



The CENTRE for EDUCATION
in MATHEMATICS and COMPUTING
cemc.uwaterloo.ca

Cayley Contest

(Grade 10)

Tuesday, February 27, 2018
(in North America and South America)

Wednesday, February 28, 2018
(outside of North America and South America)



UNIVERSITY OF
WATERLOO

Time: 60 minutes

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Calculating devices are allowed, provided that they do not have any of the following features: (i) internet access, (ii) the ability to communicate with other devices, (iii) previously stored information such as formulas, programs, notes, etc., (iv) a computer algebra system, (v) dynamic geometry software.

Instructions

1. Do not open the Contest booklet until you are told to do so.
2. You may use rulers, compasses and paper for rough work.
3. Be sure that you understand the coding system for your response form. If you are not sure, ask your teacher to clarify it. All coding must be done with a pencil, preferably HB. Fill in circles completely.
4. On your response form, print your school name and city/town in the box in the upper right corner.
5. **Be certain that you code your name, age, grade, and the Contest you are writing in the response form. Only those who do so can be counted as eligible students.**
6. This is a multiple-choice test. Each question is followed by five possible answers marked **A, B, C, D, and E**. Only one of these is correct. After making your choice, fill in the appropriate circle on the response form.
7. Scoring: Each correct answer is worth 5 in Part A, 6 in Part B, and 8 in Part C.
There is *no penalty* for an incorrect answer.
Each unanswered question is worth 2, to a maximum of 10 unanswered questions.
8. Diagrams are *not* drawn to scale. They are intended as aids only.
9. When your supervisor tells you to begin, you will have *sixty* minutes of working time.
10. You may not write more than one of the Pascal, Cayley and Fermat Contests in any given year.

Do not discuss the problems or solutions from this contest online for the next 48 hours.

The name, grade, school and location, and score range of some top-scoring students will be published on our website, cemc.uwaterloo.ca. In addition, the name, grade, school and location, and score of some top-scoring students may be shared with other mathematical organizations for other recognition opportunities.

Scoring: There is *no penalty* for an incorrect answer.

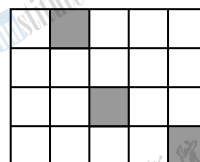
Each unanswered question is worth 2, to a maximum of 10 unanswered questions.

Part A: Each correct answer is worth 5.

1. If $3 \times n = 6 \times 2$, then n equals

(A) 6 (B) 2 (C) 9 (D) 5 (E) 4

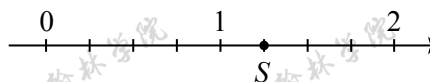
2. In the diagram, 3 of the 1×1 squares that make up the 4×5 grid are shaded. How many additional 1×1 squares need to be shaded so that one-half of all of the 1×1 squares are shaded?



(A) 5 (B) 9 (C) 7
(D) 6 (E) 8

3. In the diagram, the number line between 0 and 2 is divided into 8 equal parts. The numbers 1 and S are marked on the line. What is the value of S ?

(A) 1.1 (B) 0.75 (C) 1.2
(D) 1.25 (E) 1.15

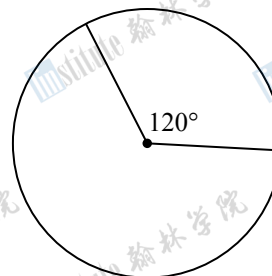


4. Which of the following is equal to 9^4 ?

(A) 3^2 (B) 3^4 (C) 3^6 (D) 3^8 (E) 3^{10}

5. In the diagram, a sector of a circle has central angle 120° . The area of the whole circle is 9π . What is the area of this sector?

(A) 2π (B) 3π (C) 4π
(D) 6π (E) $\frac{9}{2}\pi$



6. If $x = 2018$, then the expression $x^2 + 2x - x(x + 1)$ equals

(A) -2018 (B) 2018 (C) 10090 (D) -10090 (E) 4039

7. At 8:00 a.m., there were 24 cars in a parking lot.
At 9:00 a.m., there were 48 cars in the same parking lot.
What is the percentage increase in number of cars in the parking lot between 8:00 a.m. and 9:00 a.m.?

