $\max\_P$ means the maximum of each element in P. Most importantly, every row of each matrix means a certain solution.

Here's the code:

```
function [C,D, Cp, p, max_p]= solution_initial(Ak)
clc;
points = 6;
selected = 3;
RowNum = nchoosek(points,selected);
ColNum = points + selected;
C=zeros(RowNum,ColNum);
D=zeros(RowNum,1);
Cp = zeros(RowNum,points);
p=zeros(RowNum,1);
for i=1:points
    m=1;
    for j=1:4
        for k=j+1:5
            for l=k+1:6
               C(m,i)= Ak(j,i)+Ak(k,i)+Ak(l,i);
                    C(m,7)=j;
                    C(m,8)=k;
                    C(m,9)=l;
               m=m+1;
            end
        end
    end
end
for i=1:RowNum
    for u =1:points
    if C(i,u)>0
        D(i,1)=D(i,1)+1;
    end
    end
end
    Cp(:,1)=C(:,1)*50;
    Cp(:,2)=C(:,2)*80;
```

```
    Cp(:,3)=C(:,3)*30;
    Cp(:,4)=C(:,4)*55;
    Cp(:,5)=C(:,5)*35;
    Cp(:,6)=C(:,6)*20;
    for i=1:20
        for j=1:6
            p(i)=p(i)+Cp(i,j);
        end
    end
p;
max_p = max(p);
```

b. solution_cover

In Matrix C, each row means each solution. In Matrix D, each row means the sum number of the zones covered. In Matrix S, each row means the location of the ambulances. In Matrix P, each row means the sum number of the people covered by each ambulance. And $\max\_P$ means the maximum of each element in P. Most importantly, every row of each matrix means a certain solution.

Here's the code:

```
function [C,D, Cp,S, p,max_p] = solution_cover(points,locations,Ak,Pop)
clc;
X=[1:points];
S = combntns(X,locations);
len = size(S);
RowNum = nchoosek(points,locations);
C=zeros(RowNum,points);
D=zeros(RowNum,1);
Cp = zeros(RowNum,1);
p = zeros(RowNum,1);
for i = 1: len(1)
    for j = 1:len(2)
        C(i,:) = C(i,:) + Ak(S(i,j),:);
    end
end
for i=1:RowNum
    for k =1:points
    if C(i,k)>0
        D(i,1)=D(i,1)+1;
    end
    end
end
for i = 1 : points
    Cp(:,i)=C(:,i)*Pop(i);
```

```
end


for i=1:RowNum
   for j=1:points
         p(i)=p(i)+Cp(i,j);
   end
end
max_p = max(p);
```

c. solution_disaster

In Matrix C, each row means each solution. In Matrix D, each row means the sum number of the zones covered. In Matrix S, each row means the location of the ambulances. In Matrix P, each row means the sum number of the people covered by each ambulance. And $\max\_P$ means the maximum of each element in P. Most importantly, every row of each matrix means a certain solution.

Here's the code:

```
function [C,D, Cp,S, p,max_p] = solution_disaster(points,locations,Ak,Pop,Mk)
clc;
X=[1:points];
S = combntns(X,locations);
len = size(S);
RowNum = nchoosek(points,locations);
C=zeros(RowNum,points);
D=zeros(RowNum,1);
Cp = zeros(RowNum,1);
p = zeros(RowNum,1);
for i = 1: len(1)
   for j = 1:len(2)
      C(i,:) = C(i,:) + Ak(S(i,j),:);
   end
end
for i=1:RowNum
   for k =1:points
   if C(i,k)>0
      D(i,1)=D(i,1)+1;
   end
   end
end
for i = 1 : points
   Cp(:,i)=C(:,i)*Pop(i)*Mk(i);
end

for i=1:RowNum
```

```
    for j=1:points
            p(i)=p(i)+Cp(i,j);
    end
end
max_p = max(p);
```

d. The table of the data

Sichuan						
City	death number	injured number	total population	rate	death and injured total number	
1Beichuan	8605	9693	161000	11.37%	18298	
2Anxian	1571	13476	500000	3.01%	15047	
3Pingwu	1546	32145	186073	18.11%	33691	
4Jiangyou	394	10006	310000	3.35%	10400	
5Maoxian	3122	8183	90956	12.43%	11305	
6Mianzhu	11101	36822	520000	9.22%	47923	
7Shenfang	5924	31990	430000	8.82%	37914	
8Qingchuan	4663	15390	250000	8.02%	20053	
			2448029			
total number	36926	157705	278484	11.38%	194631	24328.88

MEMORANDUM

From: the Ambulance Coordinating Interest Group

To: ESC

Date: 17/11/2013

Subject: Our recommendations from our model and analysis finding

As members of an ambulance coordinating interest group, we are interested in your coordination of the medical ambulances.

Our team develops a simple model to place the three ambulances at the most efficient locations. And we work out the best solutions to locate the 3 ambulances under semi-perfect conditions regardless of the cost times in a zone but considering the cost of travel times in a zone.

Then, we further discuss the model on how we determine the location of *m* ambulances in an area which is divided into *n* zones, where *m* and *n* stand for not only 3 ambulances and 6 zones but any integer. So we substitute *m*, *n* for our new model.

Although the three ambulances might be enough to cover the whole county, but we do recommend that you increase the number of ambulances because of the uncertain factors that might affect different situations. Therefore, we developed this model to help you decide how to put more of ambulances in the county.

Based on the development of technology, we suggest that you can improve the efficiency of the engines in your ambulances to shorten the time of rescuing.

We also find that when natural disasters occur, the three ambulances will not be able to save most people in time. If the only way between the two counties collapsed in such disasters, more time will be wasted for making a detour or waiting for the cleaning of the obstacles on the road. Therefore, we recommend that you should leave at least three medical helicopters for saving people who live far away from the center of the county. Earthwork cars are also important for cleaning the obstacles on the road which landslides.

Your consideration of this suggestion would be highly appreciated.

We are looking forward to seeing more people saved in disasters and difficult situations in the county.