The CENTRE for EDUCATION
in MATHEMATICS and COMPUTING
cemc.uwaterloo.ca

Fermat Contest

(Grade 11)

Tuesday, February 27, 2018
(in North America and South America)

Wednesday, February 28, 2018
(outside of North America and South America)

UNIVERSITY OF WATERLOO

Time: 60 minutes

Calculating devices are allowed, provided that they do not have any of the following features: (i) internet access, (ii) the ability to communicate with other devices, (iii) previously stored information such as formulas, programs, notes, etc., (iv) a computer algebra system, (v) dynamic geometry software.

Instructions

1. Do not open the Contest booklet until you are told to do so.
2. You may use rulers, compasses and paper for rough work.
3. Be sure that you understand the coding system for your response form. If you are not sure, ask your teacher to clarify it. All coding must be done with a pencil, preferably HB. Fill in circles completely.
4. On your response form, print your school name and city/town in the box in the upper right corner.
5. **Be certain that you code your name, age, grade, and the Contest you are writing in the response form. Only those who do so can be counted as eligible students.**
6. This is a multiple-choice test. Each question is followed by five possible answers marked **A**, **B**, **C**, **D**, and **E**. Only one of these is correct. After making your choice, fill in the appropriate circle on the response form.
7. Scoring: Each correct answer is worth 5 in Part A, 6 in Part B, and 8 in Part C.
   There is *no penalty* for an incorrect answer.
   Each unanswered question is worth 2, to a maximum of 10 unanswered questions.
8. Diagrams are *not* drawn to scale. They are intended as aids only.
9. When your supervisor tells you to begin, you will have *sixty* minutes of working time.
10. You may not write more than one of the Pascal, Cayley and Fermat Contests in any given year.

Do not discuss the problems or solutions from this contest online for the next 48 hours.

The name, grade, school and location, and score range of some top-scoring students will be published on our website, cemc.uwaterloo.ca. In addition, the name, grade, school and location, and score of some top-scoring students may be shared with other mathematical organizations for other recognition opportunities.

Scoring: There is *no penalty* for an incorrect answer.
Each unanswered question is worth 2, to a maximum of 10 unanswered questions.

Part A: Each correct answer is worth 5.

1. The value of $2016 - 2017 + 2018 - 2019 + 2020$ is

   (A) 2012 **(B)** 2014 **(C)** 2016 **(D)** 2018 **(E)** 2020

2. On Monday, the minimum temperature in Fermatville was $-11°$C and the maximum temperature was 14°C. What was the range of temperatures on Monday in Fermatville?

   (A) 3°C **(B)** 25°C **(C)** 14°C **(D)** 11°C **(E)** 23°C

3. If $x = -2$ and $y = -1$, the value of $(3x + 2y) - (3x - 2y)$ is

   (A) -4 **(B)** 12 **(C)** 0 **(D)** 4 **(E)** 8

4. How many integers are greater than $\frac{5}{7}$ and less than $\frac{28}{3}$?

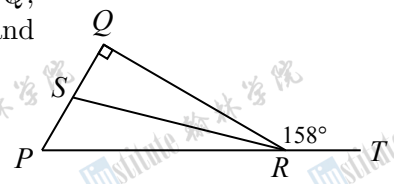
   (A) 1 **(B)** 9 **(C)** 5 **(D)** 7 **(E)** 3

5. The symbols $\heartsuit$ and ∇ represent different positive integers less than 20. If $\heartsuit \times \heartsuit \times \heartsuit = \nabla$, what is the value of $\nabla \times \nabla$?

   (A) 12 **(B)** 16 **(C)** 36 **(D)** 64 **(E)** 81

6. In the diagram, points R and S lie on PT and PQ, respectively. If $\angle PQR = 90°$, $\angle QRT = 158°$, and $\angle PRS = \angle QRS$, what is the measure of $\angle QSR$?

   (A) 34° **(B)** 22° **(C)** 68°
   (D) 11° **(E)** 79°



7. Bev is driving from Waterloo, ON to Marathon, ON. She has driven 312 km. She has 858 km still to drive. How much farther must she drive in order to be halfway from Waterloo to Marathon?