(A) 20% (B) 48% (C) 72% (D) 100% (E) 124%

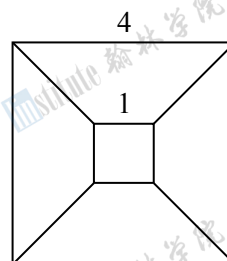
8. For what value of k is the line through the points $(3, 2k + 1)$ and $(8, 4k - 5)$ parallel to the x -axis?

(A) -1 (B) 3 (C) 2 (D) 0 (E) -4

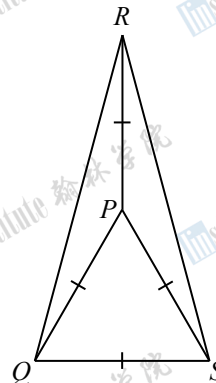
9. The three numbers $5, a, b$ have an average (mean) of 33. What is the average of a and b ?
- (A) 38 (B) 14 (C) 28 (D) 33 (E) 47
10. Glenda, Helga, Ioana, Julia, Karl, and Liu participated in the 2017 Canadian Team Mathematics Contest. On their team uniforms, each had a different number chosen from the list 11, 12, 13, 14, 15, 16. Helga's and Julia's numbers were even. Karl's and Liu's numbers were prime numbers. Glenda's number was a perfect square. What was Ioana's number?
- (A) 11 (B) 13 (C) 14 (D) 15 (E) 12

Part B: Each correct answer is worth 6.

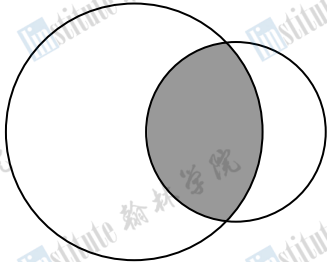
11. A large square has side length 4. It is divided into four identical trapezoids and a small square, as shown. The small square has side length 1. What is the area of each trapezoid?



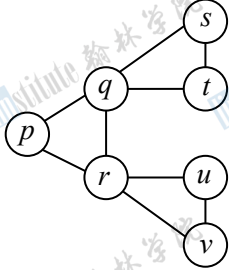
- (A) $\frac{2}{3}$ (B) 3 (C) $\frac{9}{2}$
 (D) $\frac{15}{4}$ (E) 15
12. In an unusual country, there are three kinds of coins: Exes, Wyes and Zeds. In this country, the value of 2 Exes equals the value of 29 Wyes, and the value of 1 Zed equals the value of 16 Exes. The value of 1 Zed equals the value of how many Wyes?
- (A) 3.625 (B) 1.103 (C) 232 (D) 464 (E) 928
13. The number of integer values of x for which $\frac{3}{x+1}$ is an integer is
- (A) 4 (B) 3 (C) 5 (D) 1 (E) 6
14. Including the endpoints, how many points on the line segment joining $(-9, -2)$ and $(6, 8)$ have coordinates that are both integers?
- (A) 2 (B) 7 (C) 16 (D) 11 (E) 6
15. In the diagram, $\triangle PQS$ is equilateral. Also, $\triangle PQR$ and $\triangle PSR$ are isosceles with $PQ = PR = PS$. If $\angle RPQ = \angle RPS$, the measure of $\angle QRS$ is
- (A) 30° (B) 60° (C) 15°
 (D) 20° (E) 45°



16. A ladder has 5 rungs. Elisabeth can climb up by 1 or 2 rungs at a time. In how many different ways can she climb up to the fifth rung of the ladder?
- (A) 10 (B) 9 (C) 7 (D) 6 (E) 8