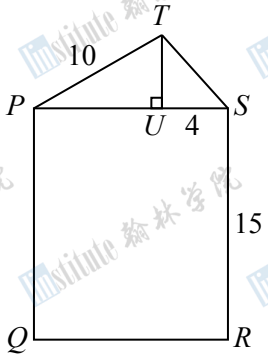
   (A) 585 km **(B)** 273 km **(C)** 312 km **(D)** 429 km **(E)** 196.5 km

8. For what value of k is the line through the points $(3, 2k + 1)$ and $(8, 4k - 5)$ parallel to the x-axis?

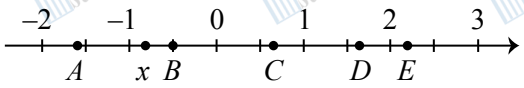
   (A) 3 **(B)** -4 **(C)** 2 **(D)** 0 **(E)** -1

9. In the diagram, $PQRS$ is a rectangle with $SR = 15$. Point T is above PS and point U is on PS so that TU is perpendicular to PS. If $PT = 10$ and $US = 4$ and the area of $PQRS$ is 180, what is the area of $\triangle PTS$?

   (A) 60 **(B)** 36 **(C)** 48
   (D) 24 **(E)** 12



10. In the diagram, the number line between -2 and 3 is divided into 10 equal parts. The integers -1, 0, 1, 2 are marked on the line as are the numbers A, x, B, C, D, E. Which number best approximates the value of x^2?



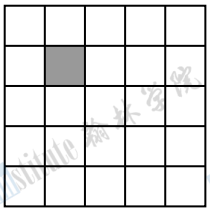
(A) A **(B)** B **(C)** C **(D)** D **(E)** E

Part B: Each correct answer is worth 6.

11. A bag contains 8 red balls, a number of white balls, and no other balls. If $\frac{5}{6}$ of the balls in the bag are white, then the number of white balls in the bag is

(A) 48 **(B)** 20 **(C)** 40 **(D)** 32 **(E)** 30

12. In the given 5×5 grid, many squares can be formed using the grid lines. How many of these squares contain the shaded 1×1 square?



(A) 15 **(B)** 16 **(C)** 11 **(D)** 12 **(E)** 14

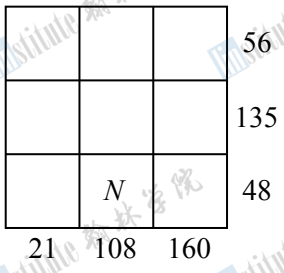
13. A digital clock shows the time 4:56. How many minutes will pass until the clock next shows a time in which all of the digits are consecutive and are in increasing order?

(A) 458 **(B)** 587 **(C)** 376 **(D)** 315 **(E)** 518

14. The line with equation $y = x$ is translated 3 units to the right and 2 units down. What is the y-intercept of the resulting line?

(A) -1 **(B)** -2 **(C)** -5 **(D)** 3 **(E)** 4

15. Francesca put the integers $1, 2, 3, 4, 5, 6, 7, 8, 9$ in the nine squares in the grid. She put one integer in each square and used no integer twice. She calculated the product of the three integers in each row and wrote the products to the right of the corresponding rows. She calculated the product of the integers in each column and wrote the products below the corresponding columns. Finally, she erased the integers from the nine squares. Which integer was in the square marked N?

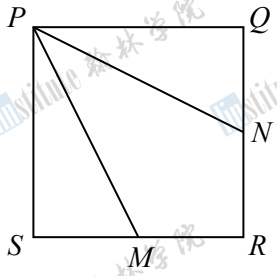


(A) 3 **(B)** 8 **(C)** 9 **(D)** 6 **(E)** 4

16. Points P and Q are two distinct points in the xy-plane. In how many different places in the xy-plane can a third point, R, be placed so that $PQ = QR = PR$?

(A) 6 **(B)** 1 **(C)** 2 **(D)** 3 **(E)** 4

17. In the diagram, square $PQRS$ has side length 2. Points M and N are the midpoints of SR and RQ, respectively. The value of $\cos(\angle MPN)$ is



(A) $\frac{4}{5}$ **(B)** $\frac{\sqrt{2}}{2}$ **(C)** $\frac{\sqrt{5}}{3}$
(D) $\frac{1}{3}$ **(E)** $\frac{\sqrt{3}}{2}$

18. Suppose that m and n are positive integers with $\sqrt{7+\sqrt{48}} = m + \sqrt{n}$. The value of $m^2 + n^2$ is

(A) 37 **(B)** 25 **(C)** 58 **(D)** 29 **(E)** 13

19. Radford and Peter ran a race, during which they both ran at a constant speed. Radford began the race 30 m ahead of Peter. After 3 minutes, Peter was 18 m ahead of Radford. Peter won the race exactly 7 minutes after it began. How far from the finish line was Radford when Peter won?