17. If $\frac{x-y}{x+y} = 5$, then $\frac{2x+3y}{3x-2y}$ equals
 (A) 1 (B) 0 (C) $\frac{2}{3}$ (D) $\frac{15}{2}$ (E) $\frac{12}{5}$
18. A quadrilateral is bounded by the lines with equations $x = 0$, $x = 4$, $y = x - 2$, and $y = x + 3$. The area of this quadrilateral is
 (A) 16 (B) 24 (C) 4 (D) $20\sqrt{2}$ (E) 20
19. In the diagram, two circles overlap. The area of the overlapped region is $\frac{3}{5}$ of the area of the small circle and $\frac{6}{25}$ of the area of the large circle. The ratio of the area of the small circle to the area of the large circle is
 (A) 18 : 125 (B) 1 : 3 (C) 5 : 12
 (D) 2 : 5 (E) 1 : 4
- 
20. Abigail chooses an integer at random from the set $\{2, 4, 6, 8, 10\}$. Bill chooses an integer at random from the set $\{2, 4, 6, 8, 10\}$. Charlie chooses an integer at random from the set $\{2, 4, 6, 8, 10\}$. What is the probability that the product of their three integers is *not* a power of 2?
 (A) $\frac{117}{125}$ (B) $\frac{2}{5}$ (C) $\frac{98}{125}$ (D) $\frac{3}{5}$ (E) $\frac{64}{125}$

Part C: Each correct answer is worth 8.

21. In the diagram, each of p, q, r, s, t, u, v is to be replaced with 1, 2 or 3 so that p, q and r are all different, q, s and t are all different, and r, u and v are all different. What is the maximum possible value of $s + t + u + v$?
 (A) 8 (B) 9 (C) 11
 (D) 7 (E) 10
- 
22. If n is a positive integer, the symbol $n!$ (read “ n factorial”) represents the product of the integers from 1 to n . For example, $4! = (1)(2)(3)(4)$ or $4! = 24$. If x and y are integers and $\frac{30!}{36^x 25^y}$ is equal to an integer, what is the maximum possible value of $x + y$?
 (A) 10 (B) 47 (C) 17 (D) 26 (E) 13
23. A container in the shape of a triangular prism stands on one of its triangular faces. Three spheres of radius 1 are placed inside the container, each touching the triangular bottom. Each sphere touches two of the rectangular faces of the container and each sphere touches the other two spheres. A fourth sphere of radius 1 is placed on top of the three spheres, touching each of the three spheres and the top of the prism. The volume of the prism is closest to
 (A) 48.00 (B) 47.75 (C) 47.50 (D) 47.25 (E) 47.00

24. There are more than 1 000 000 ways in which n identical black socks and $2n$ identical gold socks can be arranged in a row so that there are at least 2 gold socks between any 2 black socks. The sum of the digits of the smallest possible value of n is

- (A) 9 (B) 10 (C) 11 (D) 12 (E) 13

25. There are N sequences with 15 terms and the following properties:

- each term is an integer,
- at least one term is between -16 and 16 , inclusive,
- the 15 terms have at most two different values,
- the sum of every six consecutive terms is positive, and
- the sum of every eleven consecutive terms is negative.

The value of N is

- (A) 48 (B) 72 (C) 64 (D) 80 (E) 56



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For students...

Thank you for writing the 2018 Cayley Contest! Each year, more than 240 000 students from more than 75 countries register to write the CEMC's Contests.

Encourage your teacher to register you for the Galois Contest which will be written in April.

Visit our website cemc.uwaterloo.ca to find

- More information about the Galois Contest
- Free copies of past contests
- Math Circles videos and handouts that will help you learn more mathematics and prepare for future contests
- Information about careers in and applications of mathematics and computer science

For teachers...

Visit our website cemc.uwaterloo.ca to

- Register your students for the Fryer, Galois and Hypatia Contests which will be written in April
- Look at our free online courseware for senior high school students
- Learn about our face-to-face workshops and our web resources
- Subscribe to our free Problem of the Week
- Investigate our online Master of Mathematics for Teachers
- Find your school's contest results