(A) 16 m **(B)** 64 m **(C)** 48 m **(D)** 82 m **(E)** 84 m

20. For how many positive integers x is $(x-2)(x-4)(x-6)\cdots(x-2016)(x-2018) \leq 0$? (The product on the left side of the inequality consists of 1009 factors of the form $x - 2k$ for integers k with $1 \leq k \leq 1009$.)

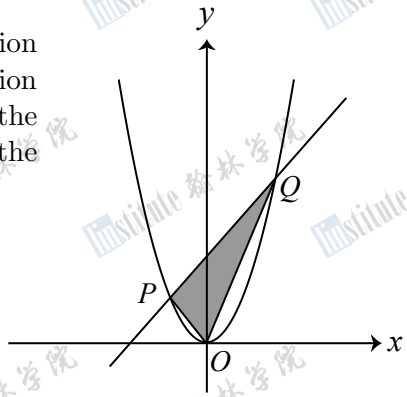
(A) 1009 **(B)** 1010 **(C)** 1514 **(D)** 1515 **(E)** 1513

Part C: Each correct answer is worth 8.

21. A sequence has terms $a_1, a_2, a_3, \ldots$. The first term is $a_1 = x$ and the third term is $a_3 = y$. The terms of the sequence have the property that every term after the first term is equal to 1 less than the sum of the terms immediately before and after it. That is, when $n \geq 1$, $a_{n+1} = a_n + a_{n+2} - 1$. The sum of the first 2018 terms in the sequence is

(A) $-x - 2y + 2023$ **(B)** $3x - 2y + 2017$ **(C)** y
(D) $x + y - 1$ **(E)** $2x + y + 2015$

22. Suppose that $k > 0$ and that the line with equation $y = 3kx + 4k^2$ intersects the parabola with equation $y = x^2$ at points P and Q, as shown. If O is the origin and the area of $\triangle OPQ$ is 80, then the slope of the line is



(A) 4 **(B)** 3 **(C)** $\frac{15}{4}$
(D) 6 **(E)** $\frac{21}{4}$

23. Suppose that a, b and c are integers with $(x-a)(x-6)+3 = (x+b)(x+c)$ for all real numbers x. The sum of all possible values of b is

(A) -12 **(B)** -24 **(C)** -14 **(D)** -8 **(E)** -16

24. Wayne has 3 green buckets, 3 red buckets, 3 blue buckets, and 3 yellow buckets. He randomly distributes 4 hockey pucks among the green buckets, with each puck equally likely to be put in each bucket. Similarly, he distributes 3 pucks among the red buckets, 2 pucks among the blue buckets, and 1 puck among the yellow buckets. Once he is finished, what is the probability that a green bucket contains more pucks than each of the other 11 buckets?

(A) $\frac{97}{243}$ **(B)** $\frac{89}{243}$ **(C)** $\frac{93}{243}$ **(D)** $\frac{95}{243}$ **(E)** $\frac{91}{243}$

25. For each positive digit D and positive integer k, we use the symbol $D_{(k)}$ to represent the positive integer having exactly k digits, each of which is equal to D. For example, $2_{(1)} = 2$ and $3_{(4)} = 3333$. There are N quadruples (P, Q, R, k) with P, Q and R positive digits, k a positive integer with $k \leq 2018$, and $P_{(2k)} - Q_{(k)} = \left(R_{(k)}\right)^2$. The sum of the digits of N is

(A) 10 **(B)** 9 **(C)** 11 **(D)** 12 **(E)** 13

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For students...

Thank you for writing the 2018 Fermat Contest! Each year, more than 240 000 students from more than 75 countries register to write the CEMC's Contests.

Encourage your teacher to register you for the Hypatia Contest which will be written in April.

Visit our website `cemc.uwaterloo.ca` to find

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- Free copies of past contests
- Math Circles videos and handouts that will help you learn more mathematics and prepare for future contests
- Information about careers in and applications of mathematics and computer science

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